FROM THE MAKERS OF THE ACT®

THE OFFICIAL ACT®

MATHEMATICS GUIDE

A Step-by-Step Guide Outlining the Preparation for the ACT®

FEATURES

- Review the entire ACT® mathematics test so you'll know what to expect
- Familiarize yourself with the types of math questions found on the ACT®
- Understand the math topics within the problems you'll solve while taking the mathematics test
- Detailed solutions and explanations for every official ACT® math question in the book

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THE
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ACT

MATHEMATICS GUIDE

ACT

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Introduction

So you want to do well on the ACT mathematics test. That's a good goal to have! Whether you've already taken the test once or are planning to do so in the near or even distant future, this book will help you achieve your goal. There are three major factors that will determine how well you do on the ACT math test:

- Focus
- Effort
- Guidance

You need to be focused and diligent in your studies and preparation for the ACT mathematics test. There is no getting around that. You need to put the time and effort into fully practicing the skills the test will be looking for. If you are reading this book, it can be assumed that you are focused on achieving your goal of success and that you are willing to put the necessary time and effort into making it happen.

The third factor, guidance, is where this book comes into play. Guidance is difficult to supply to yourself. This book covers just about every math topic that you're likely to see on the ACT mathemtics test, so by going through this book, you'll know what to expect on test day.

The ACT math test can be daunting because of the sheer number of math topics. Fortunately, the test is based on topics covered in high school, so nothing here is new. Even in high school math, the ACT mathematics test only asks questions about certain topics, and only these topics are in this book. There are topics that you may see on the test, such as exponents, and topics that you won't see, such as Venn diagrams. If you see an unfamiliar topic in this book, you'll probably see that topic again, in class, before the year is over.

The plan is simple. Start with chapter 1 and become familiar with the reporting categories on the test, which are the foundation for your math score. Continue with chapters 2 through 6 and refresh your math

progressively while working the practice ACT questions that accompany most topics. Be sure to note anything you struggle with to review later. Finally, work through chapter 7, which includes the practice ACT questions from the text along with additional questions directly from the ACT. Review chapter 8 for a detailed and thorough breakdown of the solution to each question.

If you struggle with certain math topics, you're not alone, but now you have an edge: you know the scope of the ACT math test and which topics you need to review. Most students who score well on the ACT had to practice first, and this is something that you can do. Almost no one gets a perfect score, but with the guidance in this book, you can score well enough to be a competitive applicant to a good university.

You definitely want to pick up the other ACT subject guides, and you'll eventually want to take at least one full-length practice exam from *The Official ACT Prep Guide*, but this is a good start for the math. You can and will succeed with your goal of doing well on the ACT math test. You provide the focus and the effort, and this book will provide the guidance. So read on, and let's get started.

Chapter 1:
The ACT Math Test

The ACT mathematics test is a 60-question, 60-minute test designed to assess the mathematical reasoning skills that you've acquired in courses taken up to the beginning of grade 12. Most questions are self-contained, but some may belong to a set of several questions (e.g., about the same graph or chart).

The questions cover a wide variety of concepts, techniques, and procedures that emphasize the major content areas requisite to successful performance in entry-level courses in college mathematics. Some questions require computation, but the questions are designed to emphasize your ability to reason mathematically, not your ability to compute numbers or recall complex formulas.

You may use a calculator. See www.act.org/calculator-policy.html for details about models and features that are permitted or prohibited.

Reporting Categories: Score Reporting

Nine scores are reported for the mathematics test: a total test score based on all 60 questions and eight reporting category scores based on specific mathematical knowledge and skills. The approximate number of questions and percentage of the test devoted to each reporting category is shown in Table 1.1.

Table 1.1: Number of Questions per Reporting Category

Reporting Category/Reported Score	Number of Questions	Percentage of Test
Integrating Essential Skills	24–26	40–43
Preparing for Higher Mathematics	34–36	57–60

Reporting Category/Reported Score	Number of Questions	Percentage of Test
Number and Quantity	4–6	7–10
Algebra	7–9	12–15
Functions	7–9	12–15
Geometry	7–9	12–15
Statistics and Probability	5–7	8–12
Modeling	≥ 16	≥ 27
Total Math Test Score	**60**	**100**

Reporting Categories: Topic Overview

Integrating Essential Skills measures how well you can synthesize and apply your understanding and skills from previously learned concepts to solve more complex problems. These questions ask you to address topics such as rates and percentages; proportional relationships; area, surface area, and volume; average and median; and the expression of numbers in different ways. You will be asked to solve nonroutine problems that involve combining skills in longer chains of steps in more varied contexts, while understanding conceptual connections and demonstrating fluency of skills.

Preparing for Higher Mathematics covers mathematics that you recently learned, starting with the use of algebra as a general way of expressing and solving equations. This category has its own reported score and is divided into five discrete categories, each with its own reported score:

- **Number and Quantity** asks about real and complex number systems. You will be asked to apply your understanding and fluency with rational numbers and the four basic operations (addition, subtraction, multiplication, and division) to irrational numbers by manipulating rational numbers and delving deeper into properties of the real number system. You will be asked to apply your knowledge of integer exponents to rational exponents. Questions also involve vectors and

matrices, which can be treated as number systems with properties, operations, and applications.

- **Algebra** asks you to solve, graph, and model different types of expressions. You will be asked to interpret and use various equations based on linear, polynomial, radical, and exponential relationships in the context of equations and inequalities. Questions will ask you to apply your understanding of expressions to strategically solve problems, and you will be asked to apply polynomial relationships in applications to create expressions, equations, and inequalities that represent problems and constraints in real-world contexts.
- **Functions** questions are based on the definition, notation, representation, and application of functions in linear, radical, piecewise, polynomial, and logarithmic forms. These questions provide a framework for modeling real-world phenomena, and you will be asked to interpret the characteristics of a function in the context of a problem while recognizing the difference between a model and reality. You will also be asked to manipulate and translate functions as well as interpret and use key features of graphs.
- **Geometry** asks you to apply your knowledge of shapes and solids to spatial concepts that include congruence and similarity relationships and surface area and volume measurements. You will be asked to apply your understanding of geometric objects to model and solve problems as well as find missing values in triangles, circles, and other figures. You will also be asked to apply trigonometric ratios as functions of right triangles and apply these concepts to the coordinate plane. Questions may also ask about trigonometric concepts of non-right triangles based on the law of sines and the law of cosines.
- **Statistics and Probability** is based on the distribution of data. You will be asked about data collection methods and relationship models in bivariate data. You will also be asked to calculate probabilities by recognizing the related sample spaces.

Modeling represents all questions that involve producing, interpreting, understanding, evaluating, and improving models, which are representations of complex mathematical concepts. This category is an overall measure of how well you use modeling skills across mathematical topics. Each Modeling question is also counted in other appropriate

reporting categories; thus, the Modeling category is an overall measure of how well you use modeling skills across mathematical topics.

Tips for Taking the Mathematics Test

If you use a calculator, use it wisely.

All of the mathematics problems can be solved without a calculator. Many of the problems are best done without a calculator. Use good judgment in deciding when, and when not, to use a calculator. For example, for some problems you may wish to do scratch work to clarify your thoughts on the question before you begin using a calculator to do computations.

Solve the problem.

To work out solutions to the problems, you will usually do scratch work in the space provided. You may wish to glance over the answer choices after reading the questions. However, working backwards from all five answer choices can take a lot of time and may not be effective.

Find your solution among the answer choices.

Once you have solved the problem, look for your answer among the choices. If your answer is not included among the choices, carefully reread the problem to see whether you missed important information. Pay careful attention to the question being asked. If an equation is to be selected, check to see whether the equation you think is best can be transformed into one of the answer choices provided.

Make sure you answer the question.

The solutions to many questions on the test will involve several steps. Make sure your answer accounts for all the necessary steps. Frequently, an answer choice is an intermediate result, not the final answer.

Make sure your answer is reasonable.

Sometimes an error in computation will result in an answer that is not practically possible for the situation described. Always think about your

answer to determine whether it is reasonable.

Check your answer.

You may arrive at an incorrect solution by making common errors in the problem-solving process. If there is time remaining before the end of the mathematics test, it is important that you reread the questions and check your answers to make sure they are correct.

Chapter 2:
Number and Quantity

Number and Quantity tests your ability to apply real and complex number systems in various forms, including integer and rational exponents, vectors, and matrices.

Real and Complex Number Systems

A **real number** is any number that appears on the number line, whether positive or negative. This includes every fraction, decimal, whole number, and zero.

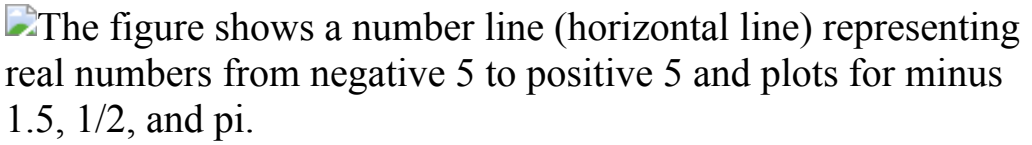
A **non-real number** is a number that can't exist in real math and therefore isn't on the number line. Two types of non-real number may appear on the ACT mathematic test: any number divided by zero and the square root of a negative number, also known as an *imaginary number* or *i*, covered later in this chapter.

A **rational number** is any number or fraction that can be expressed as a terminating or repeating decimal. For example, the fraction  inline can be expressed as the decimal 0.25, which both terminates and expresses the true value of the decimal. The fraction  inline can be expressed as the decimal  inline, which doesn't terminate but is considered rational because it repeats. The fraction  inline, equivalent to  inline, is also considered rational because the 18 in the decimal repeats.

A **non-rational** or **irrational number** is a real number that cannot be written as a fraction or a terminating or repeating decimal. Examples of non-rational numbers are  inline and  inline.

Number Line and Absolute Value

The **number line** represents the spectrum of all real numbers and symbolically extends infinitely in both directions.

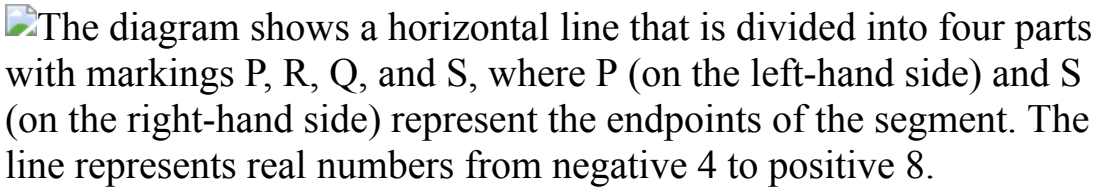

The figure shows a number line (horizontal line) representing real numbers from negative 5 to positive 5 and plots for minus 1.5, 1/2, and pi.

Absolute value represents an expression's distance from 0 on the number line. Absolute value is always positive, because a distance is always positive. Because –5 is 5 units from 0, the absolute value of –5, written as inline, is 5.

You can take the negative of an absolute value, but the absolute value itself is always positive. For example, inlineis the same thing as inline, which equals –7. The trick is to take the calculations step by step: inline, and the negative of that is –7.

Reporting Category Quiz: Preparing for Higher Mathematics | Number and Quantity

1. On the real number line, what is the midpoint of −5 and 17?
    1. −11
    2. 6
    3. 11
    4. 12
    5. 22
2. On the real number line below, with coordinates as labeled, an object moves according to the following set of instructions: from point *P* the object moves right to *Q*, then left to *R*, then right to *S*, and finally left until it returns to its original position at *P*. What is the closest estimate of the total length, in coordinate units, of the movements this object makes?

    
    The diagram shows a horizontal line that is divided into four parts with markings P, R, Q, and S, where P (on the left-hand side) and S (on the right-hand side) represent the endpoints of the segment. The line represents real numbers from negative 4 to positive 8.

1. 0
2. 4
3. 12
4. 22
5. 36

3. If the inequality inline is true, then which of the following must be true?
    1. inline
    2. inline
    3. inline
    4. inline
    5. inline

Whole Numbers

An **integer** is any number, positive, negative, or zero, that can be written without a fractional or decimal component. A **whole number** is any positive integer and doesn't include zero.

Multiples, Factors, and Prime Numbers

A **multiple** is a whole number that results from the product of two other whole numbers. For example, to find multiples of 7, multiply 7 by 1, 2, 3, and so on, resulting in 7, 14, 21, and so on. Note that every whole number is a multiple of itself: 23 is a multiple of 23.

A **factor** is a whole number that results from dividing two other whole numbers. For example, to find the factors of 30, find the pairs of numbers that multiply to 30: 5 and 6, 3 and 10, 2 and 15, and 1 and 30. Note that 1 is a factor of every whole number, and every whole number is a factor of itself. For example, factors of 52 include 1 and 52.

A **prime number** is a whole number that has exactly two factors: 1 and itself. For example, 13 is a prime number, because its only factors are 1 and 13. Note that 2 is the only even prime number, and 1 and 0 are not considered prime.

A **composite number** is a whole number that isn't prime; that is, it has more than two factors. For example, 12 is a composite number, because its factors are 1, 2, 3, 4, 6, and 12.

Prime factorization is the factoring of a composite number to its primes, including duplicates. For example, the prime factorization of 30 is inline. Note that prime factorization doesn't include 1 as a factor.

These composite numbers can be prime factored:

1. 20
2. 36
3. 48

Results:

1. inline
2. inline
3. inline

Reporting Category Quiz: Integrating Essential Skills

4. Mr. Dietz is a teacher whose salary is $22,570 for this school year, which has 185 days. In Mr. Dietz's school district, substitute teachers are paid $80 per day. If Mr. Dietz takes a day off without pay and a substitute teacher is paid to teach Mr. Dietz's classes, how much less does the school district pay in salary by paying a substitute teacher instead of paying Mr. Dietz for that day?
    1. $42
    2. $80
    3. $97
    4. $105
    5. $122
5. The following chart shows the current enrollment in all the mathematics classes offered by Eastside High School.


The figure shows a four-column table illustrating the current enrollment in all the mathematics classes offered by Eastside High School.

The school owns 2 classroom sets of 30 calculators each, which students are required to have during their mathematics class. There are 2 calculators from one set and 6 calculators from the other set that are not available for use by the students because these calculators are being repaired.

For which of the following class periods, if any, are there NOT enough calculators available for each student to use a school-owned calculator without having to share?

1. Period 2 only
2. Period 3 only
3. Period 4 only
4. Periods 3 and 4 only
5. There are enough calculators for each class period.

6. Nick needs to order 500 pens from his supplier. The catalog shows that these pens come in cases of 24 boxes with 10 pens in each box. Nick knows that he may NOT order partial cases. What is the fewest number of cases he should order?
   1. 2
   2. 3
   3. 18
   4. 21
   5. 50

Reporting Category Quiz: Preparing for Higher Mathematics | Number and Quantity

7. Kareem has 4 sweaters, 6 shirts, and 3 pairs of slacks. How many distinct outfits, each consisting of a sweater, a shirt, and a pair of slacks, can Kareem select?
   1. 13

2. 36
3. 42
4. 72
5. 216

8. How many prime numbers are there between 30 and 50?
    1. 4
    2. 5
    3. 6
    4. 7
    5. 8

Decimals and Percents

A **decimal** represents a fraction where the denominator is a multiple of 10. For example, 0.3 is equivalent to inline, and 2.19 is equivalent to inline or inline.

A **percent** represents the first two decimal places of a decimal, or a fraction over 100. For example, 21% is equivalent to 0.21 or inline. To convert a decimal amount to a percent, multiply the decimal amount by 100. For example, inline. To convert a fraction to a percent, convert the fraction so that it has a denominator of 100, and use the numerator. For example, inline.

Percent of Change

To find the **percent of change**, place the quantity of change over the starting quantity, and convert this to a fraction with a denominator of 100.

For example, to find the percent of change of the price of a calculator that increased from $20 to $25, place the quantity of change, 5, over the starting quantity for inline, and convert this to a fraction with a denominator of 100: inline, for 25%.

If that calculator goes on sale from $25 to $20, find the new percent of change: place the quantity of change, 5, over the starting quantity for

inline, and convert this to a fraction with a denominator of 100: inline, for 20%.

Note that the same $5 price change yields a different percent of change when starting from different amounts. Also, a percent of change is always positive: the price didn't change negative 20%; it went down (positive) 20%.

These are examples of numbers with percents of change:

1. From 5 to 3
2. From 50 to 66
3. From 20 to 70

Results:

1. 40%.

   The quantity of change is 2 and the starting quantity is 5:inline.

2. 32%.

   The quantity of change is 16 and the starting quantity is 50:inline.

3. 250%.

   The quantity of change is 50 and the starting quantity is 20:inline.

Percent of a Group

To find the **percent of a group,** place the number of the subgroup over the entire group and convert the fraction. For example, if 7 of the 20 job applicants have college degrees, find the percent who have college degrees:

1. The number of the subgroup: 7
2. Over the entire group:inline
3. Converted fraction:inline

7 out of 20 is 35%.

These are examples of percents of groups:

1. 6 out of 25 dogs are Labs.
2. 170 out of 200 flights are on time.
3. 28 out of 40 cars are electric.

Results:

1. 24%.

   The number of the subgroup is 6 and the entire group is 25:  .

2. 85%.

   The number of the subgroup is 170 and the entire group is 200:  .

3. 70%.

   The number of the subgroup is 28 and the entire group is 40:  .

Reporting Category Quiz: Integrating Essential Skills

9. The oxygen saturation level of a river is found by dividing the amount of dissolved oxygen the river water currently has per liter by the dissolved oxygen capacity per liter of the water and then converting to a percent. If the river currently has 7.3 milligrams of dissolved oxygen per liter of water and the dissolved oxygen capacity is 9.8 milligrams per liter, what is the oxygen saturation level, to the nearest percent?
   1. 34%
   2. 70%
   3. 73%
   4. 74%
   5. 98%

Fractions and Ratios

A **fraction** is a numerical quantity that is not a whole number, such as inline. The **reciprocal** of a fraction is the switching of its numerator and denominator. For example, the reciprocal of inline is inline. Placing a 1 on top of a fraction yields its reciprocal. For example, inline.

- Add and subtract fractions by giving them common denominators: inline.
- Multiply fractions by multiplying the numerators then the denominators: inline.
- Divide fractions by multiplying the first fraction by the reciprocal of the second fraction: inline.

If an ACT mathematics test question involves multiplying fractions that have large numerators or denominators, you can often simplify the math work by reducing and cancelling the numbers before multiplying. In this example, the numerator 500 is a multiple of the denominator 250, so reducing these numbers saves math work:inline.

A **ratio** compares the quantities of two groups as a reduced fraction. For example, if the sports center has 24 baseballs and 16 footballs, then the fraction of quantitiesinlineis reduced to a ratio of inline.

Find the actual quantities from the ratio by multiplying each ratio number by *x*, adding them together, and setting them equal to the total. For example, if the ratio of gloves to bats is 2:3 and there are 30 goods total, find the number of each good:

image

Now substitute 6 for *x* in the original equation:

image

And there are 12 gloves and 18 bats.

These are examples of ratios from quantities:

1. The ratio of classical to electric guitars is 3:4, and there are 35 guitars total. The ratio and quantity can be used to determine the numbers of classical and electric guitars.
2. The ratio of mountain bikes to road bikes is 5:3, and there are 20 mountain bikes. The ratio and number of mountain bikes can be used to determine the number of road bikes.
3. There are 15 cars and 35 trucks. The quantities of cars and trucks can be used to determine the ratio.

Results:

1. 15 and 20.

   Multiply 3 and 4 each by *x*, add them together, and set that equal to the total. After finding that inline multiply that by 3 and 4 from the ratio to determine that there are 15 classical and 20 electric guitars:

   image

2. 12.

   Multiply 5 and 3 each by *x*, add them together, and set that equal to the total. After finding that inline, multiply that by 3 to determine that there are 12 road bikes:

   image

3. 3:7.

   Set the proportion up as a fraction and reduce it to find the ratio: inline

Reporting Category Quiz: Integrating Essential Skills

10. If 12 vases cost $18.00, what is the cost of 1 vase?
    1. $0.67
    2. $1.05
    3. $1.33
    4. $1.50

5. $1.60

11. Company A sells 60 pens for $15.00. Company B sells the same type of pens in packs of 40 for $8.00. Which company's price per pen is cheaper, and what is that price?
    1. Company A, at $0.20
    2. Company A, at $0.23
    3. Company A, at $0.25
    4. Company B, at $0.20
    5. Company B, at $0.25
12. At a refinery, 100,000 tons of sand are required to produce each 60,000 barrels of a tarry material. How many tons of sand are required to produce 3,000 barrels of this tarry material?
    1. 5,000
    2. 18,000
    3. 20,000
    4. 40,000
    5. 50,000

Reporting Category Quiz: Integrating Essential Skills | Modeling

13. Of the 804 graduating seniors in a certain high school, approximately  inline are going to college and approximately  inlineof those going to college are going to a state university. Which of the following is the closest estimate for how many of the graduating seniors are going to a state university?
    1. F. 80
    2. G. 90
    3. H. 160
    4. J. 200
    5. K. 320

Reporting Category Quiz: Preparing for Higher Mathematics | Number and Quantity

14. What is the least common denominator when adding the fractions inline, inline, inline, and inline?
    1. 45
    2. 90
    3. 135
    4. 270
    5. 810
15. A ramp for wheelchair access to the gym has a slope of 5% (that is, the ramp rises 5 feet vertically for every 100 feet of horizontal distance). The entire ramp is built on level ground, and the entrance to the gym is 2 feet above the ground. What is the *horizontal* distance, in feet, between the ends of the ramp?
    1. 4
    2. 10
    3. 40
    4. 100
    5. 400

Exponents

An **exponent** refers to a base number multiplied by itself a certain number of times. For example, inline refers to the base number 5 multiplied by itself 3 times:inline. A *negative exponent*, such as inline, is the reciprocal of a positive exponent: inline.

Multiplication and Division

Exponential expressions that have the same base number can be multiplied and divided. To multiply, add the exponents: inline. To divide, subtract the exponents: inline.

These are examples of expressions that can be simplified:

1. inline
2. inline
3. inline

Results:

1. inline.

   When multiplying like terms, add the exponents: inline.

2. inline or inline.

   Add the exponents: inline or inline.

3. inline.

   When dividing like terms, subtract the exponents: inline.

Exponents of Exponents

Simplify an exponent of an exponent by multiplying the exponents:inline.

These are examples of expressions that can be simplified:

1. inline
2. inline
3. inline

Results:

1. inline.

   When taking the exponent of an exponent, multiply the exponents: inline.

2. inline or inline.

   Multiply the exponents, even when negative: inline or inline.

3. inline.

   Multiply the exponents, even as fractions: inline.

Variations on Exponents

These are variations of exponents that can be simplified:

1. inline
2. inline
3. inline
4. inline
5. inline
6. inline
7. inline
8. inline

Results:

1. 16.

   The exponent multiplies the entire expression containing the –4: inline.

2. –25.

   The exponent multiplies only the 5; the negative is outside this: inline.

3. inline.

   The negative exponent yields the reciprocal of the positive exponent: inline.

4. inline.

   An exponent of a fraction multiplies the entire fraction:inline.

5. 0.16.

   This is 0.4 multiplied by itself. Note that inline is equivalent to 0.4 and inline is equivalent to 0.16:inline.

6. inline.

   Don't just distribute the exponent for inline, which is a common mistake. Instead, multiply the expressions using the FOIL method, which is covered further in chapter 3, “Algebra”: inline.

7. 1.

   Any quantity raised to the 0 power equals 1: inline.

8. 1.

   inline.

These are additional variations that can be simplified:

1. inline
2. inline
3. inline
4. inline
5. inline
6. inline
7. inline
8. inline

Results:

1. 49.

   The exponent multiplies the entire expression containing the –7: inline.

2. –64.

   The exponent multiplies only the 8; the negative is outside this: inline.

3. inline.

The negative exponent yields the reciprocal of the positive exponent: inline.

4. inline.

   An exponent of a fraction multiplies the entire fraction: inline.

5. 0.09.

   This is 0.3 multiplied by itself:inline.

6. inline.

   Multiply the expressions using the FOIL method: inline.

7. 1.

   Any quantity raised to the 0 power equals 1: inline.

8. 1.

   inline.

Reporting Category Quiz: Preparing for Higher Mathematics | Number and Quantity

16. inline is equivalent to:

    F. inline

    G. inline

    H. inline

    J. inline

    K. inline

17. Which of the following expressions is equivalent to inline?

A. inline

B. inline

C. inline

D. inline

E. inline

18. Which real number satisfies inline?

F. 2

G. 3

H. 4

J. 4.5

K. 7

Imaginary and Complex Numbers

An **imaginary number** is represented by *i*, where inline or inline. Because a square root can only be of a positive number, the square root of a negative number isn't possible and is therefore an imaginary number. For example, inline is an imaginary number, but it can be simplified:

image

These are examples of imaginary numbers that can be written in terms of *i*:

1. inline
2. inline
3. inline

Results:

1. inline
2. inline
3. inline

To find the value of an imaginary number, use inline. To find the value of inline, use inline, and multiply the 3s separately from the *i*s:inline.

These are imaginary numbers that can be simplified:

1. inline
2. inline
3. inline
4. inline
5. inline

Results:

1. –25.

   Multiply the numbers and the *i*s separately: inline.

2. 16.

   Multiply the numbers and the *i*s separately. The 4 *i*s multiplied together become positive 1: inline.

3. –8*i*.

   Multiply them separately: inline.

4. –15.

   inline

5. 6.

   inline

A **complex number** includes the imaginary number *i* in the form inline, such as inline. You can represent the complex number as a graph on the number plane, where the horizontal axis, labeled *r*, represents the real component and the vertical axis, labeled *i*, represents the imaginary component. For example, to represent the complex number inline, plot the point 3 spaces right and 2 spaces up for the coordinates inline:

image

Distance and Midpoint

Find the **distance**, also called the **modulus**, between two graphed complex numbers with the *distance formula*, where *a* and *b* are the coordinates of one point and *s* and *t* are the coordinates of the other:

image

To find the distance between inline and inline, place 3 and 2 for *a* and *b*, respectively, and –5 and –4, as *s* and *t*, respectively, into the formula:

image

image

Find the **average** or **midpoint** in the complex plane between two graphed complex numbers with the *midpoint formula*:

image

To find the distance between inline and inline, place 3 and 2 for *a* and *b*, respectively, and –5 and –4, as *s* and *t*, respectively, into the formula:

image

image

Multiplication

Multiply or square complex numbers with the FOIL method as you would a quadratic expression:

image

Find the **conjugate** of a complex number by reversing the sign of the imaginary component. For example, the conjugate of inline is inline. When a complex number is multiplied by its conjugate, the imaginary component cancels and the real component remains. For example, multiply inline by its conjugate inline:

image

These are examples of complex numbers that can be simplified:

1. inline
2. inline
3. inline

Results:

1. inline.

   FOIL the expression and combine like terms:

   image

2.

   image

3. 25.

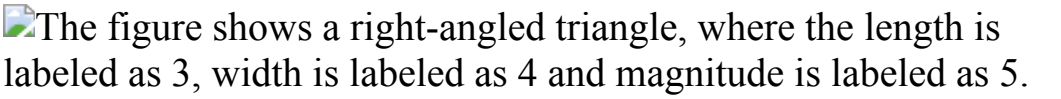
   image

Vectors

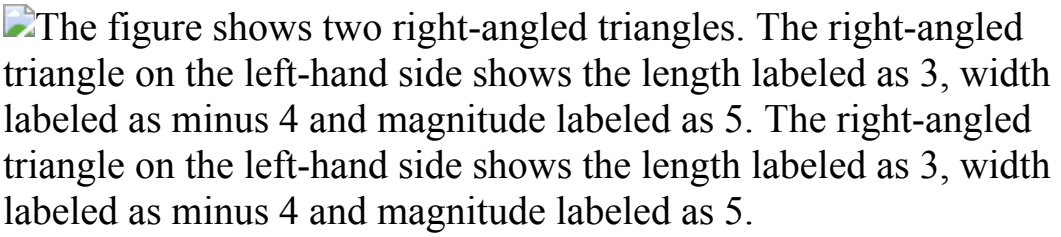
A **vector** is an object, represented by an arrow, that has both magnitude and direction. It's written in the *component form* as inline, where, from any starting point, *a* represents the number of units that it moves right and *b* represents the number of units that it moves up. In this way, the component form represents changes in its *x*- and *y*-values. For example, the vector inline moves 4 units right and 3 units up:

The distance between the starting point and ending point is the *magnitude*, which can be found using the Pythagorean theorem of its component form. Place the component numbers in the theorem as *a* and *b*, with *c* as the magnitude:

image



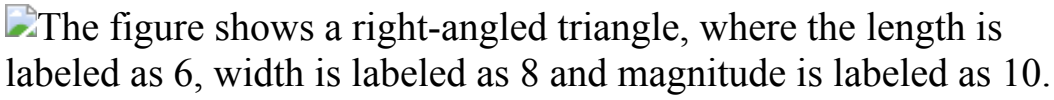
Think of the vector as a right triangle, with its component form as the sides and its magnitude as the hypotenuse. The component form can have negative values, meaning the vector moves left and/or down. However, the magnitude is always positive:



Multiplication

The vector can be multiplied by a number, known as a *scalar*. For example, to multiply the previous vector by a scalar of 2, write it as inline, in which case the vector moves 8 units right and 6 units up. Find the magnitude by placing these component numbers into the Pythagorean theorem:

image



These vectors can be multiplied:

1. Find 2**a** where vectorinline.
2. Find –3**b** where vectorinline.

Results:

1. inline.

   Multiply inline by 2: inline.

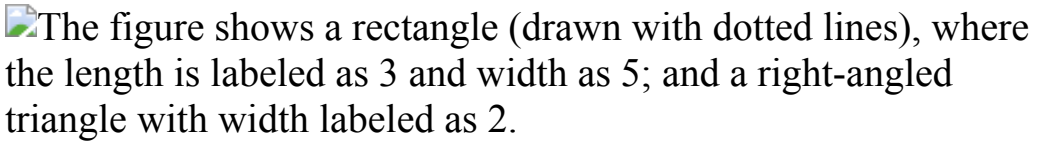
2. inline.

   Multiply inline by –3: inline.

Addition and Subtraction

To add or subtract vectors, use their component forms to separately add or subtract the *x*-values and *y*-values. For example, to add the vectors inline andinline, add the 2 and the 5 for the shift in *x*-value and the 3 and the 1 for the shift in *y*-value, like this:inline.

In other words, if vector **a** moves right 2 and up 3, and vector **b** moves right 5 and up 1, andinline, then vector **c** moves right inline and up inline:



Note that the second vector begins where the first one ends.

These are examples of vectors that can be multiplied and added:

1. Find vector **c** where vectors inline, inline, and inline.

2. Find vector **c** where vectors inline, inline, and inline.

Results:

1. inline.

   inline

2. inline.

   image

Note that the resulting magnitude isn't the sum of the two starting magnitudes. It's calculated from the component form of the new vector with the Pythagorean theorem.

Reporting Category Quiz: Preparing for Higher Mathematics | Number and Quantity

19. The component forms of vectors **u** and **v** are given by inline and inline. Given that inline, what is the component form of **w** ?
    - **F.** inline
    - **G.** inline
    - **H.** inline
    - **J.** inline
    - **K.** inline

Matrices

A **matrix** is a rectangular array of letters or numbers that represents data. The matrix facilitates data manipulation, making it ideal for use in complex applications such as statistics or computer operations.

For example, inlineis a inline matrix, because it has two rows and three columns.

Addition and Subtraction

Matrices can only be added or subtracted when they have the same dimensions. For example, a inline matrix can only be added or subtracted to another inline matrix. To do this, add (or subtract) the quantities in each corresponding position. For example, to add inline and inline, add the values in each position (top left, top right, etc.) for the result of inline.

Questions: Add or subtract these matrices:

1. inline
2. inline

Answers:

1. inline
2. inline

Multiplication

A matrix can be multiplied by a single quantity, also known as a *scalar*. To do this, distribute the scalar among the values in the matrix, and the result is a *scalar multiple*. For example, to multiply the matrix inline by the scalar 3, written asinline, multiply each value in the matrix by 3 for the scalar multiple: inline.

A matrix can be multiplied by another matrix when the number of columns in the first matrix matches the number of rows in the second matrix. For example, inline has two columns and inline has two rows, so these can be multiplied.

The resulting matrix has the number of rows from the first matrix and the number of columns from the second, so the order matters when multiplying matrices:inline, with 3 rows in the first matrix and 3 columns in the second, produces a inline answer. Now switch the order:inline, having 2 rows in the first matrix and 2 columns in the second, produces a inline answer.

To multiply inline, draw a blank inline matrix: inline. Start with the first row of the answer: Multiply 1 from the first matrix by *x* from the

second matrix, and add this to the product of 2 from the first matrix and 2x from the second matrix. Place the resulting 5x in the top left position: inline. Multiply 1 from the first matrix by y in the second matrix, and add this to the product of 2 from the first matrix and 2y in the second matrix. Place the resulting 5y in the top middle position: inline. The 1 and 2 from the first matrix are similarly multiplied by z and 2z from the second matrix, and that sum is placed in the top right position: inline.

Now for the second row of the answer: add inline to inline for 11x, inline and inline: inline. The third row is similar: add inline to inline for 17x, inline and inline: inline.

These are examples of matrices that can be multiplied:

1. inline
2. inline
3. inline

Results:

1. inline.

   Distribute the 2 by each quantity in the matrix.

2. inline.

   Three rows in the first matrix by three columns in the second produce a inline answer: inline.

3. inline.

   Two rows in the first matrix by two columns in the second produce a inline answer: inline.

Reporting Category Quiz: Preparing for Higher Mathematics | Number and Quantity

20. The number of students participating in fall sports at a certain high school is shown by the following matrix.

image

The athletic director estimates the ratio of the number of sports awards that will be earned to the number of students participating with the following matrix.

image

Given these matrices, what is the athletic director's estimate for the number of sports awards that will be earned for these fall sports?

1. 80
2. 88
3. 91
4. 92
5. 99

Determinant

The **determinant** that appears in the ACT mathematics test is based on a inline matrix, where the determinant of inline is inline. If the matrix has numbers, find the determinant by placing the numbers into the equation. For example, the determinant of inline is inline.

Here are examples:

1. The determinant of inline can be found.
2. The value of *k* can be found if the determinant of inline is 1.

Answers:

1. –5.

   To find the determinant of inline, use inline.

2. 2.

   If the determinant of inline is 1, then inline. Solve for *k*:

image

Reporting Category Quiz: Preparing for Higher Mathematics | Number and Quantity

21. By definition, the determinant inline. What is the value of inline when inline and inline?

    F. –138

    G. –42

    H. 12

    J. 42

    K. 138

Chapter 2: Quiz Answers

1. **The correct answer is B.** Draw a picture:

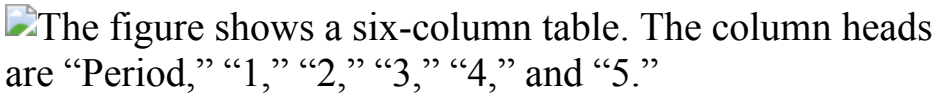
   The figure shows a number line (horizontal line) representing three real numbers: minus 5, 0 and 17.

   On the number line, −5 and 17 are 22 units apart. Half this distance is 11, so either go right 11 units from −5 or left 11 units from 17 to land at 6.

2. **The correct answer is E.** Just count the spaces. From *P* to *Q* is 9, *Q* to *R* is 6, *R* to *S* is 9, and *S* back to *P* is 12. The total is inline.
3. **The correct answer is B.** Pick a number for *a* and *b*, such as 3. If inline and inline, then inline. Pick any number at all for *a* and *b*, and you'll see this is always true. As long as inline doesn't equal inline, then *a* can't equal *b*.
4. **The correct answer is A.** Find Mr. Dietz's daily pay by dividing his salary by the number of school days: inline. If for one day, the school district pays the substitute teacher $80 instead of Mr. Dietz's

regular $122, then the school district saves the difference in these amounts: inline.

5. **The correct answer is C.** The school owns inline calculators, of which 8 are being repaired, leaving 52 available. Add up the calculators needed for each period:

   The 52 available calculators will suffice for each period except for Period 4.

6. **The correct answer is B.** Each case has inline pens. An order of 2 cases would have 480 pens, which isn't enough: Nick needs to order 3 cases to get 300 pens.
7. **The correct answer is J.** Multiply the numbers of the 3 different clothing pieces for inline distinct outfits. For each sweater, there are 6 shirts, and for each shirt, there are 3 pairs of slacks.
8. **The correct answer is G.** This one is solved by elimination. Any even number greater than 2 isn't prime, so write out the remaining odd numbers: inline.Divide these by 3, 5, and 7 and cross off anything that divides evenly. You don't have to try dividing them by anything greater than 7, because 7 times anything greater than 7 is greater than 49 and not in this list. The remaining numbers are prime: 31, 37, 41, 43, and 47, for a count of 5.
9. **The correct answer is D.** To find the oxygen saturation loss, divide the current number of milligrams of dissolved oxygen per liter of water by the dissolved oxygen capacity in milligrams per liter of water: inline. Note the key words *nearest percent*. True, this could be close to 75%, but that's not an answer choice, and it wouldn't be 73%.
10. **The correct answer is J.** To find the cost of 1 vase, divide the cost of $18 by the 12 vases: inline.
11. **The correct answer is D.** Find the price per pen for each company. For Company A, divide the pack cost of $15 by the 60 pens: inline. For Company B, divide the pack cost of $8 by the 40 pens: inline.

12. **The correct answer is A.** Set this one up as a ratio: 100,000 tons of sand per 60,000 barrels is the same as *x* tons of sand per 3,000 barrels. Reduce the fractions and cross multiply:

    image

13. **The correct answer is F.** Multiply the number of seniors graduating by the fraction going to college: inline. Next, multiply that number by the fraction going to a state university: inline. Note the key words in this question, *closest estimate*.
14. **The correct answer is B.** The least common denominator is the smallest common multiple of 2, 3, 9, and 15. Don't worry about the 3, because any number that's a multiple of 9 is also a multiple of 3.

    Factor the 2, 9, and 15 to their primes: 2, inline, and inline. Next, write down the fewest primes needed to multiply out any of these. inline has the primes to multiply out any one of 2, 9, and 15. Multiply these together for the lowest common multiple: inline.

15. **The correct answer is C.** Set this one up as a ratio: 100 feet of distance per 5 feet of rise is the same as *x* feet of distance per 2 feet of rise. Reduce the fractions and cross multiply:

    image

16. **The correct answer is H.** To find an equivalent expression, multiply the constants inline and combine the *x* terms inline and *y* terms inline.
17. **The correct answer is E.** Each value inside the parentheses is to the 4th power: inline.
18. **The correct answer is K.** Each base (2, 4, and 8) is a power of 2, so rewrite the equation with the 4 as inline and the 8 as inline, and solve for *x*:

    image

    You could also solve this one by working the straight math:

image

19. **The correct answer is G.** Start with 2**u** and –3**v**: inline and inline, respectively. Thus inline. Place this in the equation and solve for **w**:

    image

20. **The correct answer is D.** To find the number of sports awards earned, multiply the number of participants in each sport by the ratio for that sport and add the 4 products. This is a matrix multiplication:

    image

21. **The correct answer is J.** Place –3 and 2 for *x* and *y*, respectively, and solve with the arrangement inline:

    image

Chapter 3:
Algebra

Algebra expands on Number and Quantity by asking you to solve, graph, and model expressions with equations that reflect linear, polynomial, radical, and exponential relationships. It also asks you to find solutions to systems of equations.

Scientific Notation

Scientific notation is used for very small decimals or very large numbers. It's a number, usually between 1 and 10, with a single decimal place, multiplied by a power of 10.

For example, the sun is approximately 93,000,000 miles from the earth. In scientific notation, this is inline which is equivalent to inline.

The width of a carbon fiber strand is approximately 0.0003 inches, which in scientific notation is inline. This is equivalent to inline.

A shortcut method for converting scientific notation to standard notation is to move the number's decimal point by the power of the 10. In the sun example, where the distance is inline, move the 9.3 decimal point to the right 7 spaces for the numeric value of 93,000,000. In the carbon fiber example, where the width is inline, move the 3.0 decimal point to the left 4 spaces for the numeric value of 0.0003.

These are examples of numbers that can be converted from scientific notation to standard notation:

1. inline
2. inline
3. inline

Results:

1. 23,000.

   inline. Multiplying 2.3 by 10,000 is the equivalent of moving the decimal 4 spaces to the right.

2. 440,000.

   inline. Multiplying 4.4 by 100,000 is the equivalent of moving the decimal 5 spaces to the right.

3. 0.0032.

   inline. Multiplying 3.2 by 0.001 is the equivalent of moving the decimal 3 spaces to the left.

These are examples of numbers that can be converted from standard notation to scientific notation:

1. 160,000
2. 5,400,000
3. 0.00087

Results:

1. inline.

   inline. Multiplying 1.6 by 100,000 is the equivalent of moving the decimal 5 spaces to the right.

2. inline.

   inline. Multiplying 5.4 by 1,000,000 is the equivalent of moving the decimal 6 spaces to the right.

3. inline.

   inline. Multiplying 8.7 by 0.0001 is the equivalent of moving the decimal 4 spaces to the left.

Reporting Category Quiz: Preparing for Higher Mathematics | Algebra

1. The normal amount of lead in a certain water supply is inline milligrams per liter. Today, when the water was tested, the lead level found was exactly 100 times as great as the normal level, still well below the Environmental Protection Agency's action level. What concentration of lead, in milligrams per liter, was in the water tested today?

   A. inline

   B. inline

   C. inline

   D. inline

   E. inline

Equations

An **equation,** such as inline, is a pair of expressions that are equal in size, meaning one side equals the other side. A **system of equations** is two or more equations in a single instance. Note that an **inequality,** such as inline, isn't considered an equation.

An **unknown** or **variable** is a number that might not be known and is typically represented by a letter, usually x. The letter could represent a fixed value, where only one value solves the equation, or it could represent a changing value, where two or more values solve the equation. For example, where inline, only one value for x solves the equation, but where inline, two values for y solve it: 3 and –3.

A **coefficient** is the quantity of like unknowns. For example, with $3x$, the coefficient 3 tells you that there are three xs.

Some letters have standard meanings. For example:

- x and y typically represent the coordinates of a point on the rectangular coordinate system, covered further in this chapter.
- m and b typically represent the slope and y-intercept, respectively, of a linear equation, also covered further in this chapter.
- h and k typically represent the x- and y-positions, respectively, of the center of a circle or the vertex of a parabola. These are covered further in this chapter and in chapter 5, "Geometry."
- k could also represent a **constant,** which is a number that doesn't change.
- i typically represents the **imaginary number**  , which is a number that cannot exist in real math and is covered further in chapter 2, "Number and Quantity."
- a, b, and c typically represent the coefficients of a quadratic equation, covered further in this chapter, or the sides of a triangle, covered further in chapter 5.
-  , pronounced *pi*, represents the ratio of a circle's circumference to its diameter, roughly 3.14 or  , covered further in chapter 5.
-  , pronounced *theta*, represents the degree measure of an angle, often within a unit circle, also covered further in chapter 5.

Solution

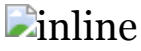
To solve an equation, move the unknown to one side of the equation, and divide both sides of the equation by the coefficient. For example, where  , subtract x from both sides of the equation for  . Divide both sides by the coefficient 2, for , and the solution is 3.

Reporting Category Quiz: Preparing for Higher Mathematics | Algebra

2. If inline, then inline
   - F. inline
   - G. inline
   - H. inline
   - J. inline
   - K. inline
3. What is the value of x that satisfies the equation inline?
   - A. -1
   - B. inline
   - C. inline
   - D. 5
   - E. inline
4. If inline, then inline?
   - F. inline
   - G. inline
   - H. inline
   - J. inline
   - K. inline

Substitution

Substitution refers to placing the given value of a letter into the equation. Substitute the numeric value for the letter or letters and simplify the equation. For example, if inline and inline, you can find the value of y by substituting 3 for x:



Reporting Category Quiz: Preparing for Higher Mathematics | Algebra

5. What is the value of the expression  for  ?

   F. 10

   G. 12

   H. 18

   J. 20

   K. 36

6. When  , what is the value of  ?

   F. 23

   G. 37

   H. 38

   J. 43

   K. 47

7. If  , then 

   F. –16

   G. –8

   H. 1

   J. 8

   K. 16

Multiplication

A letter or variable can be multiplied like a number. For example, if  , then  , because  . Note that when  , y could equal either 4 or –4, because both  and  .

Reporting Category Quiz: Preparing for Higher Mathematics | Algebra

8. If inline and inline, which of the following CANNOT be a value of inline?
   F. –15
   G. –1
   H. 1
   J. 15
   K. 113
9. If x is a real number such that inline, then inline
   F. 4
   G. 10
   H. 18
   J. 20
   K. 47
10. The expression inline is equivalent to:
   A. inline
   B. inline
   C. inline
   D. inline
   E. inline

Answers with Unknowns

The answer could include an unknown. For example, if a box holds x pieces, then 3 boxes hold $3x$ pieces, and the actual number of pieces isn't known or solved.

Reporting Category Quiz: Preparing for Higher Mathematics | Algebra

11. For all pairs of real numbers M and V where inline, inline
    F. inline
    G. inline
    H. inline
    J. inline
    K. inline
12. For all positive integers x, y, and z, which of the following expressions is equivalent to inline?
    A. inline
    B. inline
    C. inline
    D. inline
    E. inline

Construction of Equations

A word problem could require you to construct an equation using a letter as the number you're looking for. For example, if Betty has 5 more dollars than Andy, and Andy has $15, how much does Betty have? Set this up with b as the amount that Betty has:

image

When constructing an equation, treat the word *is* or any variation of it as an equal sign. For example, if Joey is 2 years older than Billy, set up the equation as inline.

Reporting Category Quiz: Preparing for Higher Mathematics | Algebra

13. So far, a student has earned the following scores on four 100-point tests this grading period: 65, 73, 81, and 82. What score must the student earn on the fifth and last 100-point test of the grading period to earn an average test grade of 80 for the 5 tests?

    F. 75

    G. 76

    H. 78

    J. 99

    K. The student cannot earn an average of 80.

14. What expression must the center cell of the table below contain so that the sums of each row, each column, and each diagonal are equivalent?

x	$8x$	$-3x$
$-2x$	?	$6x$
$7x$	$-4x$	$3x$

    F. $6x$

    G. $4x$

    H. $2x$

    J. $-2x$

    K. $-4x$

Linear Equations

A **linear equation** is any equation with one or more unknowns, usually x and y, and no exponents, such as  . The equation has infinite solutions, because for any given value of x, there's a

corresponding value of y. For example, with inline, when inline, inline; when inline, inline; and so on. These x- and y-values form a line, and each x-value and corresponding y-value falls on the line:


A graph is shown in the standard (x,y) coordinate plane, where a diagonal line is drawn with plots for (0, 3) and (1, 5).

This equation can model a simple real-life scenario. For example, if a 3-foot-tall tree is planted and grows 2 feet each year, the height of the tree can be expressed as inline, where h is the tree's height and t is the number of years. When the tree is planted, no time has passed, so inline, and the corresponding inline. Note that a linear equation model question may not necessarily be within the Modeling reporting category.

Reporting Category Quiz: Integrating Essential Skills

15. A city utility department charges residential customers $2.50 per 1,000 gallons of water and $16.00 per month for trash pickup. Which of the following expressions gives a residential customer's total monthly charges, in dollars, for use of g thousand gallons of water and trash pickup?

    F. inline

    G. inline

    H. inline

    J. inline

    K. inline

16. A company rents moving vans for a rental fee of $25.00 per day with an additional charge of $0.30 per mile that the van is driven. Which of the following expressions represents the cost, in dollars, of renting a van for 1 day and driving it m miles?

    A. inline

    B. inline

    C. inline

    D. inline

    E. inline

17. The relationship between temperature in degrees Fahrenheit, F, and temperature in degrees Celsius, C, is expressed by the formula inline. Calvin reads a temperature of inline on a Celsius thermometer. To the nearest degree, what is the equivalent temperature on a Fahrenheit thermometer?

    F. inline

    G. inline

H. inline

J. inline

K. inline

18. The length L, in meters, of a spring is given by the equation inline, where F is the applied force in newtons. What force, in newtons, must be applied for the spring's length to be 0.18 meters?

F. 0.13

G. 0.15

H. 0.225

J. 0.255

K. 0.27

System of Equations

A **system of equations** is two or more equations in a single instance. A system of equations can have zero, one, or more than one solution.

Linear Systems

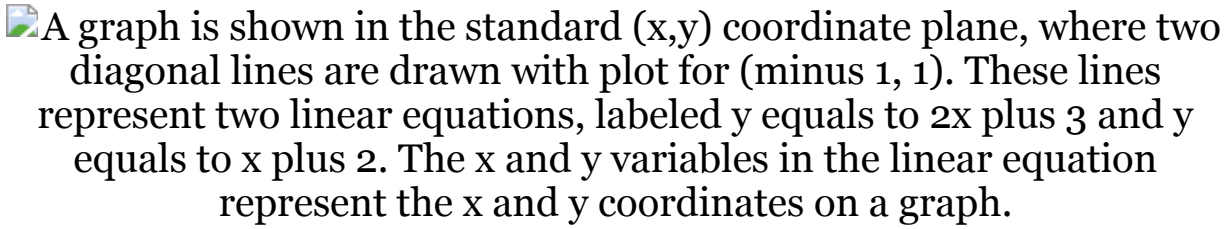
The instance can have two linear equations, and the question can ask for the solution to these equations. For example, inline and inline each has infinite solutions, but together they have one solution for x and y, because there is only one x and corresponding y value that satisfies both equations.

The simplest way to find the solution is to subtract one equation from the other:

image

Now that you know inline, place -1 for x in one of the equations and find the value of y. You should get the same result from either equation: both inline and inline result in inline. The solution to

these equations is thus inline, because those are the only x and y values that satisfy both equations. If you graph the equations, those are the coordinates where the lines intersect.


A graph is shown in the standard (x,y) coordinate plane, where two diagonal lines are drawn with plot for (minus 1, 1). These lines represent two linear equations, labeled y equals to 2x plus 3 and y equals to x plus 2. The x and y variables in the linear equation represent the x and y coordinates on a graph.

Note that when subtracting the equations, you could have to first multiply one of the equations by a coefficient or a negative. The purpose is to eliminate one unknown so that you can solve for the other one. For example, subtracting inline and inline won't eliminate an unknown, so start by multiplying the first equation by 3:

image

Now subtract them to eliminate the y:

image

These are examples of systems of linear equations that each has a single solution:

1. inline and inline
2. inline and inline

Results:

1. inline.

   First subtract the first equation from the second equation:

   image

   Then substitute the value of x into one of the equations:

   image

2. inline.

First multiply the first equation by 2 and the second one by 3, then subtract them:

 image

Next, subtract the second result from the first one:

 image

Finally, substitute the value of x into one of the equations:

 inline

Reporting Category Quiz: Preparing for Higher Mathematics | Algebra

19. What is the x-coordinate of the point in the standard  inline coordinate plane at which the 2 lines  inline and  inline intersect?

    A. 1

    B. 2

    C. 4

    D. 6

    E. 10

20. The cost of a hamburger and a soft drink together is $2.10. The cost of 2 hamburgers and a soft drink together is $3.50. What is the cost of a soft drink?

    A. $0.50

    B. $0.55

    C. $0.70

    D. $1.05

    E. $1.40

Another way to find the solution to two equations is to substitute the value of one of the unknowns for that unknown in the other equation. For example, to find the solution to inline and inline, y equals both inline and inline, so the two expressions equal each other. Set them up and solve for x:

 image

Then place the value of x in one of the equations and solve for y:

 image

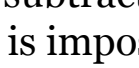
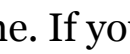
The coordinates inline.

Reporting Category Quiz: Preparing for Higher Mathematics | Algebra

21. If inline and inline, which of the following expresses y in terms of x?
    - A. inline
    - B. inline
    - C. inline
    - D. inline
    - E. inline
22. The larger of two numbers exceeds twice the smaller number by 8. The sum of twice the larger and 3 times the smaller number is 65. If x is the smaller number, which equation below determines the correct value of x?
    - F. inline
    - G. inline
    - H. inline
    - J. inline
    - K. inline

Numbers of Solutions

Note that the two equations might not have a single solution.

- If the equations represent parallel lines, then the lines don't intersect and there is no solution, because there are no corresponding values for x and y that satisfy both equations. For example,  and  represent parallel lines. If you were to subtract these, the result would be  . Because  is impossible, the system of parallel lines doesn't have a solution.
- If the equations represent the same line, then there are infinite solutions, because any corresponding value for x and y would satisfy both equations. For example,  and  represent the same line. If you were to subtract these, the result would be  . Because the two equations represent one line, there are infinite solutions.

Look at the slopes of the lines. If the slopes are different, the lines intersect, and there is a single solution for the system of equations. If the slopes are the same, the lines are either parallel or identical, and there is either no solution or infinite solutions.

Reporting Category Quiz: Preparing for Higher Mathematics | Algebra

23. A system of linear equations is shown below.

image

Which of the following describes the graph of this system of linear equations in the standard inline coordinate plane?

A. Two distinct intersecting lines

B. Two parallel lines with positive slope

C. Two parallel lines with negative slope

D. A single line with positive slope

E. A single line with negative slope

Variations on Linear Equations

Linear equations can have certain variations.

- A y value without an x value represents a horizontal line. For example, inline is a horizontal line that crosses the y axis at 3. This line has a slope of 0.
- An x value without a y value represents a vertical line. For example, inline is a vertical line that crosses the x axis at 5. This line has an undefined slope.
- An expression, whether linear or non-linear, can be an inequality. For example, inline and inline represent the area beneath the line, while inline and inline represent the area above the line. An expression of "less than" or "greater than," $<$ or $>$, respectively, is graphed with a dashed line, while an expression of "or equal to," inline or inline, is graphed with a solid line.

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24. Which of the following systems of inequalities is represented by the shaded region of the graph below?

 A graph is shown in the standard (x,y) coordinate plane, where a diagonal line is drawn with plot for (3, minus 6). A line is also drawn, which is parallel to y-axis and intersecting the diagonal line at point (3, minus 6), with plot for (3, 0).

A. inline and inline

B. inline or inline

C. inline and inline

D. inline or inline

E. inline and inline

Non-Linear Systems

A system of equations can feature non-linear equations, in which case the system can have more than one solution. For example, the line and parabola cross twice, giving the system of equations two solutions.

 A graph is shown in the standard (x,y) coordinate plane, where a parabola is drawn which intersects a point on both x-axis and y-axis. A diagonal line is drawn which intersects the parabola and a point on both x-axis and y-axis

A line and parabola may cross once, as shown in the drawing on the left, giving the system of equations one solution; or they may not cross at all, as shown in the drawing on the right, in which case the system has no solution.

Illustration shows two different graphs. A graph is shown in the standard (x,y) coordinate plane (on the left-hand side), where a parabola is drawn which intersects a point on both x-axis and y-axis. A diagonal line is drawn which intersects the parabola and a point on both x-axis and y-axis. A graph is shown in the standard (x,y) coordinate plane (on the right-hand side), where a parabola is drawn which intersects a point on both x-axis and y-axis. A diagonal line is drawn which intersects a point on both x-axis and y-axis.

Quadratic Equations and Parabolas

A linear equation, such as inline, has no exponents and single corresponding values of x and y, but a **quadratic equation** has inline, giving each value of y either zero, one, or two possible values of x. For example, with the quadratic equation inline, when inline, x has two possible values, 3 or −3; when inline, x has one possible value, 0; and when inline, x has zero possible values. A quadratic equation graphs into a **parabola:**

A graph is shown in the standard (x,y) coordinate plane, where a parabola, labeled y equals to x superscript 2, is drawn which intersects the x-axis.

Note that inline can be used in place of y, making inline the same as inline; this is covered further in chapter 4, “Functions.”

Reporting Category Quiz: Preparing for Higher Mathematics | Algebra

25. Which of the following is the graph, in the standard inline coordinate plane, of inline?

A. A graph is shown in the standard (x,y) coordinate plane, where a diagonal line is drawn with plots for (0, 1) and (1, 3).

B. A graph is shown in the standard (x,y) coordinate plane, where a diagonal line is drawn with plot for (1, 3).

C. A graph is shown in the standard (x,y) coordinate plane, where a diagonal line is drawn with plots for (0, 2) and (1, 3).

D. A graph is shown in the standard (x,y) coordinate plane, where a parabola is drawn which intersects the point (0, 1); and plots for (minus 1, 3) and (1, 3).

E. A graph is shown in the standard (x,y) coordinate plane, where a parabola is drawn which intersects the origin point and plots for (minus 1, 3) and (1, 3).

Quadratic Factors

A quadratic equation having a leading coefficient of 1, meaning it begins with inline, not inline or inline, can be factored simply. For example, inline factors into inline. To find the possible values of x, set y equal to 0, and inline factors into inline, meaning x could equal -3 or 1. If you graph the equation inline, the parabola crosses the x-axis (where inline) at -3 and 1.


A graph is shown in the standard (x,y) coordinate plane, where a parabola is drawn which intersects the point (1, 0) and plot for (minus 3, 0). The parabola represents a linear equation, labeled y equals to x superscript 2 plus 2 x minus 3.

One simple method to factor a quadratic with a leading coefficient of 1 is to find two values whose sum is the x coefficient and product is

the constant term. (The *constant term* is the number on the end that doesn't have an x.) With the example inline, the x coefficient is 2 and the constant term is –3, so the two values are 3 and –1, because inline and inline. Place these values into the factored expressions: inline and inline.

Another method to factor the quadratic is to *complete the square*. With the same example inline, look for an expression that, when squared, FOILs to a product containing inline: in this case, inline. However, the FOIL of inline is inline, so for this result, add 4 to both sides:

image

Because inline, change the left side to inline and take the square root:

image

To check whether the equation was factored correctly, multiply the factors and see whether the product matches the original equation. Use the FOIL (**f**irst, **o**uter, **i**nner, **l**ast) method to multiply inline back to inline:

image

These are examples of quadratic equations that each has one or two possible solutions for x:

1. inline
2. inline
3. inline

Results:

1. inline.

   The two values that add to 3 and multiply to –10 are 2 and –5; place these into the factored expressions:

   image

2. inline.

   The two values that add to –1 and multiply to –12 are 3 and –4; place these into the factored expressions:

   image

3. inline.

This is an example where the quadratic has a single value of *x*. inline and inline, so each number is 3: place 3s into the factored expression, and combine them into a single squared value:

image

Reporting Category Quiz: Preparing for Higher Mathematics | Algebra

26. Which of the following quadratic equations has solutions inline and inline?

    F. inline

    G. inline

    H. inline

    J. inline

    K. inline

The inline in the quadratic inline is known as the *quadratic term*, and the 2*x* is known as the *linear term*. If the quadratic doesn't have a linear term, such as inline, it can be solved by taking the square root. Isolate the quadratic term and take the square root of both sides:

image

If the quadratic equation cannot be factored simply, for example, if the leading coefficient is greater than 1, the **quadratic formula** is another option:

 image

The coefficients and numeric value from the quadratic equation are *a*, *b*, and *c* in the equation. For example, the equation inline with coefficients showing is inline, so the values of *a*, *b*, and *c* are 1, 2, and –3, respectively. Place these into the quadratic formula and simplify:

 image

You get the same solutions for *x* whether you factor the equation or solve it with the quadratic formula.

Note the expression inline within the quadratic equation: if inline equals 0, *x* has one solution, and if inline is negative, the equation cannot be solved or factored.

Multiplication

To multiply a quadratic by a coefficient, distribute the coefficient among the terms. For example, to multiply 3 by inline, written as inline, distribute the 3 among the terms for inline.

To multiply two expressions into a quadratic, use the FOIL method. For example, to multiply inline by inline, multiply the first terms (each *x*), the inner terms (the –2 and *x*), the outer terms (the *x* and –3), and the last terms (the –2 and –3). Add the results for the answer:

 image

To square an expression, write it out as two separate expressions for multiplying. For example, to simplify inline, write it out as inline and multiply using the FOIL method:

 image

Note that when multiplying expressions that are identical except for the signs between the terms, the middle terms cancel out. For example, inline and inline are identical except that one has +5 and the other –5. When these are multiplied, the 5*x* and –5*x* cancel, and the resulting quadratic has no middle term:

 image

This is called a **math conjugate** and also applies to factoring: the factors of a perfect square minus a perfect square are identical expressions of square roots but with differing signs. Common forms of this include  , which factors to  ,  , which factors to  , and  , which factors to  .

Reporting Category Quiz: Preparing for Higher Mathematics | Algebra

27. inline is equivalent to:

F. inline

G. inline

H. inline

J. inline

K. inline

28. Which of the following expressions is equivalent to inline?

F. inline

G. inline

H. inline

J. inline

K. inline

29. For all *x*, inline

F. inline

G. inline

H. inline

J. inline

K. inline

30. inline

A. inline

B. inline

C. inline

D. inline

E. inline

There are three common quadratic forms. The first is the *standard form*, which is the now-familiar inline, or inline. The second is the *vertex form*, which is inline, or with this example, inline. Note that inline in almost every ACT mathematics test parabola, so you can treat these as inline and inline. The advantage of the vertex form is that it gives you the parabola's vertex as *h* and *k*. If you FOIL the squared expression in the vertex form, it matches the standard form.

image

The resulting graph of inline matches the graph of inline.

A graph is shown in the standard (x,y) coordinate plane, where a parabola is drawn which intersects a point on both x-axis and y-axis. The parabola represents a linear equation, labeled left parenthesis x plus 1 right parenthesis superscript 2 minus 4.

The third common quadratic form is the *factored form*, which is inline. For example, the factored form of inline is inline. The advantage of the factored form is that it tells you the *x*-intercepts as *a* and *b*.

Roots

A **square root** refers to a quantity that, when squared, yields the starting quantity. For example, inline, because inline. A **cube root** is similar, except the quantity is cubed to yield the starting quantity. For example, inline, because inline.

A fractional exponent yields a root. For example, inline and inline . The denominator of the fraction becomes the *index number* (the small number outside the radical), and the numerator of the fraction stays as the exponent. For example, inline.

Square Roots

A square root always yields a positive number, because a quantity times itself is never negative. The square root of any negative number is therefore not a real number and is referred to as *i* for *imaginary*. Where inline represents a real number, inline and is imaginary. Remember that inline and inline.

Note that when inline, *x* can equal either 3 or –3, because you're not taking the square root of inline or 9: you're finding values of *x* that satisfy the equation. However, inline can only be 3, and not –3, because a square root can only be of a positive number.

To take the square root of a fraction, take the square roots of the numerator and denominator as separate numbers. For example, to take the square root of inline, take the square roots of 4 and 25 separately: inline.

Multiplication and Division

You can multiply and divide square root quantities, but you cannot add or subtract them. For example, inline, but inline cannot be simplified. However, if the square root quantities are the same, they can be counted: inline, the same way that inline.

Multiply square root quantities by multiplying the quantities within. In the same way that inline, multiply the square root quantities inline.

Divide square root quantities with the same method, by dividing the quantities within. In the same way that inline, divide the square root quantities inline.

These are examples of roots that can be simplified:

1. inline
2. inline
3. inline

Results:

1. inline.

Multiply the numbers under the radical: inline.

2. inline.

   Count the like radicals: inline.

3. inline.

   Divide the numbers under the radical: inline

   If the roots have coefficients, multiply or divide the coefficients separately from the roots: inline.

Simplification

To simplify a square root, factor the quantity and simplify the perfect squares. For example, to simplify inline, factor it and simplify:

inline

These are examples of roots that can be simplified:

1. inline
2. inline
3. inline

Results:

1. inline.

   Factor out the perfect square, which is 25: inline.

2. inline.

   Factor out the perfect square, which is 16: inline.

3. inline.

   Factor out the perfect square, which is 4: inline.

Cube Roots

A cube root can yield a positive or negative number, because a quantity times itself three times stays positive or negative. For

example, inline and inline, because inline.

Multiplication and Division

Similar to square root quantities, cube root quantities can be multiplied, divided, and counted, but not added or subtracted.

These are examples of roots that can be simplified:

1. inline
2. inline
3. inline

Results:

1. inline.

   Multiply the numbers under the radicals: inline.
2. inline.

   Add the like radicals: inline.
3. inline.

   Divide the numbers under the radicals: inline.

Simplification

To simplify a cube root, factor the quantity and simplify the perfect cubes. For example, to simplify inline, factor it out and simplify:

 image

These are examples of roots that can be simplified:

1. inline
2. inline
3. inline

Results:

1. inline.

   Factor out the perfect cube, which is 8: inline.

2. inline.

   Factor out the perfect cube, which again is 8: inline.

3. inline.

   Factor out the perfect cube, which is 27: inline.

Higher Roots

No index number indicates a square root and a 3 indicates a cube root, as in inline and inline. However, the index number can be any number, such as 4 or 5. It refers to the number of times the result is multiplied by itself to get the starting number. For example, inline, because inline.

Logarithmic Equations

A **logarithmic equation** is the inverse function of an exponential equation. This means that the logarithm of a number is the exponent needed to produce that number from the base. For example, the logarithm of 16 from a base of 2 is 4, because the number 2 needs an exponent 4 to produce 16. This is written as inline, which is the same as inline.

To solve for an unknown in a logarithmic equation, rewrite the equation in its exponential form. For example, rewrite inline as inline.

These are examples of logarithmic equations where the unknowns can be solved by rewriting the equations in their exponential forms:

1. inline
2. inline
3. inline

Results:

1. 4.

   Rewrite the equation in its exponential form:

    image

2. 32.

   Rewrite the equation in its exponential form:

    image

3. 5.

   Rewrite the equation in its exponential form:

    image

Reporting Category Quiz: Integrating Essential Skills

31. The value of  is between which of the following pairs of consecutive integers?

    A. 0 and 1

    B. 4 and 5

    C. 5 and 6

    D. 6 and 7

    E. 9 and 10

Chapter 3: Quiz Answers

1. **The correct answer is D.** Multiply inline by 100, which is equivalent to inline: inline.
2. **The correct answer is F.** Move the *x*s to the left and the numbers to the right, then divide both sides by the *x* coefficient.

    image
3. **The correct answer is D.** First distribute the coefficient among the values in the expression, then move the *x*s to the left and the numbers to the right. Finally, divide both sides by the *x* coefficient.

    image
4. **The correct answer is G.** The *x* can stay on the right: subtract inline from both sides.

   inline
5. **The correct answer is H.** Substitute 2 for *g* in the equation:

    image
6. **The correct answer is K.** Substitute 6 for inline in the equation:

    image
7. **The correct answer is K.** First find the value of inline. Subtract *b* and multiply by –1:

    image

   Then, substitute –2 for inline in the second expression:

    image
8. **The correct answer is K.** If inline and inline, inline or –7, and inline or –8, giving inline four possibilities:

image

The only one that's not a correct answer is 113.

9. **The correct answer is H.** If inline, then inline. Substitute 4 in the equation:

   image

10.

    The correct answer is A. Distribute the *a* among the expression: inline.

11. **The correct answer is J.** Set up the equation and solve for *V*:

    image

12. **The correct answer is A.** With the fraction inline, the *z*s cancel, leaving inline. Another way to work this question is to substitute numbers for the letters, such as inline, inline, and inline. Then look for the answer choice equivalent to inline.

13. **The correct answer is J.** For an average of 80 on 5 tests, the student needs inline points. Set up the equation with *x* as the fifth test:

    image

14. **The correct answer is H.** You don't need to check each row, column, and diagonal: just set one complete row equal to the one with the missing value. Using the first row:

    image

    You could eliminate the *x*s and work with the coefficients:

    image

15. **The correct answer is F.** This is a standard linear equation with *g* as the *x* and the value of the entire equation as the *y*. Set it up as inline and add the numbers from the question. The customer pays a flat $16.00 regardless of the water used, so use $16.00 for *b*. The customer pays $2.50 per 1,000 gallons of water, and *g* represents 1,000 gallons, so use $2.50 for *m*. Last,

place *g* for *x*, and the equation looks like this: inline. Eliminate the *y* for the answer.

16. **The correct answer is A.** This is a standard linear equation with *m* as the *x* and the value of the entire equation as the *y*. Because *m* is usually the slope of the equation, for this explanation the slope will be *e*. Set up the equation as inline and add the numbers from the question. The customer pays a flat $25.00 regardless of the miles driven, so use $25.00 for *b*. The customer pays $0.30 per mile, so use $0.30 for *e*. Last, place *m* for *x*, and the equation looks like this: inline. Eliminate the *y* for the answer.
17. **The correct answer is K.** Place 38 for *C* in the equation and solve for *F*:

    image

    Note that the question reads, to the *nearest* degree.
18. **The correct answer is H.** Place 0.18 for *L* in the equation and solve for *F*:

    image
19. **The correct answer is B.** Subtract the first equation from the second one and solve for *x*:

    image
20. **The correct answer is C.** First, set up the two equations with *h* as the cost of a hamburger and *d* as the cost of a soft drink:

    image

    Next, subtract the first equation from the second one:

    inline

    Finally, place the value of *h* into the simpler equation:

    image
21. **The correct answer is A.** Substitute to eliminate the *t*. First, isolate the *t* in one equation:

 image

Next, substitute *t* with 5-*y* in the other equation:

 image

Finally, isolate the *y*:

 image

22. **The correct answer is J.** First, set up two equations with *b* as the larger number and *x* as the smaller number. Use the words *exceeds* and *is* as the equal signs:

     image

    Next, place the value of *b* from the first equation into the *b* in the second equation, so that the *b* is eliminated, and the result should match an answer choice:

     image

23. **The correct answer is A.** The slopes of the lines are the *x* coefficients when *y* equals the equation. Because the slopes are different, the lines intersect.

24. **The correct answer is A.** Narrow down the answer. The shaded region represents constraints from line 1 *and* line 2, which narrows the answer down to choices **A, C,** and **E.** The shaded region is *below* the diagonal line, so the answer contains  and has to be choice A. Just to be sure, the shaded region is to the right of the vertical line, so the answer also contains , also in choice **A.**

25.

    The correct answer is A. Factor out the *x* to reduce the fraction:

     image

    With no exponent, the graph is a line, which narrows the answer down to choices **A, B,** and **C**. The slope is 2 and *y*-intercept is 1, so the answer is **A**.

26. **The correct answer is J.** If the solutions to the equation are inline and inline, then the factors of the equation are inline . Multiply these using the FOIL method for the answer:

    image

    The two middle terms contain *x*, so this is factored out within the equation.

27. **The correct answer is J.** Distribute the negative among the second expression, and combine like terms:

    image

28. **The correct answer is H.** Distribute the 3*x* among the expression:

    image

29. **The correct answer is K.** Set the expression times itself and multiply using the FOIL method:

    image

30. **The correct answer is B.** Set the expression times itself and multiply using the FOIL method:

    image

31. **The correct answer is D.** Set the equation equal to *x* and rewrite it in its exponential form:

    image

Chapter 4:
Functions

The questions in this category test your ability to work with **functions**. Questions can ask you to manipulate and translate linear, radical, piecewise, polynomial, and logarithmic functions. They can also ask you to find and apply features of graphs.

Series, Sequences, and Consecutive Numbers

A **series** or **sequence** is the adding of many quantities, one after the other, to a given quantity, possibly in a repeating pattern. To discern a series or a sequence, look for the pattern or difference between each of the numbers. For example, to find the next number in series inline, the common difference is 3, so you know that the next number after 10 is 13.

These are examples of series and sequences that can be completed:

1. inline
2. inline
3. inline

Results:

1. 17.

   The common difference between each number is 5. Start with 2, and repeatedly add 5.

2. 63.

   The common difference between each number doubles: 4, 8, and 16. From this, you can discern that the next difference is 32:

   image

3. 5.

This sequence is the repetition of the numbers 2, 5, 8, 5, and 10.

Consecutive numbers are of a specific series where numbers follow each other in order, typically with a common difference of 1 between each number: inline. Consecutive numbers specified as *even* or *odd* have a common difference of 2 between each number: inline or inline. Consecutive numbers can also be negative: inline.

Reporting Category Quiz: Preparing for Higher Mathematics | Functions

1. What 2 numbers should be placed in the blanks below so that the difference between consecutive numbers is the same?

   inline

   A. 23, 29

   B. 24, 34

   C. 25, 33

   D. 26, 35

   E. 27, 31

2. Which of the following statements describes the total number of dots in the first n rows of the triangular arrangement illustrated below?

   The figure shows 5 rows, where dots are arranged in the triangular pattern (in an increasing order). From top to bottom, the first rows shows "1 dot," the second row shows "3 dots," the third row shows "5 dots," the fourth row shows "7 dots," and the fifth row shows "9 rows."

   A. This total number is always equal to 25 regardless of the number of rows.

   B. This total is equal to twice the number of rows.

   C. This total is equal to 5 times the number of rows.

   D. This total is equal to the square of the number of rows.

   E. There is no consistent relationship between this total and the number of rows.

3. Which of the following statements is NOT true about the arithmetic sequence inline?

   A. The fifth term is −3.

B. The sum of the first 5 terms is 35.

C. The eighth term is –18.

D. The common difference of consecutive terms is –5.

E. The common ratio of consecutive terms is –5.

Slope-Intercept Form of a Linear Equation

The **slope-intercept form** of a linear equation is the form inline, where m is the slope and b is the y-intercept. With the line inline, the slope is 3 and the y-intercept is 5.

A linear equation can appear in a different form, such as inline. To find the slope and y-intercept, convert it to the slope-intercept form by setting the equation equal to y:

 image

With this line, the slope is –2 and the y-intercept is 3.

Note that parallel lines have identical slopes, and perpendicular lines have negative reciprocal slopes. For example, inline is parallel to inline but perpendicular to inline.

Reporting Category Quiz: Preparing for Higher Mathematics | Functions

4. What is the slope-intercept form of inline?

   F. inline

   G. inline

   H. inline

   J. inline

   K. inline

5. What is the slope of any line parallel to the line inline?

   A. −7

   B. inline

   C. inline

   D. 6

   E. 7

Slope and Intercept from a Line in the Coordinate Plane

Find the slope and *y*-intercept from the drawing to re-create the equation. Place the slope and *y*-intercept as the *m* and *b*, respectively, into the slope-intercept equation inline. The *y*-intercept is the point where the line crosses the *y*-axis. In this example, the line crosses the *y*-axis at 3, so inline. The slope can be found using either *rise over run* or the *slope formula* inline. Using rise over run, the line rises 3 and runs 2. Because the line goes down, the slope is negative, so inline.

A graph is shown in the standard (x,y) coordinate plane, where a diagonal line is drawn with plots for (0, 3) and (2, 0).

The slope formula captures the rise over run from any two points on the line. $y_2 - y_1$ refers to one *y*-coordinate minus the other, and $x_2 - x_1$

refers to one *x*-coordinate minus the other. In the following drawing, coordinate 2 is $(2,0)$ and coordinate 1 is $(0,3)$, so $y_2 - y_1$ is $0-3$ and $y_2 - y_1$ is $2-0$. It doesn't matter which point is coordinate 2 or 1, as long as the *x*s and *y*s are consistent.

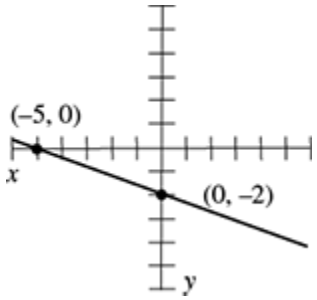
Place these *x*- and *y*-values into the formula:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0-3}{2-0} \\ &= -\frac{3}{2} \end{aligned}$$

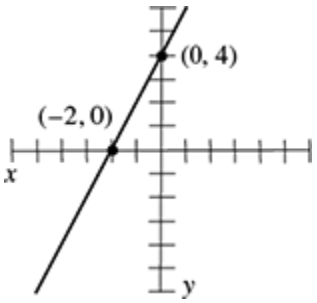
The equation for this line is thus $y = -\frac{3}{2}x + 3$.

These are examples of lines from which the equations can be found:

1.



2.



Results:

1. $y = -\frac{2}{5}x - 2$.

Find b from the y-intercept, which is -2, so $b=-2$. Find the slope from the slope formula:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2-0}{0-(-5)} \\ &= -\frac{2}{5} \end{aligned}$$

2. $y=2x+4$.

Find b from the y-intercept, which is 4, so $b=4$. Find the slope from the slope formula:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-0}{0-(-2)} \\ &= 2 \end{aligned}$$

Reporting Category Quiz: Preparing for Higher Mathematics | Functions

6. What is the slope of the line given by the equation $14x-11y+16=0$?

   A. -11

   B. $-\frac{14}{11}$

   C. $-\frac{14}{11}$

   D. $\frac{14}{11}$

   E. 14

Patterns

A linear equation can model a real-life scenario. For example, sound travels at the constant speed of 1,125 feet per second, so to calculate

the distance in feet, *d*, that a sound travels over a certain number of time in seconds, *t*, use the equation $d = 1{,}125t$. This is equivalent to a linear equation with a slope of 1,125 and an intercept of 0.

Reporting Category Quiz: Preparing for Higher Mathematics | Functions

7. When Jeff starts a math assignment, he spends 5 minutes getting out his book and a sheet of paper, sharpening his pencil, looking up the assignment in his assignment notebook, and turning to the correct page in his book. The equation $t = 10p + 5$ models the time, *t* minutes, Jeff budgets for a math assignment with *p* problems. Which of the following statements is necessarily true according to Jeff's model?

   F. He budgets 15 minutes per problem.

   G. He budgets 10 minutes per problem.

   H. He budgets 5 minutes per problem.

   J. He budgets 10 minutes per problem for the hard problems and 5 minutes per problem for the easy problems.

   K. He budgets a 5-minute break after each problem.

If the pattern is complicated, simply place the input or inputs into the equation. For example, to use the speed of sound equation $d = 1{,}125t$ to find the distance that a sound travels in 200 seconds, place 200 for *t*: $d = 1{,}125(200) = 225{,}000$ feet.

Reporting Category Quiz: Preparing for Higher Mathematics | Functions

8. What is $\sin\frac{\pi}{12}$ given that $\frac{\pi}{12}=\frac{\pi}{3}-\frac{\pi}{4}$ and that $(\sin\alpha-\beta)=(\sin\alpha)(\cos\beta)-(\cos\alpha)(\sin\beta)$?

   (Note: You may use the following table of values.)

θ	$\sin\theta$	$\cos\theta$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$

F. $\frac{1}{4}$

G. $\frac{1}{2}$

H. $\frac{\sqrt{3}-2}{4}$

J. $\frac{\sqrt{3}-\sqrt{2}}{2}$

K. $\frac{\sqrt{3}-\sqrt{2}}{2}$

Graphed Equations in the f(x) Form

A function can be a graphed equation where ***f(x)*** takes the place of y. For example, the functions $y=2x^2+1$ and $f(x)=2x^2+1$ are the same. On the coordinate system, x is the lateral coordinate and $f(x)$, like y, is the vertical coordinate. The value of x is placed into the $f(x)$. For

example, if $x=3$, the function would appear as $f(3)=2x^2+1$ or $f(3)=2(3)^2+1$, so in this case $f(3)=19$.

These are variations of functions that can be simplified where $f(x)=2x+1$.

1. $f(3)$
2. $\frac{1}{f(4)}$
3. $2f(5)$
4. $f(t)$

Results:

1. 7.

   Place the 3 for *x* in the function: $f(3)=2(3)+1=7$.

2. $\frac{1}{2}$.

   Solve for $f(4)=2(4)+1=9$ and place this under the 1.

3. 22.

   Solve for $f(5)=2(5)+1=11$ and multiply this by 2.

4. $f(t)=2t+1$.

   Place the *t* for *x* in the function.

Reporting Category Quiz: Preparing for Higher Mathematics | Functions

9. If $f(x)=x^2-2$, then $f(x+h)=?$

F. x^2+h^2

G. x^2-2+h

H. x^2+h^2-2

J. $x^2+2xh+h^2$

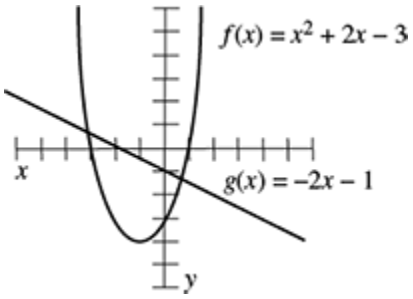
K. $x^2+2xh+h^2-2$

A function can use letters other than *f* and *x*, such as *g* (*h*). Also, a function can be nested within another function. For example, if $f(x)=x^2-3$ and $g(h)=2h-1$, the question can ask the value of the nested functions $f(g(h))$ when $h=5$. To solve this:

1. Find $g(h)$ by placing 5 for *h*: $g(5)=2(5)-1=9$.
2. Because $g(5)=9$, place 9 for $f(t)$: $f(g(5))=f(9)$.
3. Solve the final equation by placing 9 for *x*: $f(9)=(9)^2-3=78$.

System of Functions

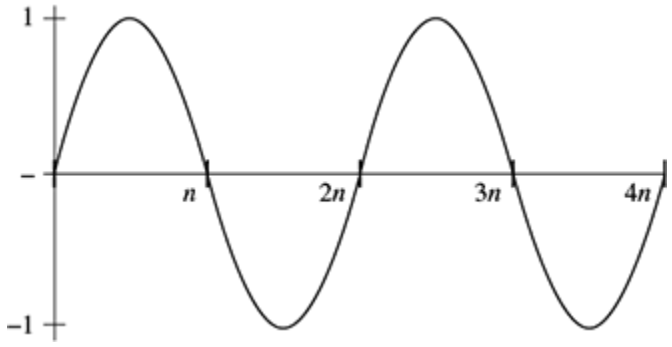
A **system of functions** refers to two or more functions in a single instance. The number of solutions refers to the number of points where the functions cross. For example, this drawing of a system of functions shows two functions with two solutions, or two points where $f(t)=2t+1$.



Finding the coordinates of the solutions is covered further in chapter 3, "Algebra."

Trigonometric Functions

A **trigonometric function** is the graph of a continuous, smooth periodic oscillation. The simplest form is $y = \sin x$ or $f(x) = \sin x$, also known as a **sine wave**, shown in the following:



The terms describing a sine wave are *period*, *amplitude*, and *frequency*:

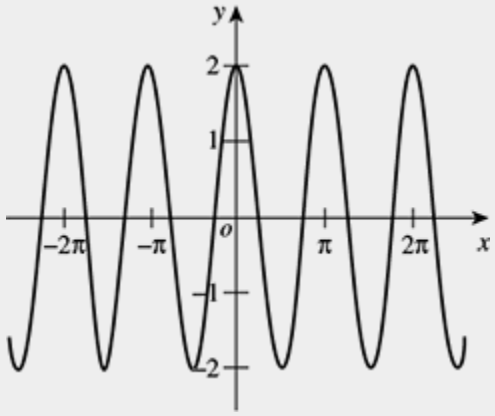
- **Period**, also known as **phase**, is the distance along the *x*-axis for the function to complete one full cycle. The function in the drawing of $y = \sin x$ has a period of 2π. An *x*-coefficient reduces the period: for example, the graph of $y = \sin(2x)$ has a period of π.
- **Frequency** is the number of cycles, or periods, in a given interval. The interval shown of 4π has a frequency of 2. An *x*-coefficient increases the frequency. For example, the graph of $y = \sin(2x)$ in the same interval of 4π has a frequency of 4.
- **Amplitude** is the distance from the mean, in this case, the *x*-axis, to the maximum. The function in the drawing of $y = \sin x$ has an amplitude of 1. A coefficient on the $0-3$ increases the

amplitude. For example, the graph of $y = 2\sin x$ has an amplitude of 2.

Note that the graph of $y = \cos x$ produces a similar trigonometric function known as a **cosine wave.**

Reporting Category Quiz: Preparing for Higher Mathematics | Functions; Modeling

10. A trigonometric function with equation $y = a\sin(bx + c)$, where a, b, and c are real numbers, is graphed in the standard (x, y) coordinate plane below. The period of this function $f(x)$ is the smallest positive number p such that $f(x + p) = f(x)$ for every real number x. One of the following is the period of this function. Which one is it?



A. $\frac{\pi}{2}$

B. π

C. 2π

D. 4π

E. 2

Chapter 4: Quiz Answers

1. **The correct answer is C.** The difference between 17 and 41 is 24. That these are *consecutive numbers* indicates a common difference, so the space of 24 means that the common difference is 8: the missing numbers are 25 and 33.
2. **The correct answer is D.** The total of 25 dots is the square of the number of rows of 5, and without the last row, the revised total of 16 dots is the square of the new number of rows of 4.
3. **The correct answer is E.** The common difference between each number is –5, making each statement true except for the last one: The common ratio of the numbers 17, 12, 7, and 2 is not –5.
4. **The correct answer is H.** To find the slope-intercept form, set the equation equal to y:

$$\begin{aligned} 8x - y - 6 &= 0 \\ 8x - 6 &= y \\ y &= 8x - 6 \end{aligned}$$

5. **The correct answer is B.** To find the slope of any parallel line, start by setting the equation equal to y:

$$\begin{aligned} 7x + 9y &= 6 \\ 9y &= -7x + 6 \\ y &= -\frac{7}{9}x + \frac{2}{3} \end{aligned}$$

   Any line with a slope of $-\frac{7}{9}$ is parallel to this line.

6. **The correct answer is J.** To find the slope, set the equation up in the *slope-intercept form*, which isolates the y. The slope is the x-coefficient.

$$14x-11y+16=0$$
$$14x-16=11y$$
$$\frac{14}{11}x-\frac{16}{11}=y$$
$$y=\frac{14}{11}x-\frac{16}{11}$$

7. **The correct answer is G.** The equation inline indicates that for each increment of *p*, the value of *t* increases by 10.
8. **The correct answer is K.** Don't worry about understanding the function. Place the values from the table into the equation and simplify. If $\frac{\pi}{12}=\frac{\pi}{3}-\frac{\pi}{4}$, then inline. Consider inline as equivalent to inline, so place the values of inline and inline from the table for inline and inline, respectively, in the equation:

   image

9. **The correct answer is K.** Place the inline for *x* in the function:

   image

10. **The correct answer is B.** The distance along the *x*-axis for this function to complete one full cycle is inline.

Chapter 5:
Geometry

Geometry questions are based on two-dimensional and three-dimensional shapes. They ask about the composition of objects and ask you to find the missing values in triangles, circles, and other figures. These questions also include trigonometric ratios.

Coordinate Geometry

Coordinate Geometry is a method of using algebraic equations to describe points, lines, and shapes on a (*x*, *y*) *coordinate plane*, also known as the *x*-*y rectangular grid*. This is a two-dimensional area defined by a horizontal *x*-axis and a vertical *y*-axis that intersect at the **origin**, which has coordinates (0, 0), and forms **Quadrants I, II, III,** and **IV.**


A graph is shown in the standard (x,y) coordinate plane, which is divided into four quadrants labeled I, II, III, and IV. The coordinate is labeled as (0, 0).

Each point on the grid is labeled using an ordered pair (*x*, *y*), with the *x*-value indicating the lateral position and the *y*-value indicating the vertical position. For example, this point has an *x*-value of 3 and a *y*-value of 2, for the coordinates (3, 2):


A graph is shown in the standard (x,y) coordinate plane. The coordinate is labeled as (3, 2).

Lines and Angles

Lines and angles comprise most basic shapes.

A **line** is straight and continuous.

Image of a bidirectional arrow.

A **segment** is part of a line and can be indicated by its endpoints. This segment, with endpoints *A* and *B*, is indicated as inline.

The diagram shows a horizontal line, where A (on the left-hand side) and B (on the right-hand side) represents the endpoints of the segment.

An **angle** is formed by the intersection of two lines or segments and can also be indicated by its endpoints. This angle, with endpoints *C*, *D*, and *E*, is indicated as inline.

The figure shows an angle formed by the intersection of two lines or segments. This angle, with endpoints C, D, and E, is indicated as CDE.

Complementary angles total inline.

The figure shows a right-angled triangle, where a diagonal line is drawn from the origin point. The angle above the origin point is labeled as “60 degree” and below the origin point is labeled as “30 degree.”

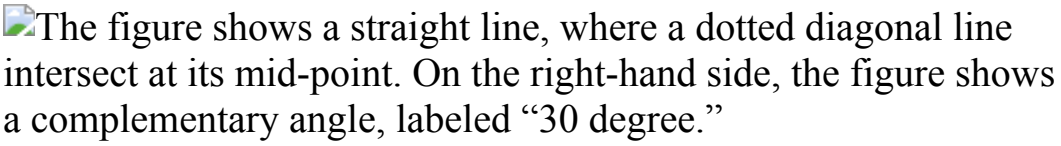
Supplementary angles total inline.

The figure shows a straight line, where a diagonal line is drawn from the mid-point. The angle on the left-hand side of the mid-point is labeled as “150 degree” and the angle on the right-hand side of the mid-point is labeled as “30 degree.”

A **triangle** has angles totaling inline and can be indicated by its endpoints. This triangle, with endpoints *F*, *G*, and *H*, is indicated as inline .

The figure shows an isosceles triangle FGH.

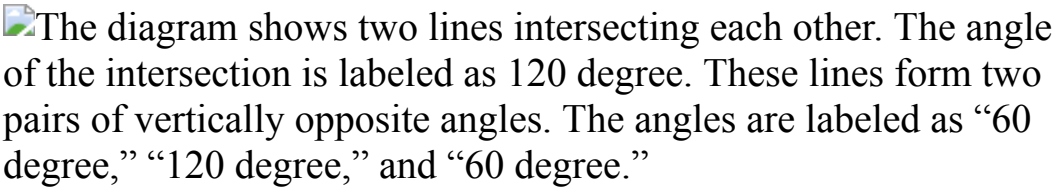
A **bisector** is a segment that *bisects*, or divides, an angle or another segment into two equal parts.


The figure shows a straight line, where a dotted diagonal line intersect at its mid-point. On the right-hand side, the figure shows a complementary angle, labeled “30 degree.”

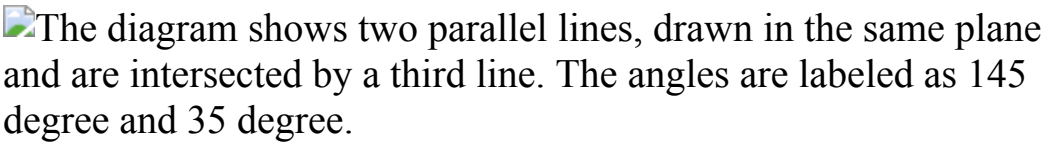
A **perpendicular bisector** is a segment that bisects another segment at inline.


The diagram shows two perpendicular lines, intersecting each other at right angle.

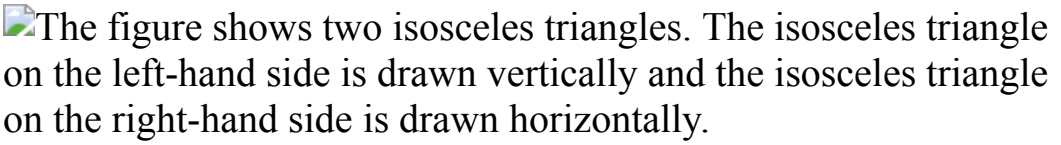
Any two lines or segments that cross create four angles. The opposite angles, also known as **vertical angles**, are equal, and all four angles total inline.


The diagram shows two lines intersecting each other. The angle of the intersection is labeled as 120 degree. These lines form two pairs of vertically opposite angles. The angles are labeled as “60 degree,” “120 degree,” and “60 degree.”

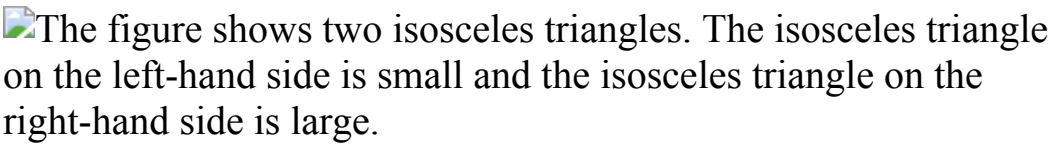
A **transversal** is a line that crosses two or more lines. If the two crossed lines are parallel, the transversal creates two identical sets of four angles.


The diagram shows two parallel lines, drawn in the same plane and are intersected by a third line. The angles are labeled as 145 degree and 35 degree.

Congruent shapes are identical, though one shape can be rotated.

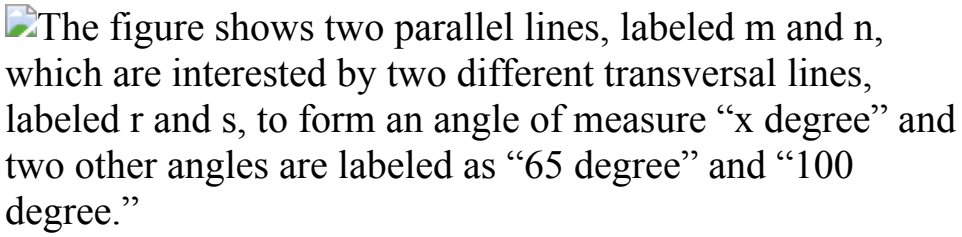

The figure shows two isosceles triangles. The isosceles triangle on the left-hand side is drawn vertically and the isosceles triangle on the right-hand side is drawn horizontally.

Similar shapes have the same angles and side-length ratios but have different sizes.

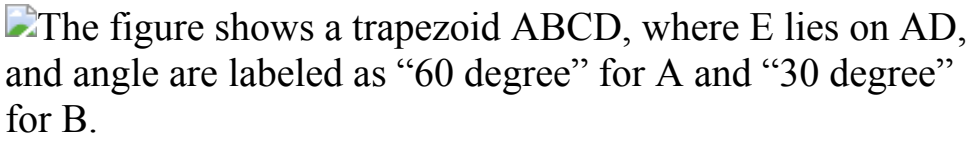

The figure shows two isosceles triangles. The isosceles triangle on the left-hand side is small and the isosceles triangle on the right-hand side is large.

Reporting Category Quiz: Preparing for Higher Mathematics | Geometry

1. In the figure below, lines *m* and *n* are parallel, transversals *r* and *s* intersect to form an angle of measure *x*°, and 2 other angle measures are as marked. What is the value of *x*?

   
   The figure shows two parallel lines, labeled m and n, which are interested by two different transversal lines, labeled r and s, to form an angle of measure “x degree” and two other angles are labeled as “65 degree” and “100 degree.”

   1. 15
   2. 25
   3. 35
   4. 65
   5. 80
2. In the figure below, *ABCD* is a trapezoid, *E* lies on inline, and angle measures are as marked. What is the measure of inline?

   
   The figure shows a trapezoid ABCD, where E lies on AD, and angle are labeled as “60 degree” for A and “30 degree” for B.

   F. 15°

   G. 25°

   H. 30°

   J. 35°

   K. 45°

Two-Dimensional Shapes

Two-dimensional shapes are the foundation of all geometry and the basis of many questions in the Geometry reporting category.

A **polygon** is any complete shape having more than two sides and only straight sides.

The **angle total** is the sum of the angles of a polygon. The angle total can be found with the equation inline, where *n* is the number of sides. For example, to find the angle total of this hexagon, use 6 for *n* in the equation:

The figure shows a polygon with six sides knows as hexagon.

image

The **perimeter** is the length of the outline of a shape. Find the perimeter by taking the sum of the side lengths.

Triangles

A **triangle** is a three-sided shape, and its angles always total 180°. With any triangle, the sum of any two side lengths is greater than the third side length.

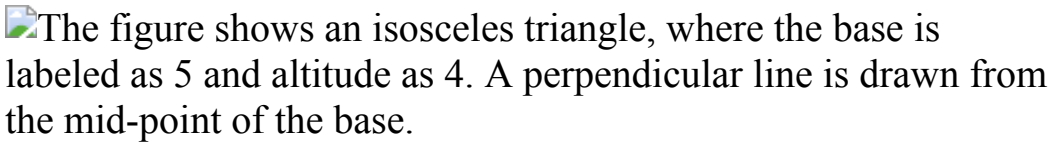
The figure shows an isosceles triangle, where the sides are labeled as “7,” “6” and “5.”

Reporting Category Quiz: Integrating Essential Skills

3. A triangle with a perimeter of 66 inches has one side that is 16 inches long. The lengths of the other two sides have a ratio of 2:3. What is the length, in inches, of the *longest* side of the triangle?
    1. 16
    2. 20
    3. 30
    4. 40
    5. 50

Area

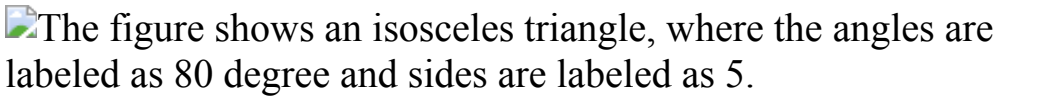
The area of any triangle can be found with the equation inline, where *b* is the base and *h* is the height, represented by a line perpendicular to the base. For example, to find the area of this triangle, place the base and height of 5 and 4 into the *b* and *h* of the equation:



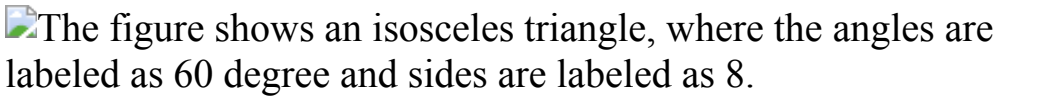
Standard Forms

There are certain standard forms of triangles.

An **isosceles triangle** is a triangle that has two identical sides and angles. If two sides are identical, two angles will also be identical.

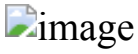


An **equilateral** triangle is a type of isosceles triangle where each side is identical and each angle is 60°.



The area of an equilateral triangle can be found with the equation inline, where *s* is a side length. For example, to find the area of this triangle, place the side length of 8 into the *s* of the equation:

A **right triangle** is a triangle where one angle is 90°, usually indicated by the right-angle symbol.



The side opposite the right angle is the longest side of the triangle and called the *hypotenuse*. Use the Pythagorean theorem to find the third side length from the two other side lengths, where *c* indicates the hypotenuse: inline. With this right triangle, place the side lengths 3 and 4 as the *a* and *b* in the Pythagorean theorem:

> The figure shows a right-angled triangle, where the base is labeled as “4,” height is labeled as “3” and hypotenuse is labeled as “c.”
>
> image

Note that although inline could mean that inline or inline, a distance or length is always positive, so use only the positive answer.

The hypotenuse of the right triangle can also appear as inline, where inline.

> The figure shows two right-angled triangles. The right-angled triangle on the left-hand side represents the base labeled as “b,” height labeled as “a” and hypotenuse labeled as “c.” The right-angled triangle on the right-hand side represents the base labeled as “b,” height labeled as “a” and hypotenuse labeled as “start root a subscript 2 plus b subscript 2 end root.”

Reporting Category Quiz: Preparing for Higher Mathematics | Geometry

4. In the figure below, *B* is on inline, *E* is on inline, inline is parallel to inline, and inline is congruent to inline. What is the measure of inline?

   > The figure shows an isosceles triangle BEF. Two parallel lines (AC and DF) are drawn, where B is on AC, E is on DF, AC is parallel to DF, and BE is congruent to BF.

   F. 35°

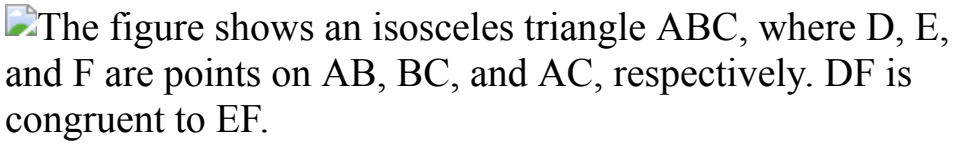
   G. 135°

H. 145°

J. 155°

K. 215°

5. In inline below, *D, E,* and *F* are points on inline, inline, and inline, respectively, and inline is congruent to inline. What is the *sum* of the measures of the angles marked *x* and *y*?

   
   The figure shows an isosceles triangle ABC, where D, E, and F are points on AB, BC, and AC, respectively. DF is congruent to EF.

   F. 40°

   G. 80°

   H. 90°

   J. 100°

   K. 130°

6. Which of the following lists gives 2 of the 3 interior angle measurements of a triangle for which the third angle measurement would be equal to 1 of the 2 given measurements?

   F. 20°, 40°

   G. 30°, 60°

   H. 40°, 100°

   J. 45°, 120°

   K. 50°, 60°

Right Triangle Ratios

Some right triangles have common ratios:

- The **3:4:5** triangle has a side-length ratio of 3:4:5 as in the previous example. As a ratio, the side lengths of the triangle could also be 6:8:10 or 9:12:15. These numbers work in the Pythagorean theorem, but they're easier to calculate as a ratio: inline.
- The **5:12:13** triangle has a side-length ratio of 5:12:13. As a ratio, the side lengths could also be inline.
- The inline triangle has angles measuring inline, inline, and inline and a side-length ratio of inline, where the hypotenuse ratio factor is 2. Note that the side lengths can appear as inline, which is consistent with the ratio.

  The figure shows two right-angled triangles. The right-angled triangle on the left-hand side represents the base labeled as “1,” height labeled as “start root 3 end root” and hypotenuse labeled as “2.” The angles are labeled as “30 degree” and “60 degree.” The right-angled triangle on the right-hand side represents the base labeled as “1/2,” height labeled as “start fraction start root 3 end root over 2 end fraction” and hypotenuse labeled as “1.” The angles are labeled as “30 degree” and “60 degree.”

- The inline triangle has angles measuring 45°, 45°, and 90° and a side-length ratio of inline, where the hypotenuse ratio factor is inline.

  The figure shows a right-angled triangle, where the base is labeled as “1,” height is labeled as “1” and hypotenuse is labeled as “start root 2 end root.” The angles are labeled as 45 degree.

Reporting Category Quiz: Integrating Essential Skills

7. A ladder is 10 ft long and reaches 8 ft up a wall, as shown below. How many feet is the bottom of the ladder from the base of the wall?

   The figure shows a right-angled triangle, where the height is labeled as “8” and hypotenuse is labeled as “10.”

F. 2

G. 3

H. 6

J. inline

K. inline

8. The ratio of the side lengths for a triangle is exactly 12:14:15. In a second triangle similar to the first, the shortest side is 8 inches long. To the nearest tenth of an inch, what is the length of the longest side of the second triangle?
    1. 11.0
    2. 10.0
    3. 9.3
    4. 6.4
    5. Cannot be determined from the given information.

Reporting Category Quiz: Preparing for Higher Mathematics | Geometry

9. Members of the fire department lean a 30-foot ladder against a building. The side of the building is perpendicular to the level ground so that the base of the ladder is 10 feet away from the base of the building. To the nearest foot, how far up the building does the ladder reach?
    1. 10
    2. 20
    3. 28
    4. 31
    5. 40
10. Which of the following sets of 3 numbers could be the side lengths, in meters, of a $30°-60°-90°$ triangle?
    1. 1, 1, 1
    2. 1, 1, inline

3. 1, inline, inline
4. 1, inline, inline
5. 1, inline, 2

11. In the figure below, *ABCD* is a square and *E, F, G,* and *H* are the midpoints of its sides. If inline = 12 inches, what is the perimeter of *EFGH,* in inches?

    F. 24

    G. inline

    H. inline

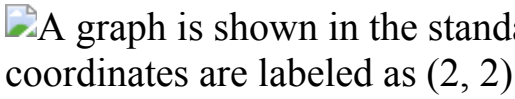
    J. inline

    K. 72

On the Coordinate Plane

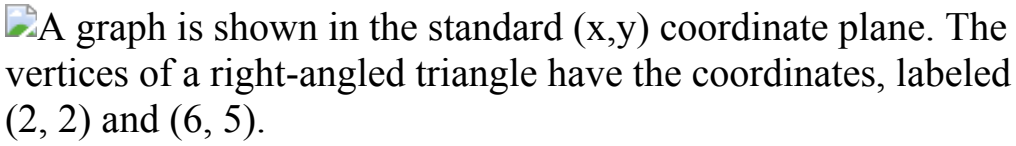
A right-triangle question can ask the distance or midpoint between two points on the coordinate plane. The points on the grid form a right triangle, and the side lengths can be determined by the (*x*, y) coordinates.

With any two points on the coordinate plane, you can find the distance and the midpoint.

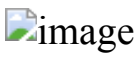


Distance

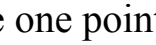
Draw right triangle connecting the points and use the Pythagorean theorem to calculate the distance. For example, to find the distance between these two points, draw a right triangle:

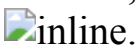


Now use the lengths of the two shorter sides and the Pythagorean theorem to find the third side, which is the hypotenuse:



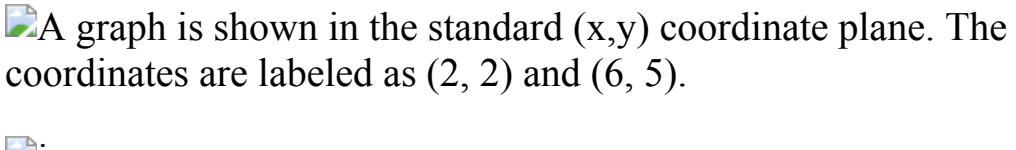
The *distance formula* automatically measures the right triangle and places the side lengths into the Pythagorean theorem. The distance formula is 

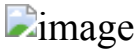
, where one point is represented by 

and the other by 

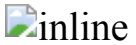
.

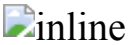
To use the distance formula to find the distance between these points, place the point coordinates into the formula:



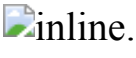


Reporting Category Quiz: Preparing for Higher Mathematics | Geometry

12. What is the distance in the standard 

coordinate plane between the points (1, 0) and (0, 5)?
    1. 4
    2. 6
    3. 16
    4. 36
    5. 

Midpoint

To find the midpoint between two points, use the *midpoint formula*, which separately measures the distances between the *x*- and *y*-coordinates. The midpoint formula is 

.

To find the midpoint between these two points, place the point coordinates into the formula:

image

The distances and midpoints between these points can be found using the respective formulas:

1. A graph is shown in the standard (x,y) coordinate plane. The coordinates are labeled as (negative 2, 3) and (4, negative 5).

2. A graph is shown in the standard (x,y) coordinate plane. The coordinates are labeled as (negative 3, negative 2) and (1, 2).

Results:

1. Distance is 10 and midpoint is (1, −1).

   image

   image

2. Distance is inline and midpoint is (−1, 0).

   image

   image

Quadrilaterals

A **quadrilateral** is any four-sided shape. Its angles always total 360°.

The figure shows a quadrilateral.

Perimeter

Find the perimeter by adding the side lengths. Certain quadrilaterals, including the square and rectangle, have opposite sides that are the same length.

Reporting Category Quiz: Integrating Essential Skills

13. Your friend shows you a scale drawing of her apartment. The drawing of the apartment is a rectangle 4 inches by 6 inches. Your friend wants to know the length of the shorter side of the apartment. If she knows that the length of the longer side of the apartment is 30 feet, how many feet long is the shorter side of her apartment?
    1. 9
    2. 20
    3. 24
    4. 30
    5. 45

Reporting Category Quiz: Preparing for Higher Mathematics | Geometry

14. A rectangular lot that measures 150 ft by 200 ft is completely fenced. What is the approximate length, in feet, of the fence?

    F. 300

    G. 350

    H. 400

    J. 700

    K. 1,400

15. In the figure shown below, each pair of intersecting line segments meets at a right angle, and all the lengths given are in inches. What is the perimeter, in inches, of the figure?

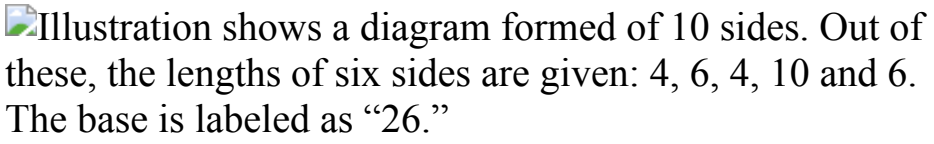


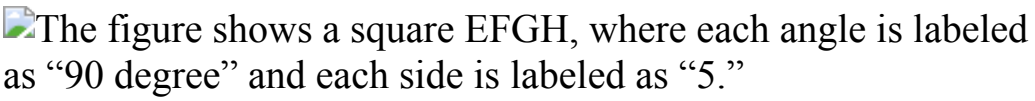
Illustration shows a diagram formed of 10 sides. Out of these, the lengths of six sides are given: 4, 6, 4, 10 and 6. The base is labeled as “26.”

1. 40
2. 52
3. 56
4. 66
5. 80

Area

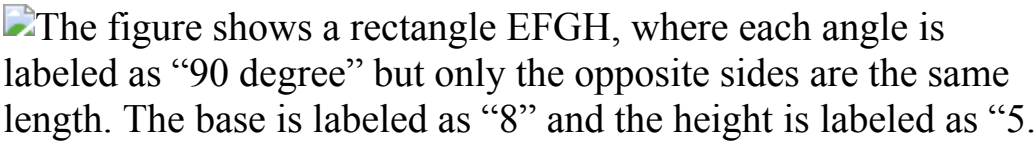
Certain quadrilaterals have different equations to find the areas. Common quadrilaterals and equations include the following:

- A **square**, where each angle is 90° and each side is the same length. Note that the drawing usually doesn't show a right-angle box in each corner: if three angles are right angles, the fourth must also be. The side length is referred to as *s*, and you can find the area by squaring the side length:

  
  The figure shows a square EFGH, where each angle is labeled as “90 degree” and each side is labeled as “5.”

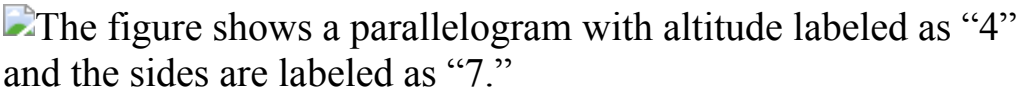
  
  image

- A **rectangle,** where each angle is 90° but only the opposite sides are the same length. The longer side is the length of the rectangle, or *l*, and the shorter side is the width, or *w*. Find the area by multiplying *l* by *w*:

  
  The figure shows a rectangle EFGH, where each angle is labeled as “90 degree” but only the opposite sides are the same length. The base is labeled as “8” and the height is labeled as “5.”

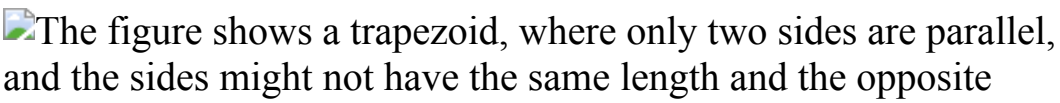
  
  image

- A **parallelogram,** where opposite angles are the same measure and opposite sides are parallel and the same length. One of the side lengths is the *base*, or *b*, and its distance from its opposite side length is the *height*, or *h*, usually represented by a dashed line with a right-angle box. Find the area by multiplying the base by the height:

  
  The figure shows a parallelogram with altitude labeled as “4” and the sides are labeled as “7.”

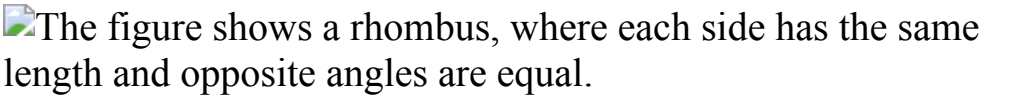
  image

- A **trapezoid,** where only two sides are parallel, and the sides might not have the same length and the opposite angles might not be the same. The parallel sides are called *bases*, or *b1* and *b2*, and the distance between them is the *height*, or *h*. If a side is perpendicular to the bases, then its length is the height; otherwise, the height can be represented by a dashed line with a right-angle box. Find the area by multiplying the average of the two bases by the height:

  
  The figure shows a trapezoid, where only two sides are parallel, and the sides might not have the same length and the opposite angles might not be the same. The base is labeled as “10” at the bottom and “8” on the top and the leg is labeled as “5.”

  image

- A **rhombus** is a type of parallelogram where each side has the same length and opposite angles are equal. Find the area by multiplying the lengths of the *diagonals*, or d_1 and d_2, and dividing by 2:

  
  The figure shows a rhombus, where each side has the same length and opposite angles are equal.

  image

These are examples of quadrilaterals where the areas can be found:

1. A square with a side length of 6

2. A rectangle with a length of 5 and a width of 3
3. A parallelogram with a base of 5 and a height of 4
4. A trapezoid with bases of 3 and 7 and a height of 6
5. A rhombus with diagonals of 6 and 8

Results:

1. 36.

   Place the side length of 5 into the equation:

2. 15.

   Place the length and width of 5 and 3 into the equation:

3. 20.

   Place the base and height of 5 and 4 into the equation:

4. 30.

   Place the bases and height of 3, 7, and 6, respectively, into the equation:

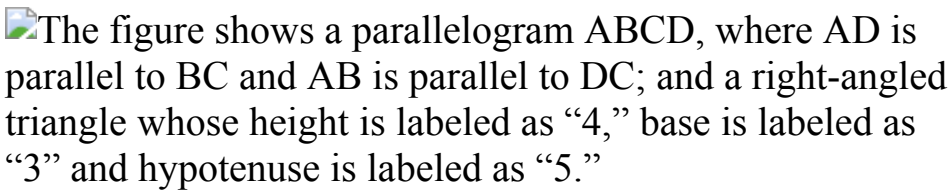
   

5. 24.

   Place the diagonals of 6 and 8 into the equation:

16. Parallelogram *ABCD,* with dimensions in inches, is shown in the diagram below. What is the area of the parallelogram, in square inches?


The figure shows a parallelogram ABCD, where AD is parallel to BC and AB is parallel to DC; and a right-angled triangle whose height is labeled as “4,” base is labeled as “3” and hypotenuse is labeled as “5.”

1. 18
2. 36
3. 39
4. 45
5. 72

17. If a rectangle measures 54 meters by 72 meters, what is the length, in meters, of the diagonal of the rectangle?

F. 48

G. 53

H. 90

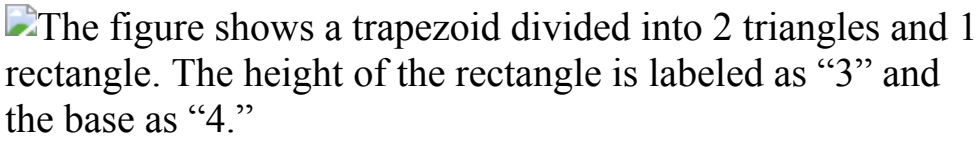
J. 126

K. 252

18. Shannon is planning to tile a rectangular kitchen countertop that is 24 inches wide and 64 inches long. She determined that 1 tile will be needed for each 4-inch-by-4-inch region. What is the minimum number of tiles that will be needed to completely cover the countertop to its edges?
    1. 44
    2. 88
    3. 96
    4. 176
    5. 384

Reporting Category Quiz: Preparing for Higher Mathematics | Geometry; Modeling

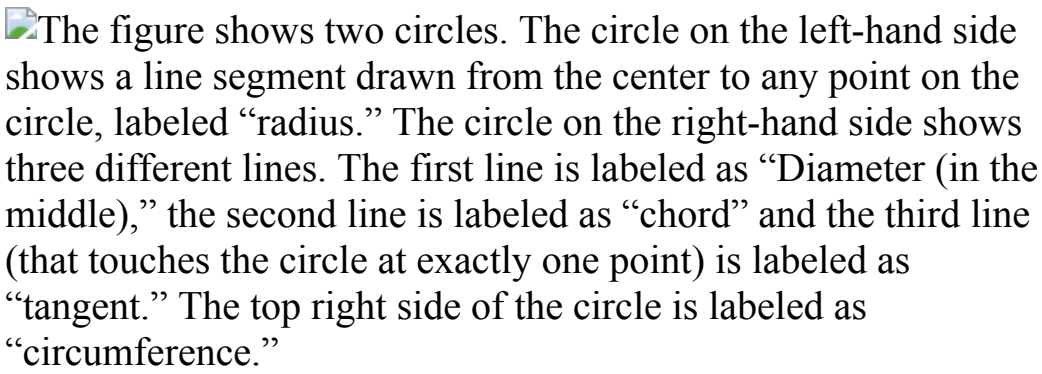
19. The trapezoid below is divided into 2 triangles and 1 rectangle. Lengths are given in inches. What is the combined area, in square inches, of the 2 shaded triangles?

    
    The figure shows a trapezoid divided into 2 triangles and 1 rectangle. The height of the rectangle is labeled as “3” and the base as “4.”

    1. 4
    2. 6
    3. 9
    4. 12
    5. 18

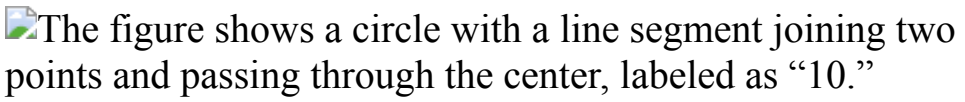
Circles

A **circle** is a shape where each point on the shape is the same distance from the center. This distance is called the **radius**. A **chord** is a line or segment that cuts across the circle. The **diameter** is a chord that cuts across the center and is the width of the circle. The **circumference** is the distance around the circle, and a **tangent** is a line or segment that touches the circle at exactly one point.


The figure shows two circles. The circle on the left-hand side shows a line segment drawn from the center to any point on the circle, labeled “radius.” The circle on the right-hand side shows three different lines. The first line is labeled as “Diameter (in the middle),” the second line is labeled as “chord” and the third line (that touches the circle at exactly one point) is labeled as “tangent.” The top right side of the circle is labeled as “circumference.”

Circumference and Area

The ratio of the circumference to the diameter is approximately 3.14, also known as pi or π. To get the circumference from the diameter, multiply the diameter by π. For example, this circle has a diameter of 10:



The radius is half the diameter, so to get the circumference from the radius, multiply the radius and π by 2:

Because $\pi \approx 3.14$, the circumference is actually closer to 31.4, but answers to ACT mathematics test questions are typically in terms of π.

To get the area of the circle, multiply π by the square of the radius:

Reporting Category Quiz: Preparing for Higher Mathematics | Geometry

20. The ratio of the radii of two circles is 4:9. What is the ratio of their circumferences?

    F. 2:3

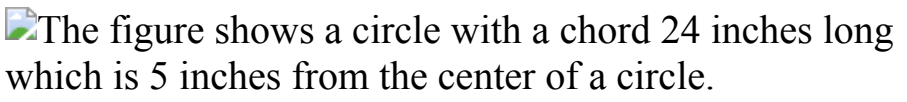
    G. 4:9

    H. 16:81

    J. 4:8π

    K. 9:18π

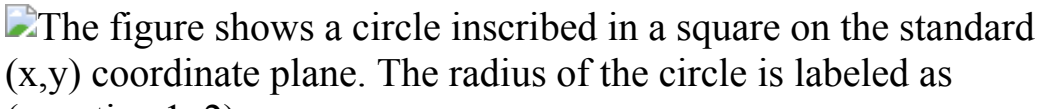
21. A chord 24 inches long is 5 inches from the center of a circle, as shown below. What is the radius of the circle, to the nearest tenth of an inch?

    
    The figure shows a circle with a chord 24 inches long which is 5 inches from the center of a circle.

    1. 29.0
    2. 24.5
    3. 16.9
    4. 13.0
    5. 10.9

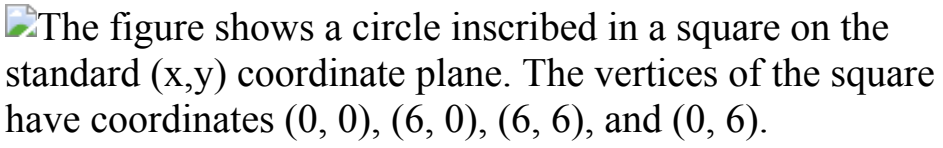
Circles on the Coordinate Plane

Circles also appear on the inline coordinate plane. The equation for a circle is inline, where *h* and *k* are the *x*- and *y*-coordinates, respectively, of the center and *r* is the radius. For example, the equation for this circle with a center of (−1, 2) and radius 4 is inline.


The figure shows a circle inscribed in a square on the standard (x,y) coordinate plane. The radius of the circle is labeled as (negative 1, 2).

Reporting Category Quiz: Preparing for Higher Mathematics | Geometry

22. A circle in the standard inline coordinate plane is tangent to the *x*-axis at 5 and tangent to the *y*-axis at 5. Which of the following is an equation to the circle?
    1. inline
    2. inline
    3. inline
    4. inline
    5. inline
23. In the standard inline coordinate below, the vertices of the square have coordinates (0, 0), (6, 0), (6, 6), and (0, 6). Which of the following is an equation of the circle that is inscribed in the square?

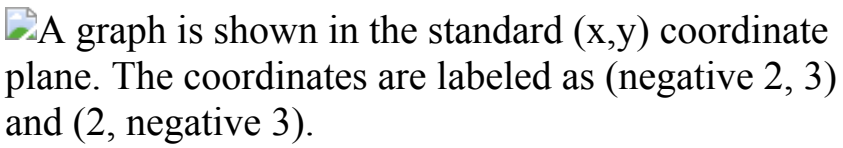

The figure shows a circle inscribed in a square on the standard (x,y) coordinate plane. The vertices of the square have coordinates (0, 0), (6, 0), (6, 6), and (0, 6).

1. inline
2. inline
3. inline
4. inline
5. inline

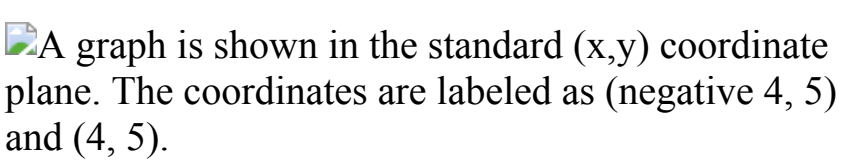
Geospatial Elements in the Coordinate Grid

Coordinate grid questions in the Algebra reporting category ask about the result of an equation, but in the Geometry reporting category they ask about the placement and movement of points, lines, or shapes. Answer these questions by adjusting the *x*- or *y*-coordinates according to these concepts:

- **Reflection** across an axis: A point, line, or shape reflected across an axis appears as if the axis is a mirror with the reflection on the opposite side.
  - A reflection across the *x*-axis has the same *x*-coordinates but negative *y*-coordinates. For example, a point with coordinates (2, 3) reflected across the *x*-axis has the coordinates (2, −3).

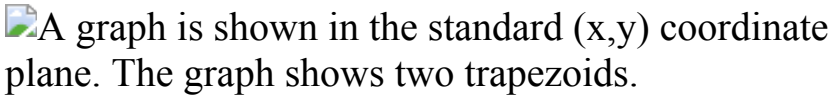
    
    A graph is shown in the standard (x,y) coordinate plane. The coordinates are labeled as (negative 2, 3) and (2, negative 3).

  - A reflection across the *y*-axis has negative *x*-coordinates but the same *y*-coordinates. For example, a point with coordinates (4, 5) reflected across the *y*-axis has the coordinates (−4, 5).

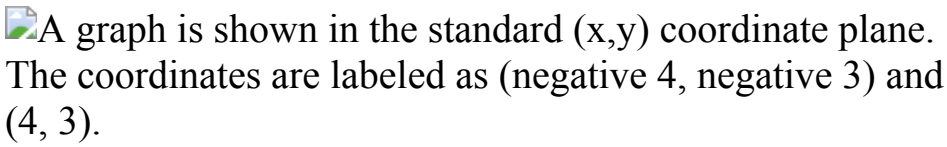
    
    A graph is shown in the standard (x,y) coordinate plane. The coordinates are labeled as (negative 4, 5) and (4, 5).

    In this example, the trapezoid is reflected across the *y*-axis. Note that the *y*-coordinates are the same but the *x*-coordinates are

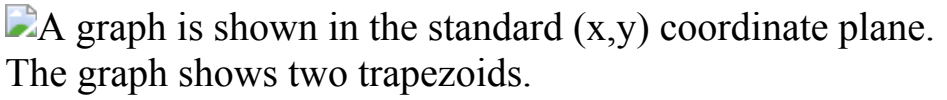
negative.


A graph is shown in the standard (x,y) coordinate plane. The graph shows two trapezoids.

- **Rotation** about the origin: A point, line, or shape rotated about the origin of the graph appears as if the origin is an anchor and the items rotate about it, like the hands of a clock, only the items rotate counterclockwise. An item that rotates 180° results in negative *x*- and *y*-coordinates. For example, a point with coordinates (4, 3) rotates 180° to coordinates (−4, −3).


A graph is shown in the standard (x,y) coordinate plane. The coordinates are labeled as (negative 4, negative 3) and (4, 3).

In this example, the trapezoid is rotated 180°. Note that the resulting shape is upside-down but otherwise unchanged and not flipped.


A graph is shown in the standard (x,y) coordinate plane. The graph shows two trapezoids.

Reporting Category Quiz: Preparing for Higher Mathematics | Geometry

24. A triangle, $\triangle ABC$, is reflected across the *x*-axis to have the image $\triangle A'B'C'$ in the standard inline coordinate plane; thus, A reflects to A'. The coordinates of point A are (c, d). What are the coordinates of point A'?

F. $(c, -d)$

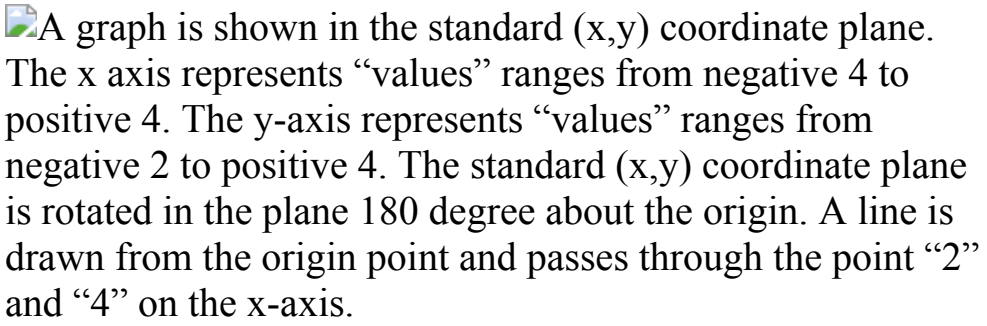
G. $(-c, d)$

H. $(-c, -d)$

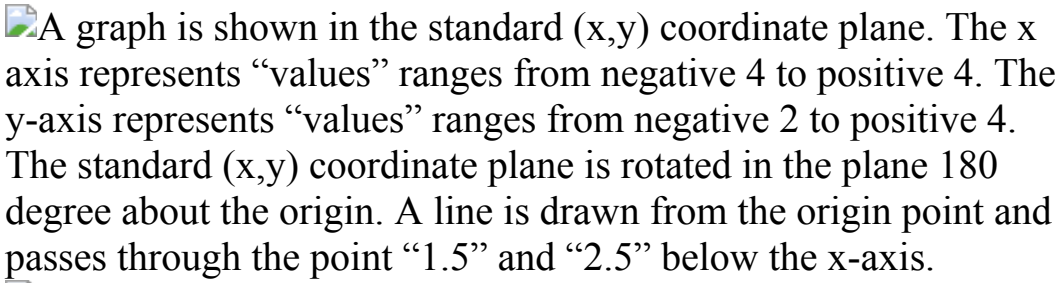
J. (d, c)

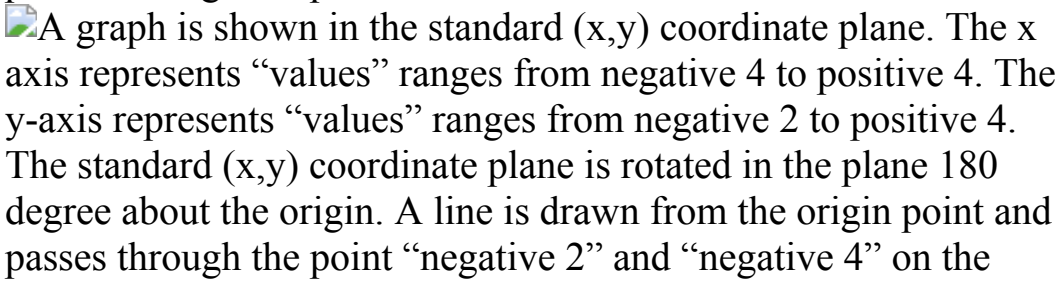
K. Cannot be determined from the given information

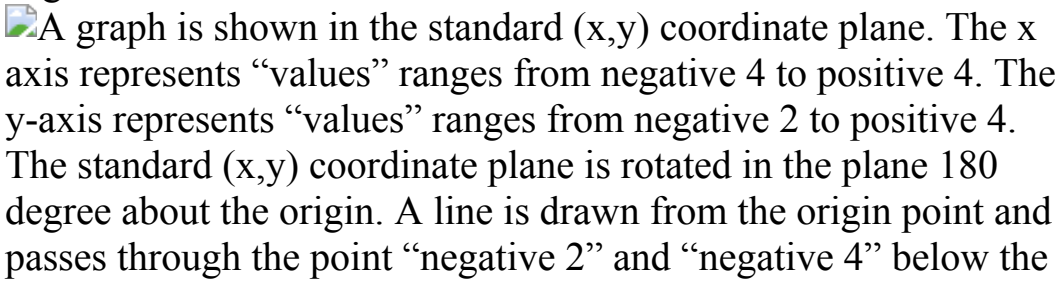
25. The graph shown in the standard inline coordinate plane below is to be rotated 180° about the origin.

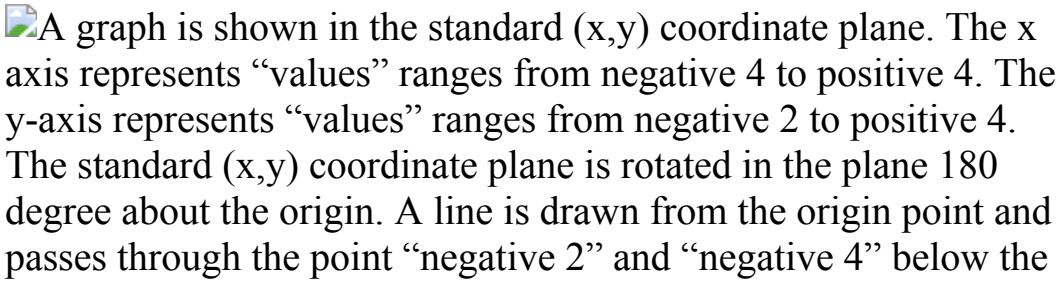


One of the following graphs is the result of this rotation. Which one is it?

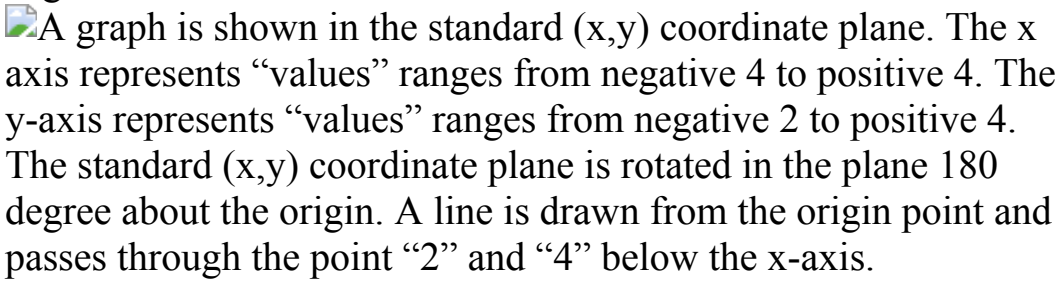
1. 

2. 

3. 

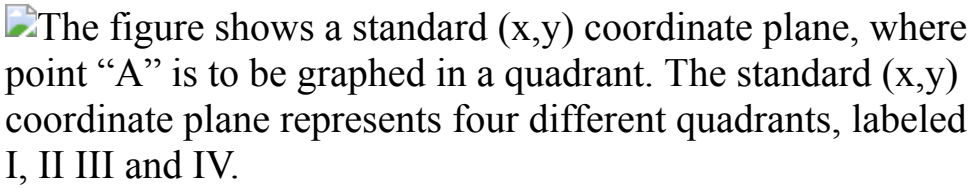


4.



5.

26. Point *A* is to be graphed in a quadrant, not on an axis, of the standard inline coordinate plane below.



If the *x*-coordinate and the *y*-coordinate of point *A* are to have opposite signs, then point *A must* be located in:

1. Quadrant II only.
2. Quadrant IV only.
3. Quadrant I or III only.
4. Quadrant I or IV only.
5. Quadrant II or IV only.

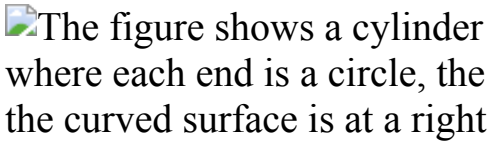
Three-Dimensional Shapes

The ACT mathematics test questions on three-dimensional shapes often ask about the volume. Following are some common shapes and formulas. If the question asks about an uncommon shape, it provides the formula that you need to solve it.

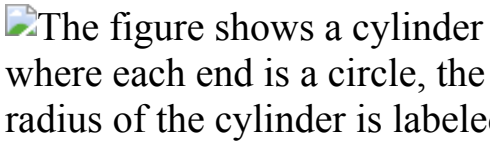
Volume

Common shapes and volume equations include the following:

- A **cylinder** or *right-circular cylinder* is like a can of soup, where each end is a circle, the circular sides are parallel, and the curved surface is at a right angle to the circular sides.

  
  The figure shows a cylinder (right-circular cylinder), where each end is a circle, the circular sides are parallel, and the curved surface is at a right angle to the circular sides.

  This cylinder has a radius of 3 and a height of 5. Find the volume by multiplying the area of a circular side, πr^2, by the height, h:

  
  The figure shows a cylinder (right-circular cylinder), where each end is a circle, the circular sides are parallel. The radius of the cylinder is labeled as 3 and the height as 5.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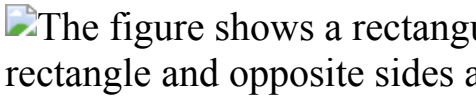
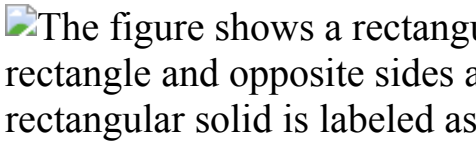
- A **rectangular solid** or *rectangular prism* is like a shoebox, where each side is a rectangle and opposite sides are parallel.

  
  The figure shows a rectangular solid, where each side is a rectangle and opposite sides are parallel.

  This rectangular solid has a length of 5, a width of 3, and a height of 2. Find the volume by multiplying these together as l, w, and h:

  
  The figure shows a rectangular solid, where each side is a rectangle and opposite sides are parallel. The length of the rectangular solid is labeled as 5, the width as 3 and the height as 2.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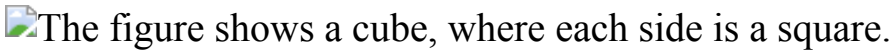
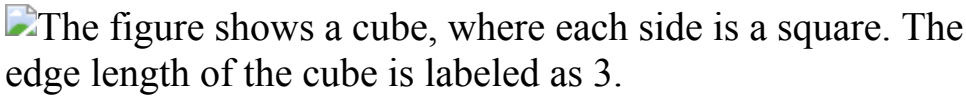
- A **cube** is a type of rectangular solid where each side is a square. Because the length, width, and height are equal, each is called an *edge*. A six-sided die is an example of a cube.

  
  The figure shows a cube, where each side is a square.

  This cube has an edge length, or *e*, of 3. Find the volume by cubing the edge length:

  
  The figure shows a cube, where each side is a square. The edge length of the cube is labeled as 3.

  
  image

The following are examples of three-dimensional shapes for which the volumes can be found:

1. A right-circular cylinder with a radius of 4 and a height of 5
2. A rectangular solid with a length, width, and height of 3, 4, and 5, respectively
3. A cube with an edge length of 4
4. A cube with an edge length of 
   inline

Results:

1. 80π.

   Place the radius and height of 4 and 5, respectively, into the equation:

   
   image

2. 60.

   Place the length, width, and height of 3, 4, and 5 into the equation:

   
   image

3. 64.

Place the edge length of 4 into the equation:

image

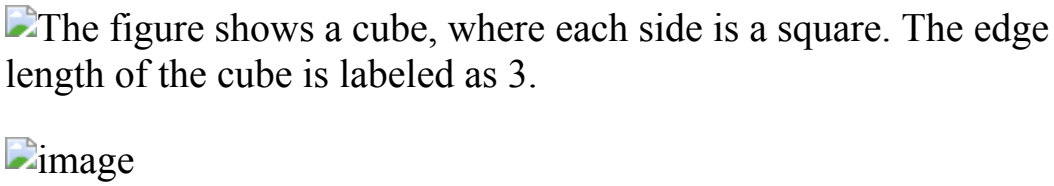
4. 5.

Place the edge length of inline into the equation:

image

Surface Area

An ACT mathematics test question can also ask for the surface area of a cube. Find this by squaring the edge and multiplying by 6:



image

The following are examples of cubes for which the surface areas can be found:

1. A cube with an edge length of 2
2. A cube with an edge length of inline

Results:

1. 24.

Place the edge length of 2 into the equation:

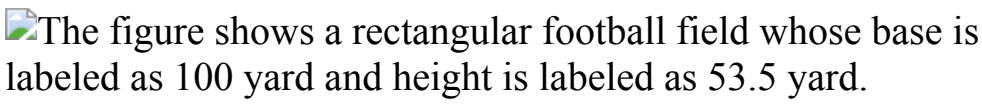
image

2. 42.

Place the edge length of inline into the equation:

image

27. After a snowstorm, city workers removed an estimated 10,000 cubic yards of snow from the downtown area. If this snow were spread in an even layer over the entire rectangular football field shown below, about how many yards deep would the layer of snow be?

    1. Less than 1
    2. Between 1 and 2
    3. Between 2 and 3
    4. Between 3 and 4
    5. More than 4
28. A formula for the volume V of a sphere is inline. If the radius of a spherical rubber ball is inline inches, what is its volume to the nearest cubic inch?
    1. 5
    2. 7
    3. 8
    4. 16
    5. 65
29. When Angela was cleaning her refrigerator, she found 2 bottles of catsup. Looking at the labels, she noticed that the capacity of the larger bottle was twice the capacity of the smaller bottle. She estimated that the smaller bottle was about inline full of catsup and the larger bottle was about inline full of catsup. She poured all the catsup from the smaller bottle into the larger bottle. Then, about how full was the larger bottle?
    1. inline full
    2. inline full
    3. inline full
    4. Completely full
    5. Overflowing

Trigonometry

Trigonometry is the study of the relationship between the side lengths and the angles of triangles. Most trigonometry questions in the ACT mathematics test are based on right triangles.

Right Triangles

A right triangle is any triangle where one angle measure is 90°. The side across from the 90° angle is the *hypotenuse*, and the other two sides are the *opposite* and *adjacent*. *Opposite* refers to *across from* and *adjacent* refers to *next to*, so the sides that are opposite or adjacent depend on the angle in question. In this triangle, for angle A, side inline is opposite and side inline is adjacent; but these switch for angle B, where side inline is opposite and side inline is adjacent.

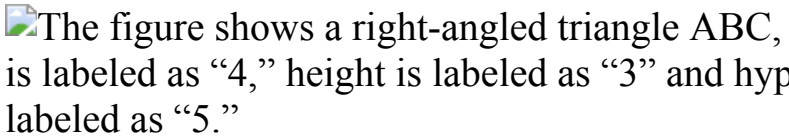

The figure shows a right-angled triangle ABC.

Sine, Cosine, and Tangent

Many trigonometry questions on the ACT mathematics test ask about the **sine**, **cosine**, and **tangent** functions of angles of a right triangle. The easiest way to remember the trigonometric relationships is with the acronym *SOH CAH TOA*, where *O*, *A*, and *H* refer to the side lengths of the *opposite*, *adjacent*, and *hypotenuse*:

- *SOH* refers to inline, or inline
- *CAH* refers to inline, or inline
- *TOA* refers to inline, or inline

In this right triangle, the sine of angle A is inline, because the side length opposite angle A is 4 and the hypotenuse is 5. The cosine of angle A is inline, because the side length adjacent to angle A is 3, and the tangent of angle A is inline.


The figure shows a right-angled triangle ABC, where the base is labeled as “4,” height is labeled as “3” and hypotenuse is labeled as “5.”

However, for angle *B*, the opposite and adjacent sides switch, so the respective sine, cosine, and tangent of angle *B* are different:

- inline
- inline
- inline

There are two trigonometric identities relevant to this concept:

- The sine of a non-right angle equals the cosine of the other non-right angle; in this case, the sine of *A* equals the cosine of *B*:

  inline

- For any angle in a right triangle, the square of the sine plus the square of the cosine equals 1:

  image

It is important to note that *SOH CAH TOA* and these two identities only apply to a right triangle.

Reporting Category Quiz: Preparing for Higher Mathematics | Geometry

30. For right triangle $\triangle RST$ shown below, what is tan *R*?

    The figure shows a right-angled triangle RST, where the base is labeled as “r,” height is labeled as “t” and hypotenuse is labeled as “s.”

    F. inline

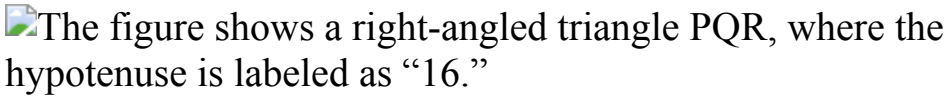
    G. inline

    H. inline

    J. inline

    K. inline

31. The hypotenuse of the right triangle $\triangle PQR$ shown below is 16 feet long. The sine of inline is inline. About how many feet long is inline?

    F. 8.0

    G. 9.6

    H. 12.4

    J. 14.3

    K. 15.4

Cosecant, Secant, and Cotangent

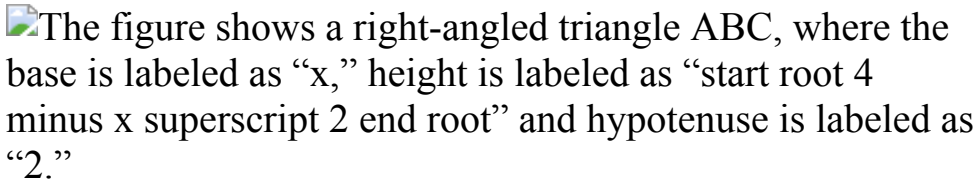
Cosecant, **secant**, and **cotangent** are the respective reciprocals of sine, cosine, and tangent. When an ACT mathematics test question involves one of these reciprocal functions, it's usually easier to convert the function back to its original form. These are the functions and their reciprocals:

- Cosecant: inline
- Secant: inline
- Cotangent: inline

The easiest way to remember that *cosecant* is the reciprocal of *sine* and *secant* is the reciprocal of *cosine* is that the *c* is the reciprocal of the *s*, and vice versa: inline and inline.

Reporting Category Quiz: Preparing for Higher Mathematics | Geometry

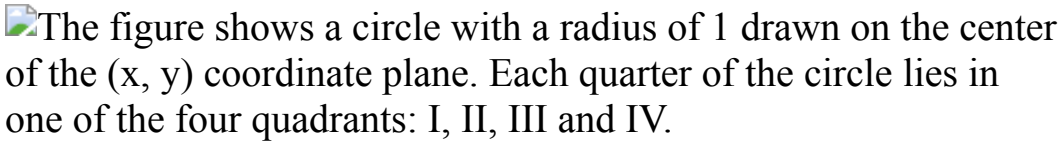
32. The lengths, in feet, of the sides of right triangle $\triangle ABC$ are as shown in the diagram below, with inline. What is the cotangent of inline, in terms of x?



1. inline
2. inline
3. inline
4. inline
5. inline

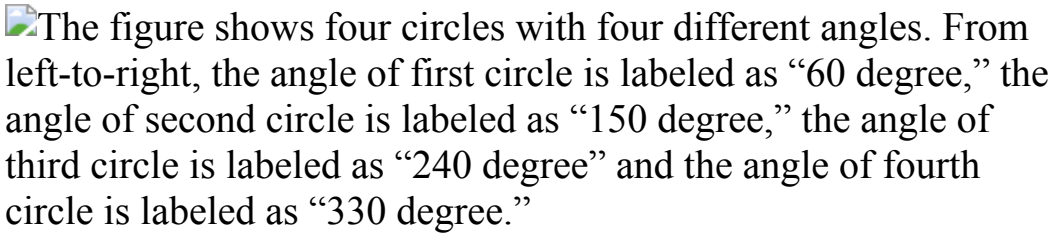
Unit Circle

The **unit circle** is a circle with a radius of 1 drawn on the center of the (*x, y*) coordinate plane. Each quarter of the circle lies in one of the four quadrants:



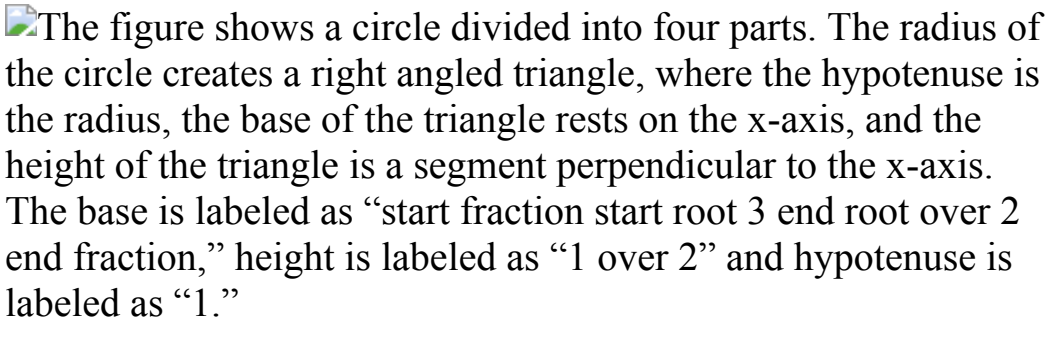
Degrees and Radians

Any radius of the circle creates an angle, measured counterclockwise from the 3:00 position on the *x*-axis. The full circle is 360°, and the angle is known as theta, or θ. For example, these drawings show θ equaling 60°, 150°, 240°, and 330°.



The radius of the circle also creates a right triangle, where the hypotenuse is the radius, the base of the triangle rests on the *x*-axis, and the height of the triangle is a segment perpendicular to the *x*-axis. For example, in this

unit circle, θ = 30°, forming a right triangle with side lengths 1, 0.5, and inline:

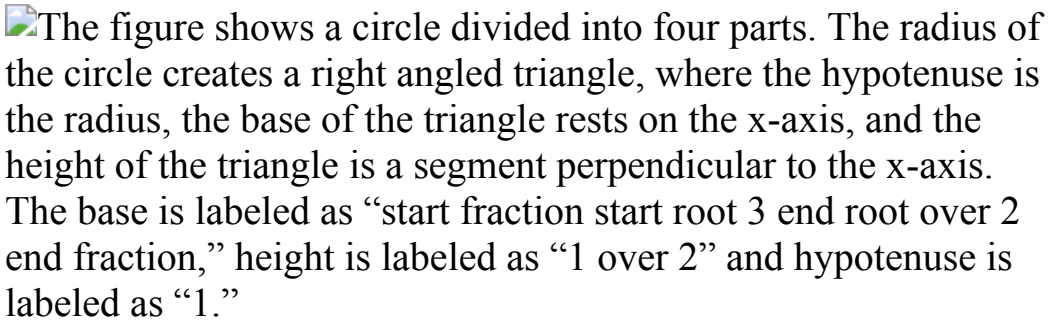


With this right triangle, you can find the sine, cosine, and tangent of θ using *SOH CAH TOA*, where the *opposite* is the height and the *adjacent* is the base. Using the previous example:

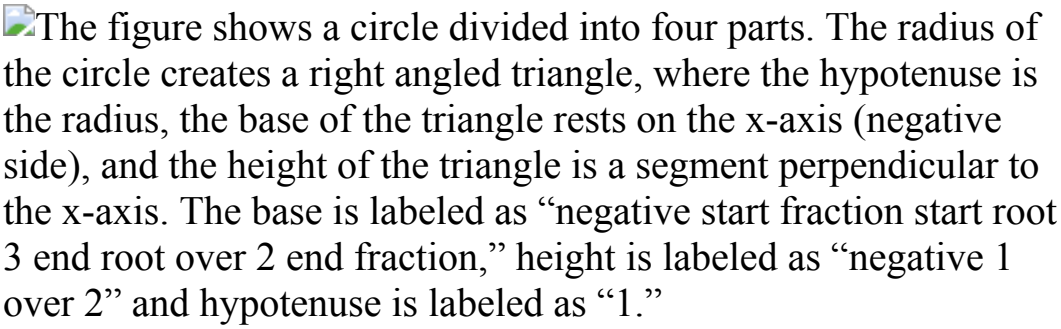
- inline
- inline
- inline

Quadrants

In this unit circle, the *xy*-coordinates of the point where the radius intersects the circle are inline.
When using *SOH CAH TOA*, the opposite and adjacent are inline and inline, respectively, which are positive values:



However, the *xy*-coordinates of this point could be negative, as in this example:



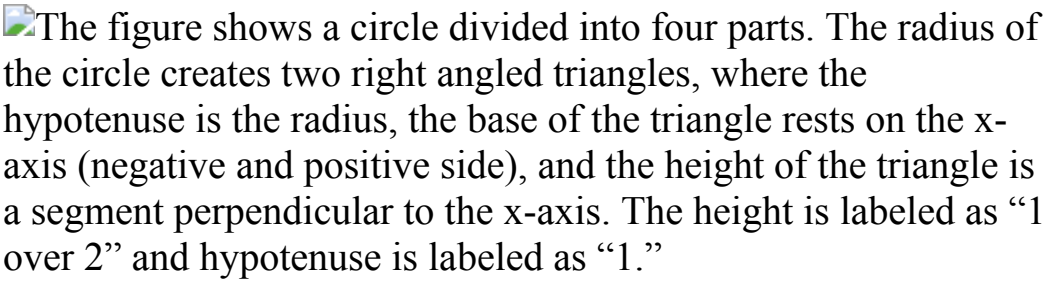
Because the *xy*-coordinates of the point are inline, the opposite and adjacent are negative, but the hypotenuse is always positive. Use these negative values to find the sine, cosine, and tangent of θ:

- inline
- inline
- inline

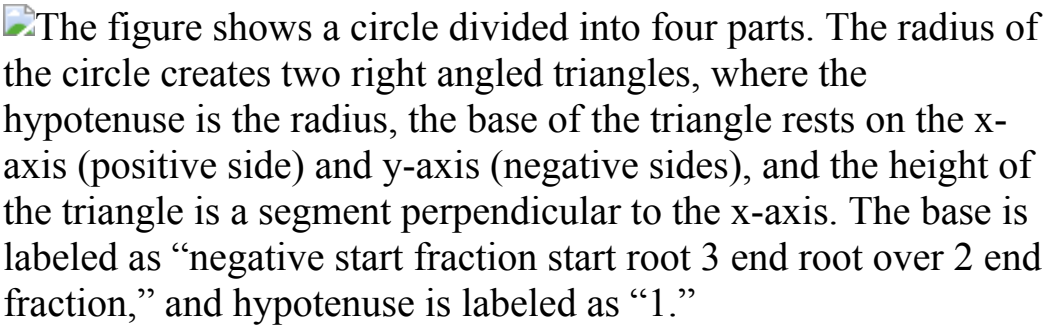
In this example, although the sine and cosine are now negative, the tangent stays positive.

Sine, cosine, and tangent can each be positive or negative, based on the quadrant. This is because although the *hypotenuse* always stays positive, the *opposite* and *adjacent* can be positive or negative:

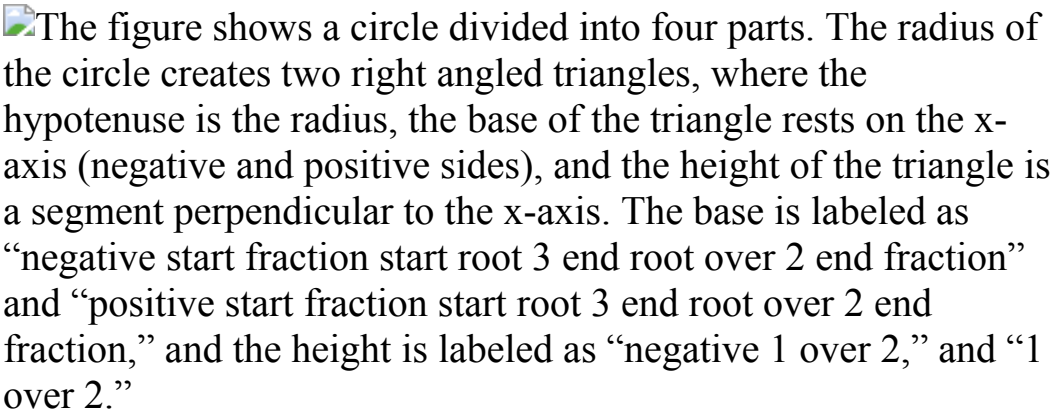
- Sine is always positive in Quadrants I and II, where the *opposite* is positive. If inline, θ could be in either Quadrant I or Quadrant II:

- Cosine is always positive in Quadrants I and IV, where the *adjacent* is positive. If inline, θ could be in either Quadrant I or Quadrant IV:


The figure shows a circle divided into four parts. The radius of the circle creates two right angled triangles, where the hypotenuse is the radius, the base of the triangle rests on the x-axis (positive side) and y-axis (negative sides), and the height of the triangle is a segment perpendicular to the x-axis. The base is labeled as “negative start fraction start root 3 end root over 2 end fraction,” and hypotenuse is labeled as “1.”

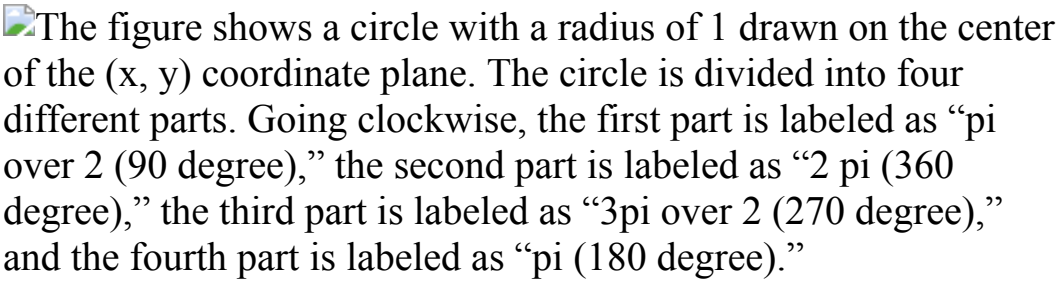
- Tangent is always positive in Quadrants I and III, where the *opposite* and *adjacent* are either both positive or both negative. If tan , θ could be in either Quadrant I or Quadrant III:

  
  The figure shows a circle divided into four parts. The radius of the circle creates two right angled triangles, where the hypotenuse is the radius, the base of the triangle rests on the x-axis (negative and positive sides), and the height of the triangle is a segment perpendicular to the x-axis. The base is labeled as “negative start fraction start root 3 end root over 2 end fraction” and “positive start fraction start root 3 end root over 2 end fraction,” and the height is labeled as “negative 1 over 2,” and “1 over 2.”

With any sine, cosine, or tangent of an angle, the angle could be in two different quadrants on the unit circle. To discern the position of the angle, you also need to know the quadrant, which the question gives you in terms of π radians.

Degrees and Radians

The unit circle angle can be expressed in either degrees or π radians, where π = 180°. The full circle is 360° or 2π radians:


The figure shows a circle with a radius of 1 drawn on the center of the (x, y) coordinate plane. The circle is divided into four different parts. Going clockwise, the first part is labeled as “pi over 2 (90 degree),” the second part is labeled as “2 pi (360 degree),” the third part is labeled as “3pi over 2 (270 degree),” and the fourth part is labeled as “pi (180 degree).”

- To convert an angle measure of degrees to radians, multiply the measure by π and divide by 180°. For example, to convert the measure 720° to radians: inline.
- To convert a measure of radians to degrees, multiply the measure by 180° and divide by π. For example, to convert the measure 3π to degrees: inline.

An ACT mathematics test question can use π radians to tell you the quadrant:

- inline tells you that θ is in Quadrant I.
- inline tells you that θ is in Quadrant II.
- inline tells you that θ is in Quadrant III.
- inline tells you that θ is in Quadrant IV.

Reporting Category Quiz: Preparing for Higher Mathematics | Geometry

33. If inline and inline, then inline

    F. inline

    G. inline

    H. inline

    J. inline

    K. inline

34. If the value, to the nearest thousandth, of cos θ is –0.385, which of the following could be true about θ?
    1. inline

2. inline
3. inline
4. inline
5. inline

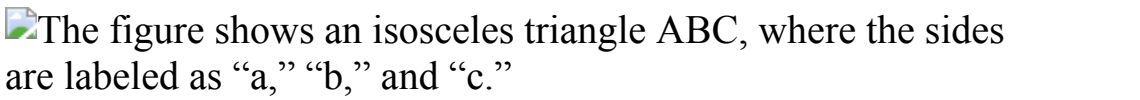
Laws of Sines and Cosines

The laws of sines and cosines apply to all triangles, including non-right triangles.

Law of Sines

The **law of sines** states that given a triangle with angles *A*, *B*, and *C*, and opposite sides *a*, *b*, and *c*, respectively, the ratios of the sine of each angle to its opposite side length are equal:

image



Although the law of sines features three ratios, you usually need to use only two. If one angle is 90°, simplify the ratio by replacing sin 90° with 1, because sin 90° = 1.

Law of Cosines

The **law of cosines** is a function of three side lengths and one angle measure:

image

Square root both sides to show the value of *c*:

image

Note that cos 90° = 0, so when $C = 90°$, as in a right triangle, $2ab\cos C$ goes away, and you're left with the Pythagorean theorem: inline or

inline. Don't worry about memorizing the law of cosines, because the ACT mathematics test question usually provides it for you, and all you do is place the side lengths *a* and *b* and the angle measure *C* from the triangle in the equation.

Chapter 5: Quiz Answers

1. **The correct answer is C.** The angle supplemental to 100° is 80°, which along with the 65° angle form a triangle whose third angle, opposite the *x*°, is 35°.
2. **The correct answer is K.** The shape is a trapezoid, so segments inline and inline are parallel. If inline, inline. The drawing tells you that inline, so inline, making inline. inline and inline are supplemental to the angle in question, meaning all three total 180°. You now know that one is 30° while the other is 105°, so the mystery angle is 45°.
3. **The correct answer is C.** If the perimeter is 66 inches and one side is 16 inches, then the other two sides total 50 inches. Find the actual number of inches from the ratio of 2:3 by using the equation:

   image

4. **The correct answer is H.** If inline is parallel to inline, then inline, which is inline, is equal to inline, also 35°. If inline is congruent to inline, then the triangle is isosceles, making angles inline and inline each 35°. The angle in question is supplementary to inline, so its measure is 145°.
5. **The correct answer is H.** The triangles are congruent: inline is congruent to inline, and they share side inline, so by the Pythagorean theorem, the third sides inline and inline are congruent. Therefore the measures of the two smaller angles inline and inline are the same, as are the two larger angles, inline and inline. One small angle, such as *x*°, plus one large angle, such as *y*°, totals 90°.
6. **The correct answer is H.** Find two angle measures where if one were added twice, the three total 180°. Add 40° twice for the correct angle total: 40° + 40° + 100° = 180°.

7. **The correct answer is H.** This is a 3:4:5 triangle multiplied by 2: 2(3 : 4 : 5) = 6 : 8 : 10. You could also use the Pythagorean theorem:

8. **The correct answer is B.** Compare the ratio of the shortest sides to the ratio of the longest sides:

9. **The correct answer is C.** Draw a right triangle with a base of 10 and hypotenuse of 30, and use the Pythagorean theorem to the height:

   Note the key word *nearest* in the question.

10. **The correct answer is E.** The side lengths of a 30°−60°−90° triangle are in the ratio  . Note that the side lengths don't have to be in that order.
11. **The correct answer is G.** Each triangle in the corner of the square is a 45°−45°−90° triangle with a side-length ratio of  . Take triangle  : if  , then  , and hypotenuse  . The four sides of the square are equal, so the perimeter of *EFGH* is  .
12. **The correct answer is E.** Place the coordinates into the distance formula:

13. **The correct answer is B.** Compare the ratio of the longer side to the ratio of the shorter side:

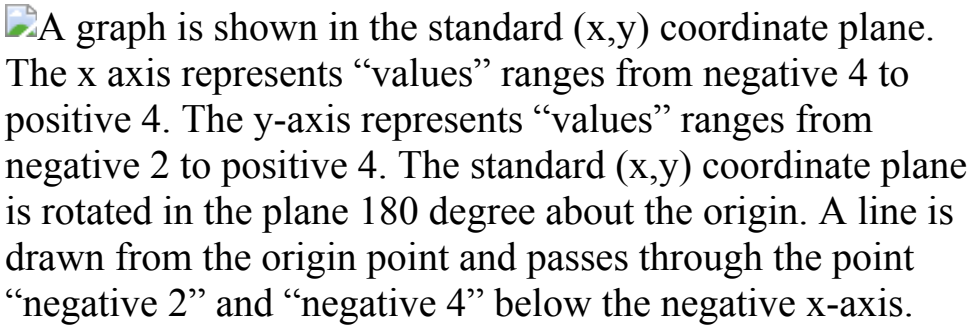
14. **The correct answer is J.** The perimeter is the sum of all four sides, and the opposite sides are equal. This means two sides measure 150 ft each and two other sides measure 200 ft each: 2(150) + 2(200) = 700 ft.

15. **The correct answer is E.** The bottom is 26 inches; therefore, the segments of the top also total 26 inches. The segments of the left side total 14 inches, therefore the segments of the right side also total 14 inches: 2(26) + 2(24) = 80. The 10 on the top and the 6 on the right are extraneous information.
16. **The correct answer is B.** The base is the sum of 3 and 6 at the bottom of the drawing. Multiply this base by the height: 9 × 4 = 36.
17. **The correct answer is H.** The diagonal of the rectangle creates a right triangle with side lengths of 54 and 72 meters. You could use the Pythagorean theorem, but this is a 3:4:5 triangle multiplied by 18: 18 (3 : 4 : 5) = 54 : 72 : 90.
18. **The correct answer is C.** If each tile is 4 inches square, then 6 tiles line the 24-inch width and 16 tiles line the 64-inch width. Shannon now has a grid of 6 by 16 tiles: 6 × 16 = 96.
19. **The correct answer is B.** Subtract the area of the rectangle from the area of the trapezoid. In square inches, the rectangle is 4 × 3 = 12 and the trapezoid is inline. Subtract these for a difference of 6.
20. **The correct answer is G.** The circumference of a circle can be found with $2\pi r$, so the circumferences of circles with radii 4 and 9 are 8π and 18π, respectively. Set these as a fraction and reduce: inline.
21. **The correct answer is D.** Isolate the right triangle, where the height is 5, the base is 12, and the hypotenuse is r. You could use the Pythagorean theorem, but this is one of the right triangle ratios with side lengths of 5:12:13.
22. **The correct answer is D.** A circle tangent to the x-axis at 5 and the y-axis at 5 exists in Quadrant I:

    A graph is shown in the standard (x,y) coordinate plane. The x axis represents “values” ranges from 5 to 10. The y-axis represents “values” ranges from 5 to 10. A circle is drawn, which lies on the x-axis and touches the coordinate (5, 5).

    The circle has a center at coordinates (5, 5) and a radius of 5, so its equation is inline.

23. **The correct answer is A.** The circle has a center at coordinates (3, 3) and a radius of 3, so its equation is inline.
24. **The correct answer is F.** A point reflected across the *x*-axis has the same *x*-coordinate but the negative of the *y*-coordinate. The point (*c*, *d*) reflected across the *x*-axis thus has the coordinates (*c*, −*d*).
25. **The correct answer is D.** An image rotated 180° appears upside-down on the opposite side of the graph. Note also that the original graph has an endpoint at coordinates (0, 0), which is the "anchor" of the rotation; thus, the rotated image not only is upside-down but also has an endpoint at those coordinates.

    
    A graph is shown in the standard (x,y) coordinate plane. The x axis represents "values" ranges from negative 4 to positive 4. The y-axis represents "values" ranges from negative 2 to positive 4. The standard (x,y) coordinate plane is rotated in the plane 180 degree about the origin. A line is drawn from the origin point and passes through the point "negative 2" and "negative 4" below the negative x-axis.

26. **The correct answer is E.** A point with coordinates having opposite signs, such as (−2, 2), is in Quadrant II, and (2, −2) is in Quadrant IV.
27. **The correct answer is B.** This is a rectangular solid with yards as units. You're given the length and width of the football field along with the volume of snow, so use the equation to find the height, which in this case is the depth:

    image

28. **The correct answer is C.** Place the radius of inline, or inline, for *r* in the equation:

    image

29. **The correct answer is C.** inline of a smaller bottle is less than inline of a larger bottle, so the catsup from the smaller bottle can be added to the larger bottle without filling or overflowing it. The larger bottle would then be more than inline full, so it couldn't be inline or inline full: it could only be inline full.

30. **The correct answer is G.** Tangent is *opposite* over *adjacent*. The sides opposite and adjacent to inline are *r* and *t*, respectively, so inline.
31. **The correct answer is G.** Sine is *opposite* over *hypotenuse*. The sine of inline is inline, so the lengths of the base and hypotenuse are in the ratio 3:5. Set this ratio as a fraction equal to the hypotenuse of 16 and base of *x*:

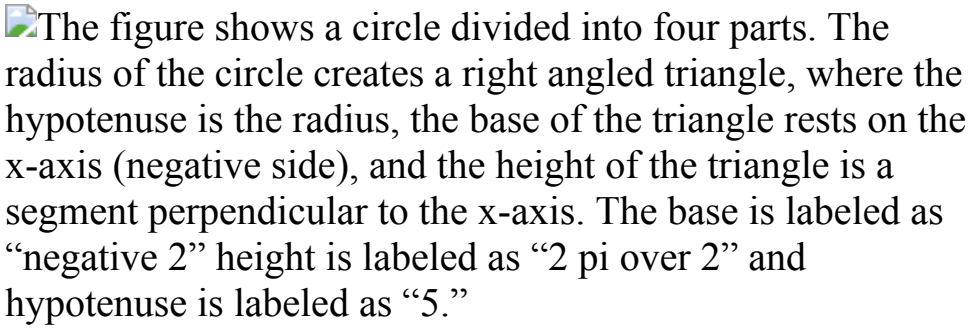
    image

32. **The correct answer is E.** Tangent is *opposite* over *adjacent*. The sides opposite and adjacent to inline are *x* and inline, respectively, so inline. Take the reciprocal for the cotangent: inline.
33. **The correct answer is J.** inline places θ in Quadrant III. inline tells you that the ratio of the side length opposite θ to the hypotenuse is 3:5, so the side length adjacent to θ is 4.

    The figure shows a circle divided into four parts. The radius of the circle creates a right angled triangle, where the hypotenuse is the radius, the base of the triangle rests on the x-axis (negative side), and the height of the triangle is a segment perpendicular to the x-axis. The base is labeled as "negative 4," height is labeled as "negative 3" and hypotenuse is labeled as "5."

    Because these are in Quadrant III, the opposite and adjacent side lengths are negative: inline.

34. **The correct answer is D.** If inline, then it could be written as inline or inline. Cosine is *adjacent* over *hypotenuse*, and hypotenuse is always positive, so the adjacent must be negative, placing θ in Quadrant II or III. The answer choices include only Quadrants I and II, so θ is in Quadrant II. Now draw a picture with the adjacent and hypotenuse in the ratio of 2:5:



inline, and θ is between inline and inline.

Chapter 6:
Statistics and Probability

Questions in the **Statistics and Probability** reporting category ask about probabilities of events and interpretation of data distributions, data collection methods, and data relationships. They also ask about averages and medians.

Probability

Probability refers to the likelihood of an event occurring. In its simplest form, it's a fraction with the number of desired outcomes as the numerator and the total possible outcomes as the denominator:  .

- If a box of 9 pens contains 2 red pens, the probability that you pull a red pen is  .

The probability that something will *not* occur is 1 minus the probability that it does occur.

- The probability that you will not pull a red pen is  . This is also true because 7 of the pens aren't red.

The probabilities of one outcome *or* another outcome is the sum of each independent probability.

- If the box has 2 red pens and 3 blue pens, the probability that you pull a red *or* a blue pen is  .

The probabilities of one outcome *and* another outcome is the product of each independent probability.

- The probability that you pull a red pen today is  . If you place it back in the box, the probability that you pull a red pen tomorrow is

still inline. The probability that you pull a red pen on *both* days is inline.

Probability is always a number between 0 and 1. A probability of 0 means that the event will not occur, and a probability of 1 means that the event will definitely occur.

- If a box of 10 sodas contains *only* diet sodas, then the probability that you pull a regular soda is inline, or 0.
- From the same box, the probability that you pull a diet soda is inline, or 1.

Reporting Category Quiz: Preparing for Higher Mathematics | Statistics and Probability

1. If a marble is randomly chosen from a bag that contains exactly 8 red marbles, 6 blue marbles, and 6 white marbles, what is the probability that the marble will NOT be white?
    6. inline
    7. inline
    8. inline
    9. inline
    10. inline

Sets of Numbers

The ACT mathematics test questions can ask for a simple analysis on a **set of numbers**, such as inline.

Average/Arithmetic Mean

The **average**, also called the **mean** or **arithmetic mean**, of a set of numbers refers to the sum of the terms divided by the number of terms: inline. For example, to take the average of the 5 numbers inline, divide the sum by 5:

image

The question can provide the average but ask for one of the numbers in the set. For example, the average of inline is 8. In this case, set up the equation with *x* as the missing number and solve for *x*:

image

These are examples of averages with one of the numbers missing:

1. The set of numbers inline has an average of 7.
2. The set of numbers inline has an average of 3.

Results:

1. 5.
   - Set up the equation with *x* as the missing number:

     image

2. 11.
   - Set up the equation with *x* as the missing number:

     image

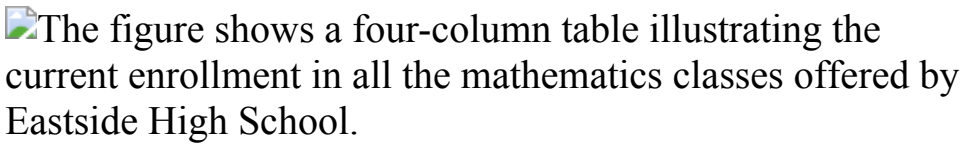
Reporting Category Quiz: Integrating Essential Skills

2. A certain type of notebook costs $2.50 before sales tax is added. When you buy 9 of these notebooks you receive 1 additional notebook free. What is the average cost per notebook for the 10 notebooks before sales tax is added?
   1. $2.78
   2. $2.50
   3. $2.30
   4. $2.25
   5. $2.15
3. Kaya drove 200 miles in 5 hours of actual driving time. By driving an average of 10 miles per hour faster, Kaya could have saved how many hours of actual driving time?
   1. inline

2. inline
3. inline
4. 1
5. 4

Reporting Category Quiz: Preparing for Higher Mathematics | Statistics and Probability

4. The following chart shows the current enrollment in all the mathematics classes offered by Eastside High School.

   
   The figure shows a four-column table illustrating the current enrollment in all the mathematics classes offered by Eastside High School.

   What is the average number of students enrolled per section in Algebra I?

   6. 24
   7. 25
   8. 26
   9. 27
   10. 29
5. A company earned a profit of $8.0 million each year for 3 consecutive years. For each of the next 2 years the company earned a profit of $9.0 million. For this 5-year period, what was the company's average yearly profit, in millions of dollars?
   6. 8.2
   7. 8.25
   8. 8.4
   9. 8.5
   10. 8.6

Median and Mode

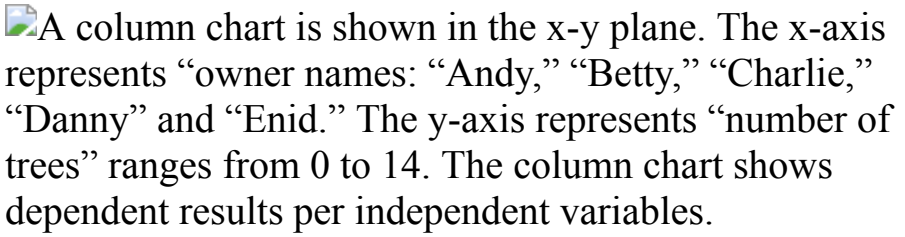
The **median** of a set of numbers is the middle value in the set. For example, the median of the set inline is 6. If the numbers are out of order, place them in order before taking the middle number. If there are two middle numbers, such as inline, take the average of the two middle numbers: inline.

The **mode** of a set of numbers is the most commonly occurring value in the set. For example, the mode of inline is 8. If the set of numbers has two values that occur the most number of times, it is called **bimodal**, meaning it has two modes. The modes of inline are 5 and 9.

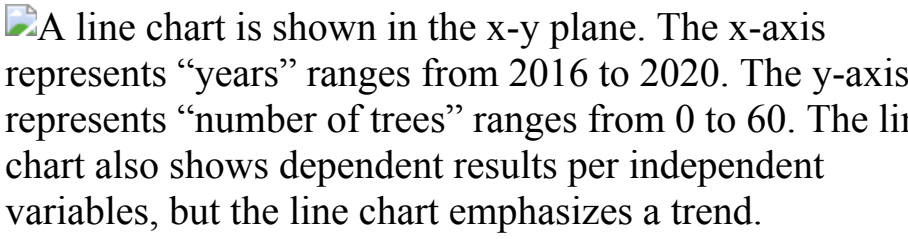
Charts

The ACT mathematics test features a few simple charts in the Statistics and Probability reporting category. The charts appearing in the ACT science test are far more extensive, but the following are the charts you're likely to see in the ACT math test:

- The **column chart** shows dependent results per independent variables:

  
  A column chart is shown in the x-y plane. The x-axis represents “owner names: “Andy,” “Betty,” “Charlie,” “Danny” and “Enid.” The y-axis represents “number of trees” ranges from 0 to 14. The column chart shows dependent results per independent variables.

- The **line chart** also shows dependent results per independent variables, but the line chart emphasizes a trend:

  
  A line chart is shown in the x-y plane. The x-axis represents “years” ranges from 2016 to 2020. The y-axis represents “number of trees” ranges from 0 to 60. The line chart also shows dependent results per independent variables, but the line chart emphasizes a trend.

- The **bar chart** is like the column chart, only the *x*- and *y*-axes are reversed:


A bar chart is shown in the x-y plane. The x-axis represents “number of trees” ranges from 0 to 14. The y-axis represents “owner names: “Andy,” “Betty,” “Charlie,” “Danny” and “Enid.” The chart shows dependent results per independent variables.

The **pie chart** shows each value as a slice of the pie having either a number or a percent of the total. The entire chart is either the total number of items or 100%:


The figure shows two different pie charts. The pie-chart on the left-hand side is divided into five parts. Going clockwise, the first part is labeled as “Enid (9),” the second part is labeled as “Danny (5),” the third part is labeled as “Charlie (11),” the fourth part is labeled as “Betty (13)” and the fifth part is labeled as “Andy (12).” The pie-chart on the right-hand side is divided into five parts. Going clockwise, the first part is labeled as “Enid (18),” the second part is labeled as “Danny (10),” the third part is labeled as “Charlie (22),” the fourth part is labeled as “Betty (26)” and the fifth part is labeled as “Andy (24).”

Reporting Category Quiz: Preparing for Higher Mathematics | Statistics and Probability

6. The graph below shows the number of cars assembled last year in 4 cities, to the nearest 5,000 cars. According to the graph, what fraction of the cars assembled in all 4 cities were assembled in Coupeville?


The figure shows a two-column table illustrating the number of cars assembled last year in 4 cities, to the nearest 5,000 cars. The column heads are “city” and “cars assembled.”

1. 
inline
2. 
inline
3. 
inline
4. 
inline
5. 
inline

7. Douglas wants to draw a circle graph showing the favorite colors of his friends. When he polled his friends asking each their favorite color, 25% of his friends said red; 30% of his friends said blue; 20% of his friends said green; 10% of his friends said purple; and the remaining friends said colors other than red, blue, green, and purple. The colors other than red, blue, green, and purple will be grouped together in an Other sector. What will be the degree measure of the Other sector?
    1. inline
    2. inline
    3. inline
    4. inline
    5. inline

Chapter 6: Quiz Answers

1. **The correct answer is K.** The probability that the marble is not white is 1 minus the probability that it is white. There are 6 white and 20 total marbles:

    image

2. **The correct answer is D.** Divide the total cost of the notebooks by the number of notebooks:

    image

3. **The correct answer is D.** First find Kaya's average speed in miles per hour by dividing the total miles driven by the total time:

    image

    Next, increase her speed by 10 miles an hour, and divide the total miles driven by the new speed:

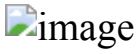
    image

If Kaya drove 10 miles per hour faster, she could have made the trip in 4 hours instead of 5, saving 1 hour.

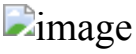
4. **The correct answer is H.** Divide the total number of Algebra I students by the number of sections:

5. **The correct answer is H.** Divide the total profit by the number of years:

6. **The correct answer is B.** Place the number of cars assembled in Coupeville over the total number of cars assembled in all 4 cities. Though you could use the car icons to count the actual number of cars, you could also just count the number of icons:
   - Total car icons: 10
   - Coupeville car icons:

7. **The correct answer is B.** The total of all the sectors is 100%, which is  on the circle graph. First add the percents given and subtract the sum from 100:

   Next, convert this to a degree measure on the circle graph:

Chapter 7:
Practice Questions

Following is a pool of 160 practice ACT mathematics test questions. This includes the practice questions from the text along with additional questions directly from the ACT.

You may use a calculator, but these problems can be solved without one. The questions challenge your ability to understand and work with the math concepts, not crunch numbers.

Answers and explanations are in chapter 8.

1. On level ground, a vertical rod 12 feet tall casts a shadow 4 feet long, and at the same time a nearby vertical flagpole casts a shadow 12 feet long. How many feet tall is the flagpole?

   A. 4

   B. 8

   C. 12

   D. 20

   E. 36

2. Kalino earned 85, 95, 93, and 80 points on the 4 tests, each worth 100 points, given so far this term. How many points must he earn on his fifth test, also worth 100 points, to average 90 points for the 5 tests given this term?

   F. 87

   G. 88

   H. 90

   J. 92

   K. 97

3. If x =−5, what is the value of inline ?

A. −6

B. −4

C. 4

D. inline

E. 19

4. Kaya ran inline miles on Monday and inline miles on Tuesday. What was the total distance, in miles, Kaya ran during those 2 days?

F. inline

G. inline

H. inline

J. inline

K. inline

5. Consider the 3 statements below to be true.

All insects that are attracted to honey are ants.

Insect I is not an ant.

Insect J is attracted to honey.

Which of the following statements is necessarily true?

A. Insect I is an ant not attracted to honey.

B. Insect I is an ant attracted to honey.

C. Insect I is attracted to honey.

D. Insect J is not attracted to honey.

E. Insect J is an ant.

6. What is the value of the expression inline when inline and inline ?

F. −2

G. 2

H. inline

J. $2i$

K. inline

7. Tickets for a community theater production cost $6 each when bought in advance and $8 each when bought at the door. The theater group's goal is at least $2,000 in ticket sales for opening night. The theater group sold 142 opening-night tickets in advance. What is the minimum number of tickets they need to sell at the door on opening night to make their goal?

   A. 143

   B. 144

   C. 192

   D. 250

   E. 357

8. Mark and Juanita own a sandwich shop. They offer 3 kinds of bread, 5 kinds of meat, and 3 kinds of cheese. Each type of sandwich has a combination of exactly 3 ingredients: 1 bread, 1 meat, and 1 cheese. How many types of sandwiches are possible?

   F. 11

   G. 15

   H. 30

   J. 45

   K. 120

9. If inline, then x = ?

   A. inline

   B. inline

   C. inline

   D. inline

   E. inline

10. In the figure below, *A, D, C,* and *E* are collinear. inline, inline , and inline are all the same length, and the angle measure of $\angle ABD$ is as marked. What is the degree measure of $\angle BCE$?

    The figure shows an isosceles triangle BDE inscribe in an isosceles triangle ABC, where ADB and BDC are a linear pair.

    F. 50°

    G. 100°

    H. 105°

    J. 130°

    K. 160°

11. If inline, then $f(-2) = ?$

    A. −54

    B. −18

    C. 18

    D. 36

    E. 38

12. What is the least common multiple of 30, 20, and 70 ?

    F. 40

    G. 42

    H. 120

    J. 420

    K. 42,000

13. While doing a problem on his calculator, Tom meant to divide a number by 2, but instead he accidentally multiplied the number by 2. Which of the following calculations could Tom then do to the result on the calculator screen to obtain the result he originally wanted?

    A. Subtract the original number

B. Multiply by 2

C. Multiply by 4

D. Divide by 2

E. Divide by 4

14. The 8-sided figure below is divided into 5 congruent squares. The total area of the 5 squares is 125 square inches. What is the perimeter, in inches, of the figure?


Illustration shows 8-sided figure, divided into 5 congruent squares. The total area of the 5 squares is 125 square inches.

F. 25

G. 60

H. 80

J. 100

K. 125

15. Hai has $100 available to buy USB drives to back up data for his business computers. Each USB drive has a price of $8, and Hai will pay a sales tax of 7% of the total price of the USB drives. What is the maximum number of USB drives Hai can buy?

A. 11

B. 12

C. 13

D. 14

E. 15

16. A certain computer performs 1.5×10^8 calculations per second. How many seconds would it take this computer to perform 6.0×10^{16} calculations?

F. $2.5 \times 10{-}^9$

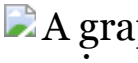
G. 9.0×10^0

H. 4.0×10^2

J. 4.0×10^8

K. 9.0×10^{24}

17. One of the following is an equation of the linear relation shown in the standard (x,y) coordinate plane below. Which equation is it?


A graph is shown in the standard (x,y) coordinate plane. The x axis represents “values” ranges from negative 4 to positive 2. The y-axis represents “values” ranges from negative 4 to positive 12. A diagonal line is drawn which crosses the coordinate (x,y).

A. $y = 5x$

B. $y = 2x$

C. $y = 5x + 2$

D. $y = 2x - 5$

E. $y = 2x + 5$

18. A square is circumscribed about a circle of 7-foot radius, as shown below. What is the area of the square, in square feet?


The figure shows a circle of 7-foot inscribed in a square.

F. 49

G. 56

H. 98

J. 49π

K. 196

19. Two workers were hired to begin work at the same time. Worker A’s contract called for a starting salary of $20,000 with an increase of $800 after each year of employment. Worker B’s contract called for a starting salary of $15,200 with an increase of $2,000 after each year of employment. If x represents the number of full years’ employment (that is, the number of yearly increases each worker has received), which of the following equations could be solved to determine the number of years until B’s yearly salary equals A’s yearly salary?

A. $20{,}000 + 800x = 15{,}200 + 2{,}000x$

B. $20{,}000 + 2{,}000x = 15{,}200 + 800x$

C. $(20{,}000 + 800)x = (15{,}200 + 2{,}000)x$

D. $(2{,}000 + 800)x = 20{,}000 - 15{,}200$

E. $(2{,}000 - 800)x = 20{,}000 + 15{,}200$

20. A ramp for loading trucks is 13 feet long and covers 12 feet along the level ground, as shown below. How many feet high is the highest point on the ramp?

The figure shows a ramp (a right-angled triangle) for loading trucks which is 13 feet long and covers 12 feet along the level ground.

F. 1

G. 2

H. 4

J. 5

K. inline

21. The expression inline is equivalent to:

A. $x + 1$

B. $x + 15$

C. $x + 19$

D. $x + 23$

E. $x + 27$

22. If 115% of a number is 460, what is 75% of the number?

F. 280

G. 300

H. 320

J. 345

K. 400

23. When $(2x - 3)^2$ is written in the form inline, where a, b, and c are integers, $a + b + c = ?$

A. −17

B. −5

C. 1

D. 13

E. 25

24. What is the area, in square feet, of the figure below?

The figure shows an "L" shaped plot with sides labeled "15 feet," 25 feet" and 5 feet."

F. 60

G. 80

H. 275

J. 375

K. 450

25. Barb is going to cover a rectangular area 8 feet by 10 feet with rectangular paving blocks that are 4 inches by 8 inches by 2 inches to make a flat patio. What is the minimum number of paving blocks she will need if all the paving blocks will face the same direction?

(Note: Barb will not cut any of the paving blocks.)

A. 80

B. 360

C. 601

D. 960

E. 1,213

26. What is the slope of the line represented by the equation inline ?

F. −14

G. inline

H. inline

J. 6

K. 14

27. Let m and n be 2 positive integers, such that $m < n$ Which of the following compound inequalities *must* be true?

A. inline

B. inline

C. inline

D. inline

E. inline

28. Two similar triangles have perimeters in the ratio 3:5. The sides of the smaller triangle measure 3 cm, 5 cm, and 7 cm, respectively. What is the perimeter, in centimeters, of the larger triangle?

F. 15

G. 18

H. 20

J. 25

K. 36

29. Thomas and Jonelle are playing darts in their garage using the board with the point values for each region shown below. The radius of the outside circle is 10 inches, and each of the other circles has a radius 2 inches smaller than the next larger circle. All of the circles have the same center. Thomas has only 1 dart left to throw and needs at least 30 points to win the game. Assuming that his last dart hits at a random point within a single region on the board, what is the percent chance that Thomas will win the game?

The figure shows a dart board. The board represents five concentric circles. The radius of the outside circle is 10 inches, and each of the other circles has a radius 2 inches smaller than the next larger circle.

A. 36%

B. 30%

C. 16%

D. 9%

E. inline

30. When asked his age, the algebra teacher said, “If you square my age, then subtract 23 times my age, the result is 50.” How old is he?

F. 23

G. 25

H. 27

J. 46

K. 50

31. The distance, *d*, an accelerating object travels in *t* seconds can be modeled by the equation inline, where a is the acceleration rate, in meters per second per second. If a car accelerates from a stop at the rate of 20 meters per second per second and travels a distance of 80 meters, about how many seconds did the car travel?

A. Between 1 and 2

B. Between 2 and 3

C. Between 3 and 4

D. 4

E. 8

32. Which of the following is the set of all real numbers *x* such that inline ?

F. The empty set

G. The set containing all real numbers

H. The set containing all negative real numbers

J. The set containing all nonnegative real numbers

K. The set containing only zero

Use the following information to answer questions 33–35.

A survey in a study skills class asked the 20 students enrolled in the class how many hours (rounded to the nearest hour) they had spent studying on the previous evening. The 20 responses are summarized by the histogram below.

A histogram is shown in the x-y plane. The x-axis represents "hours" ranges from 0 to 5. The y-axis represents "number of students" ranges from 0 to 6. The histogram illustrates how many hours student spent on studying.

33. What fraction of the students responded that they had spent less than 3 hours studying?

A. inline

B. inline

C. inline

D. inline

E. inline

34. The teacher decides to show the data in a circle graph (pie chart). What should be the measure of the central angle of the sector for 3 hours?

F. 18°

G. 20°

H. 36°

J. 72°

K. 90°

35. To the nearest tenth of an hour, what is the average number of hours for the 20 survey responses?

A. 2.0

B. 2.1

C. 2.3

D. 2.5

E. 3.0

36. Pentagons have 5 diagonals, as illustrated below.

The figure shows a pentagon with a star within it.

How many diagonals does the octagon below have?

The figure shows a hexagon.

F. 8

G. 16

H. 20

J. 30

K. 40

37. The bottom of the basket of a hot-air balloon is parallel to the level ground. One taut tether line 144 feet long is attached to the center of the bottom of the basket and is anchored to the ground at an angle of 72°, as shown in the figure below. Which of the following expressions gives the distance, in feet, from the center of the bottom of the basket to the ground?

The figure shows a right-angled triangle, where the angle is labeled as “72 degree” and hypotenuse is labeled as “144 feet.” A hot-air balloon is placed on the top side of the right-angled triangle.

A.  inline

B. inline

C. 144 tan,72°

D. 144 cos,72°

E. 144 sin,72°

38. The coordinates of the endpoints of inline, in the standard (x,y) coordinate plane, are $(-8,-3)$ and $(2,3)$. What is the x-coordinate of the midpoint of inline ?

F. −6

G. −3

H. 0

J. 3

K. 5

39. Let inline and inline. What is the value of inline ?

A. −10

B. −1

C. 2

D. 7

E. 10

40. What are the values of θ, between 0 and 2π, when $\tan,\theta = -1$?

F. inline

G. inline

H. inline

J. inline

K. inline

41. For the complex number i and an integer x, which of the following is a possible value of i^x ?

A. 0

B. 1

C. 2

D. 3

E. 4

42. A can of soda pop has the shape of a right circular cylinder with an inside height of 6 inches and an inside diameter of 2 inches. When you pour the soda pop from the full can into a cylindrical glass with an inside diameter of 3 inches, about how many inches high is the soda pop in the glass?

(Note: The volume of a right circular cylinder is $\pi r^2 h$.)

F. inline

G. 4

H. 5

J. inline

K. 8

43. The height and radius of the right circular cylinder below are given in meters. What is the volume, in cubic meters, of the cylinder?

The figure shows a cylinder (right-circular cylinder), where each end is a circle, the circular sides are parallel. The radius of the cylinder is labeled as 5 and the height as 6.

A. 30π

B. 31π

C. 150π

D. 180π

E. 900π

44. Lines l_1 and l_2 intersect each other and 3 parallel lines, l_3, l_4, and l_5, at the points shown in the figure below. The ratio of the perimeter of ΔABC to the perimeter of ΔAFG is 1:3. The ratio of

inline to inline is 2:3. What is the ratio of inline to inline ?

The figure shows three parallel lines, labeled l subscript 3, l subscript 4, and l subscript 5, which are interested by two different lines, labeled l subscript 1 and l subscript 2 at seven different points: A, B, C, D, E, F and G.

F. 1:1

G. 1:2

H. 1:3

J. 2:1

K. 3:1

45. A rocket lifted off from a launch pad and traveled vertically 30 kilometers, then traveled 40 kilometers at 30° from the vertical, and then traveled 100 kilometers at 45° from the vertical, as shown in the ÿgure below. At that point, the rocket was how many kilometers above the height of the launch pad?

The figure shows a rocket lifted off from a launch pad and traveled vertically 30 kilometers, then traveled 40 kilometers at 30 degree from the vertical, and then traveled 100 kilometers at 45 degree from the vertical.

A. 100

B. 170

C. 190

D. inline

E. inline

46. Machine A produces 500 springs a day. The number of *defective* springs produced by this machine each day is recorded for 60 days. Based on the distribution given below, what is the expected value of the number of *defective* springs produced by Machine A in any single day?

Number, n, of defective springs produced	Probability that n defective springs are produced in any single day
0	0.70
1	0.20
2	0.05
3	0.05

F. 0.00

G. 0.45

H. 0.70

J. 1.00

K. 1.50

47. The height above the ground, h units, of an object t seconds after being thrown from the top of a building is given by the equation  . An equivalent factored form of this equation shows that the object:

A. starts at a point 2 units off the ground.

B. reaches a maximum height of 3 units.

C. reaches a maximum height of 8 units.

D. reaches the ground at 3 seconds.

E. reaches the ground at 8 seconds.

48. For all positive values of g and h, which of the following expressions is equivalent to  ?

F. 

G. 

H. 

J. 

K. g^7h^7

49. The value of inline is between which of the following pairs of consecutive integers?

A. 0 and 01

B. 4 and 05

C. 5 and 06

D. 6 and 07

E. 9 and 10

Use the following information to answer questions 50–52.

A storage facility is currently offering a special rate to customers who sign contracts for 6 months or more. According to this special rate, the first month's rent is $1, and for each month after the first month, customers pay the regular monthly rental rate. The table below shows the storage unit sizes available, the floor dimensions, and the regular monthly rental rate. All the units have the same height.

Size	Floor dimensions, in meters	Regular monthly rental rate
1	2 × 04	$030
2	4 × 04	$060
3	4 × 08	$100
4	8 × 08	$150
5	8 × 16	$200

50. Daria will sign a contract to rent a Size 3 unit for 12 months at the current special rate. The amount Daria will pay for 12 months at the current special rate represents what percent decrease from the regular rental rate for 12 months?

F. 8.25%

G. 8.33%

H. 8.42%

J. 9.00%

K. 9.09%

51. Size 5 units can be subdivided to form other sizes of units. What is the greatest number of Size 1 units that can be formed from a single Size 5 unit?

A. 2

B. 4

C. 8

D. 10

E. 16

52. Janelle, the owner of the storage facility, is considering building new units that have floor dimensions larger than Size 5 units. She will use the floor area to determine the heating requirements of these larger units. For this calculation, Janelle will use the same relationship between the unit size number and the respective floor area for Sizes 1 through 5. Which of the following expressions gives the floor area, in square meters, of a Size x storage unit?

F. inline

G. inline

H. inline

J. inline

K. inline

53. A trigonometric function with equation inline, where a, b, and c are real numbers, is graphed in the standard (x,y) coordinate plane below. The *period* of this function $f(x)$ is the smallest positive number p such that inline for every real number x. One of the following is the period of this function. Which one is it?


A graph is shown in the standard (x,y) coordinate plane. The x axis represents “values” ranges from negative 2 pi to positive 2 pi. The y-axis represents “values” ranges from negative 2 to positive 2. The graph is composed of repeating, identical cycles.

A. inline

B. π

C. 2π

D. 4π

E. 2

54. The component forms of vectors **u** and **v** are given by **u** = $\langle 5,3 \rangle$ and **v** = $\langle 2,-7 \rangle$. Given that $2\mathbf{u} + (-3\mathbf{v}) + \mathbf{w} = 0$, what is the component form of **w** ?

F. $\langle -16, 15 \rangle$

G. $\langle -4,-27 \rangle$

H. $\langle 3, 10 \rangle$

J. $\langle 4, 27 \rangle$

K. $\langle 16,-15 \rangle$

55. For how many integers x is the equation $3^{x+1} = 9^{x-2}$ true?

A. 0

B. 1

C. 2

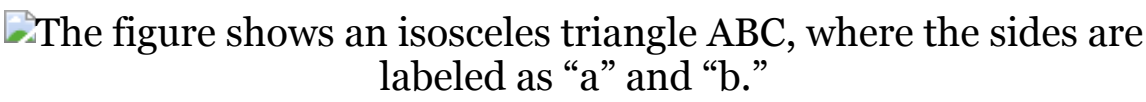
D. 3

E. An infinite number

56. In ΔABC shown below, the length of inline and the measure of θ will remain constant. The length of inline is 20 inches and the measure of $\angle C$ is equal to θ. Initially, the length of inline is 15 inches, and the length of inline is the function given by $f(t) = 15 - 2t$, where t is time, in seconds, since the length of

inline began to decrease. What is the time, t, at which the resulting triangle will have an area that is inline the area of the original triangle?

(Note: The area of a triangle is inline sin x, where a and b are the lengths of the sides that form the interior angle with measure x.)



F. 0

G. inline

H. inline

J. inline

K. inline

57. Which of the following expressions gives the number of distinct permutations of the letters in PEOPLE ?

A. 6!

B. 4(4!)

C. inline

D. inline

E. inline

58. Which of the following expressions is equivalent to inline ?

F. $(7x + 9)^2$

G. $(7x + 9i)^2$

H. $(7x - 9i)^2$

J. $(7x - 9)(7x + 9)$

K. $(7x - 9i)(7x + 9i)$

59. A bivariate data set of observed values along with a line of best ÿt for the data set are shown in the standard (x,y) coordinate

plane below. The set of 4 residuals for the model is given by inline, for i = 1, 2, 3, 4, where y_i is the observed y-value corresponding to the input x_i , and inline is on the line of best ÿt. What is the absolute value of the largest residual for this model?

A bar chart is shown in the x-y plane. The x-axis represents "values" ranges from 10 to 60. The y-axis represents "values" ranges from 20 to 180. A line is drawn in an increasing pattern from the point between 80 and 100. The line represents a linear equation, labeled y equals to 1.1x plus 93. The graph illustrates the absolute value of the largest residual.

A. 2.5

B. 6.8

C. 15.0

D. 20.0

E. 42.0

60. For the first 5 possible values of x, the table below gives the probability, $P(x)$, that a certain factory machine will make x errors on any given workday.

x errors	$P(x)$
0	0.0823
1	0.2185
2	0.2712
3	0.2046
4	0.1238

Which of the following values is closest to the probability that this machine will make at least 1 error on any given workday?

F. 0.2185

G. 0.5996

H. 0.6992

J. 0.8181

K. 0.9177

61. inline is equivalent to:

F. inline

G. inline

H. inline

J. inline

K. inline

62. Mr. Dietz is a teacher whose salary is $22,570 for this school year, which has 185 days. In Mr. Dietz's school district, substitute teachers are paid $80 per day. If Mr. Dietz takes a day off without pay and a substitute teacher is paid to teach Mr. Dietz's classes, how much less does the school district pay in salary by paying a substitute teacher instead of paying Mr. Dietz for that day?

A. $42

B. $80

C. $97

D. $105

E. $122

63. So far, a student has earned the following scores on four 100-point tests this grading period: 65, 73, 81, and 82. What score must the student earn on the fifth and last 100-point test of the grading period to earn an average test grade of 80 for the 5 tests?

F. 75

G. 76

H. 78

J. 99

K. The student cannot earn an average of 80.

64. The oxygen saturation level of a river is found by dividing the amount of dissolved oxygen the river water currently has per liter by the dissolved oxygen capacity per liter of the water and then converting to a percent. If the river currently has 7.3 milligrams of dissolved oxygen per liter of water and the dissolved oxygen capacity is 9.8 milligrams per liter, what is the oxygen saturation level, to the nearest percent?

A. 34%

B. 70%

C. 73%

D. 74%

E. 98%

65. A rectangular lot that measures 150 ft by 200 ft is completely fenced. What is the approximate length, in feet, of the fence?

F. 300

G. 350

H. 400

J. 700

K. 1,400

66. The expression  is equivalent to:

A. $ab + ac - ad$

B. $ab + ac + ad$

C. $ab + ac - d$

D. $ab + c + d$

E. $ab + c - d$

67. If $4x + 3 = 9x - 4$, then $x = ?$

F. 

G. 

H. 

J. inline

K. inline

68. What 2 numbers should be placed in the blanks below so that the difference between consecutive numbers is the same?

image

A. 23, 29

B. 24, 34

C. 25, 33

D. 26, 35

E. 27, 31

69. If x is a real number such that $x^3 = 64$, then inline

F. 4

G. 10

H. 18

J. 20

K. 47

70. A formula for the volume V of a sphere with radius r is inline. If the radius of a spherical rubber ball is 1¼ inches, what is its volume to the nearest cubic inch?

A. 5

B. 7

C. 8

D. 16

E. 65

71. If a marble is randomly chosen from a bag that contains exactly 8 red marbles, 6 blue marbles, and 6 white marbles, what is the probability that the marble will NOT be white?

F. inline

G. inline

H. inline

J. inline

K. inline

72. The number of students participating in fall sports at a certain high school is shown by the following matrix.

image

The athletic director estimates the ratio of the number of sports awards that will be earned to the number of students participating with the following matrix.

image

Given these matrices, what is the athletic director's estimate for the number of sports awards that will be earned for these fall sports?

A. 80

B. 88

C. 91

D. 92

E. 99

Use the following information to answer questions 73–74.

The following chart shows the current enrollment in all the mathematics classes offered by Eastside High School.

Course title	Section	Period	Enrollment
Pre-Algebra	A	3	23
Algebra I	A	2	24
	B	3	25

	C	4	29
Geometry	A	1	21
	B	2	22
Algebra II	A	4	28
Pre-Calculus	A	6	19

73. What is the average number of students enrolled per section in Algebra I?

    F. 24

    G. 25

    H. 26

    J. 27

    K. 29

74. The school owns 2 classroom sets of 30 calculators each, which students are required to have during their mathematics class. There are 2 calculators from one set and 6 calculators from the other set that are not available for use by the students because these calculators are being repaired. For which of the following class periods, if any, are there NOT enough calculators available for each student to use a school-owned calculator without having to share?

    A. Period 2 only

    B. Period 3 only

    C. Period 4 only

    D. Periods 3 and 4 only

    E. There are enough calculators for each class period.

75. What expression must the center cell of the table below contain so that the sums of each row, each column, and each diagonal are equivalent?

x	x	$-3x$
$-2x$	?	$6x$
$7x$	$-4x$	$3x$

F. $6x$

G. $4x$

H. $2x$

J. $-2x$

K. $-4x$

76. Point A is to be graphed in a quadrant, not on an axis, of the standard (x,y) coordinate plane below.

The figure shows a standard (x,y) coordinate plane, where point "A" is to be graphed in a quadrant. The standard (x,y) coordinate plane represents four different quadrants, labeled I, II III and IV.

If the x-coordinate and the y-coordinate of point A are to have opposite signs, then point A *must* be located in:

A. Quadrant II only.

B. Quadrant IV only.

C. Quadrant I or III only.

D. Quadrant I or IV only.

E. Quadrant II or IV only.

77. Kareem has 4 sweaters, 6 shirts, and 3 pairs of slacks. How many distinct outfits, each consisting of a sweater, a shirt, and a pair of slacks, can Kareem select?

F. 13

G. 36

H. 42

J. 72

K. 216

78. At a refinery, 100,000 tons of sand are required to produce each 60,000 barrels of a tarry material. How many tons of sand are required to produce 3,000 barrels of this tarry material?

A. 5,000

B. 18,000

C. 20,000

D. 40,000

E. 50,000

79. If a rectangle measures 54 meters by 72 meters, what is the length, in meters, of the diagonal of the rectangle?

F. 48

G. 63

H. 90

J. 126

K. 252

80. For all positive integers x, y, and z, which of the following expressions is equivalent to ?

A. 

B. 

C. 

D. 

E. 

81. What is the slope-intercept form of $8x - y - 6 = 0$?

F. $y = -8x - 6$

G. $y = -8x + 6$

H. $y = 8x - 6$

J. $y = 8x + 6$

K. $y = 6x - 8$

82. For right triangle ΔRST shown below, what is tan R?

 The figure shows a right-angled triangle RST, where the sides are labeled as “r,” “s,” and “t.”

F. inline

G. inline

H. inline

J. inline

K. inline

83. A chord 24 inches long is 5 inches from the center of a circle, as shown below. What is the radius of the circle, to the nearest tenth of an inch?

 The figure shows a circle. The radius of the circle creates a right angled triangle, where the hypotenuse is the radius, the base of the triangle rests on the x-axis, and the height of the triangle is a segment perpendicular to the x-axis. The base is labeled as “24” height is labeled as “5” and hypotenuse is labeled as “r.”

A. 29.0

B. 24.5

C. 16.9

D. 13.0

E. 10.9

84. The length L, in meters, of a spring is given by the equation inline, where F is the applied force in newtons. What force, in newtons, must be applied for the spring's length to be 0.18 meters?

F. 0.13

G. 0.15

H. 0.225

J. 0.255

K. 0.27

85. After a snowstorm, city workers removed an estimated 10,000 cubic yards of snow from the downtown area. If this snow were spread in an even layer over the entire rectangular football field shown below, about how many yards deep would the layer of snow be?

The figure shows a rectangular football field whose base is labeled as 100 yard and height is labeled as 53.5 yard.

A. Less than 1

B. Between 1 and 2

C. Between 2 and 3

D. Between 3 and 4

E. More than 4

86. The hypotenuse of the right triangle ΔPQR shown below is 16 feet long. The sine of $\angle P$ is inline. About how many feet long is inline?

The figure shows a right-angled triangle PQR, where the hypotenuse is labeled as “16.”

F. 8.0

G. 9.6

H. 12.4

J. 14.3

K. 15.4

87. The graph below shows the number of cars assembled last year in 4 cities, to the nearest 5,000 cars. According to the graph, what fraction of the cars assembled in all 4 cities were assembled in Coupeville?

The figure shows a two-column table illustrating the number of cars assembled last year in 4 cities, to the nearest 5,000 cars. The column heads are “city” and “cars assembled.”

A. inline

B. inline

C. inline

D. inline

E. inline

88. What is the x-coordinate of the point in the standard (x,y) coordinate plane at which the 2 lines $y = 2x + 6$ and $y = 3x + 4$ intersect?

A. 1

B. 2

C. 4

D. 6

E. 10

89. For all pairs of real numbers M and V where $M = 3V + 6$, V = ?

F. inline

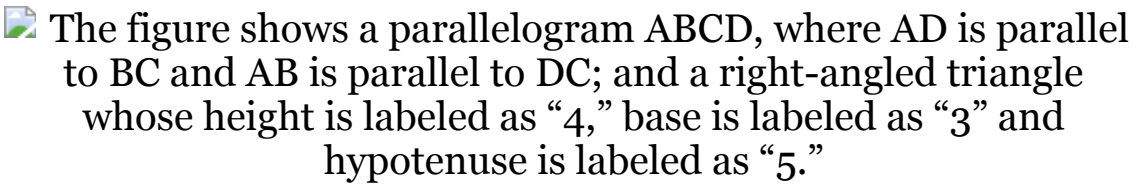
G. inline

H. $3M - 6$

J. inline

K. inline

90. Parallelogram *ABCD,* with dimensions in inches, is shown in the diagram below. What is the area of the parallelogram, in square inches?



A. 18

B. 36

C. 39

D. 45

E. 72

91. If $a = b + 2$, then $(b - a)^4 = ?$

F. −16

G. −8

H. 1

J. 8

K. 16

92. The larger of two numbers exceeds twice the smaller number by 8. The sum of twice the larger and 3 times the smaller number is 65. If x is the smaller number, which equation below determines the correct value of x ?

F. inline

G. inline

H. inline

J. inline

K. inline

93. Members of the fire department lean a 30-foot ladder against a building. The side of the building is perpendicular to the level ground so that the base of the ladder is 10 feet away from the base of the building. To the nearest foot, how far up the building does the ladder reach?

A. 10

B. 20

C. 28

D. 31

E. 40

94. The ratio of the side lengths for a triangle is exactly 12:14:15. In a second triangle similar to the first, the shortest side is 8 inches

long. To the nearest tenth of an inch, what is the length of the longest side of the second triangle?

A. 11.0

B. 10.0

C. 9.3

D. 6.4

E. Cannot be determined from the given information

95. In the figure below, *ABCD* is a trapezoid, *E* lies on inline, and angle measures are as marked. What is the measure of $\angle BDC$?


The figure shows a trapezoid ABCD, where E lies on AD, and angle are labeled as “60 degree” for A and “30 degree” for B.

F. 15°

G. 25°

H. 30°

J. 35°

K. 45°

96. In the figure shown below, each pair of intersecting line segments meets at a right angle, and all the lengths given are in inches. What is the perimeter, in inches, of the figure?


Illustration shows a diagram formed of 10 sides. Out of these, the lengths of six sides are given: 4, 6, 4, 10 and 6. The base is labeled as “26.”

A. 40

B. 52

C. 56

D. 66

E. 80

97. Of the 804 graduating seniors in a certain high school, approximately inline are going to college and approximately

inline of those going to college are going to a state university. Which of the following is the closest estimate for how many of the graduating seniors are going to a state university?

F. 80

G. 90

H. 160

J. 200

K. 320

98. What is the distance in the standard (x,y) coordinate plane between the points (1,0) and (0,5) ?

A. 4

B. 6

C. 16

D. 36

E. inline

99. The ratio of the radii of two circles is 4:9. What is the ratio of their circumferences?

F. 2:3

G. 4:9

H. 16:81

J. $4:8\pi$

K. $9:18\pi$

100. A circle in the standard (x,y) coordinate plane is tangent to the x-axis at 5 and tangent to the y-axis at 5. Which of the following is an equation of the circle?

A. inline

B. inline

C. inline

D. inline

E. inline

101. Which of the following statements describes the total number of dots in the first n rows of the triangular arrangement illustrated below?

The figure shows 5 rows, where dots are arranged in the triangular pattern (in an increasing order). From top to bottom, the first rows shows "1 dot," the second row shows "3 dots," the third row shows "5 dots," the fourth row shows "7 dots," and the fifth row shows "9 rows."

A. This total is always equal to 25 regardless of the number of rows.

B. This total is equal to twice the number of rows.

C. This total is equal to 5 times the number of rows.

D. This total is equal to the square of the number of rows.

E. There is no consistent relationship between this total and the number of rows.

102. Douglas wants to draw a circle graph showing the favorite colors of his friends. When he polled his friends asking each their favorite color, 25% of his friends said red; 30% of his friends said blue; 20% of his friends said green; 10% of his friends said purple; and the remaining friends said colors other than red, blue, green, and purple. The colors other than red, blue, green, and purple will be grouped together in an Other sector. What will be the degree measure of the Other sector?

A. 108°

B. 54°

C. 27°

D. 15°

E. 10°

103. If inline and inline, then $\tan \theta = ?$

F. inline

G. inline

H. inline

J. inline

K. inline

104. Which of the following systems of inequalities is represented by the shaded region of the graph below?


A graph is shown in the xy-plane, where a diagonal line is drawn with plot for (3, minus 6). A line is also drawn, which is parallel to y-axis and intersecting the diagonal line at point (3, minus 6), with plot for (3, 0).

A. $y \leq -2x$ and $x \geq 3$

B. $y \leq -2x$ or $x \geq 3$

C. $y \geq -2x$ and $x \geq 3$

D. $y \geq -2x$ or $x \geq 3$

E. $y \geq -2x$ and $x \leq 3$

105. If inline, then inline

F. inline

G. inline

H. inline

J. inline

K. inline

106. Which of the following is the graph, in the standard (x,y) coordinate plane, of inline?

A. A graph is shown in the xy-plane, where a diagonal line is drawn with plots for (0, 1) and (1, 3).

B. A graph is shown in the xy-plane, where a diagonal line is drawn with plot for (1, 3).

C.  A graph is shown in the xy-plane, where a diagonal line is drawn with plots for (0, 2) and (1, 3).

D.  A graph is shown in the xy-plane, where a parabola is drawn which intersects the point (0, 1); and plots for (minus 1, 3) and (1, 3).

E.  A graph is shown in the xy-plane, where a parabola is drawn which intersects the origin point and plots for (minus 1, 3) and (1, 3).

107. A triangle, ΔABC, is reflected across the x-axis to have the image $\Delta A'B'C'$ in the standard (x,y) coordinate plane; thus, A reflects to A'. The coordinates of point A are (c,d). What are the coordinates of point A' ?

F. $(c,-d)$

G. $(-c, d)$

H. $(-c,-d)$

J. (d, c)

K. Cannot be determined from the given information

108. If $x = 2t - 9$ and $y = 5 - t$, which of the following expresses y in terms of x?

A. 

B. 

C. 

D. 

E. 

109. What is  given that  and that $\sin(\alpha - \beta) =$

$(\sin \alpha)(\cos \beta) - (\cos \alpha)(\sin \beta)$?

(Note: You may use the following table of values.)

θ	$\sin \theta$	$\cos \theta$
		

inline	inline	inline
inline	inline	inline

F. inline

G. inline

H. inline

J. inline

K. inline

110. If 12 vases cost $18.00, what is the cost of 1 vase?

F. $0.67

G. $1.05

H. $1.33

J. $1.50

K. $1.60

111. Your friend shows you a scale drawing of her apartment. The drawing of the apartment is a rectangle 4 inches by 6 inches. Your friend wants to know the length of the shorter side of the apartment. If she knows that the length of the longer side of the apartment is 30 feet, how many feet long is the shorter side of her apartment?

A. 9

B. 20

C. 24

D. 30

E. 45

112. A company earned a profit of $8.0 million each year for 3 consecutive years. For each of the next 2 years the company earned a profit of $9.0 million. For this 5-year period, what was the company's average yearly profit, in millions of dollars?

F. 8.2

G. 8.25

H. 8.4

J. 8.5

K. 8.6

113. A company rents moving vans for a rental fee of $25.00 per day with an additional charge of $0.30 per mile that the van is driven. Which of the following expressions represents the cost, in dollars, of renting a van for 1 day and driving it m miles?

A. $0.30m + 25$

B. $25m + 30$

C. $30m + 25$

D. $25.30m$

E. $55m$

114. The relationship between temperature in degrees Fahrenheit, F, and temperature in degrees Celsius, C, is expressed by the formula  . Calvin reads a temperature of 38° on a Celsius thermometer. To the nearest degree, what is the equivalent temperature on a Fahrenheit thermometer?

F. 36°

G. 53°

H. 68°

J. 70°

K. 100°

115. Nick needs to order 500 pens from his supplier. The catalog shows that these pens come in cases of 24 boxes with 10 pens in each box. Nick knows that he may NOT order partial cases. What is the fewest number of cases he should order?

A. 2

B. 3

C. 18

D. 21

E. 50

116. When $a + b = 6$, what is the value of inline?

F. 23

G. 37

H. 38

J. 43

K. 47

117. The cost of a hamburger and a soft drink together is $2.10. The cost of 2 hamburgers and a soft drink together is $3.50. What is the cost of a soft drink?

A. $0.50

B. $0.55

C. $0.70

D. $1.05

E. $1.40

118. Shannon is planning to tile a rectangular kitchen countertop that is 24 inches wide and 64 inches long. She determined that 1 tile will be needed for each 4-inch-by-4-inch region. What is the minimum number of tiles that will be needed to completely cover the countertop to its edges?

A. 44

B. 88

C. 96

D. 176

E. 384

119. Which of the following lists gives 2 of the 3 interior angle measurements of a triangle for which the third angle measurement would be equal to 1 of the 2 given measurements?

F. 20°, 40°

G. 30°, 60°

H. 40°, 100°

J. 45°, 120°

K. 50°, 60°

120. A triangle with a perimeter of 66 inches has one side that is 16 inches long. The lengths of the other two sides have a ratio of 2:3. What is the length, in inches, of the *longest* side of the triangle?

A. 16

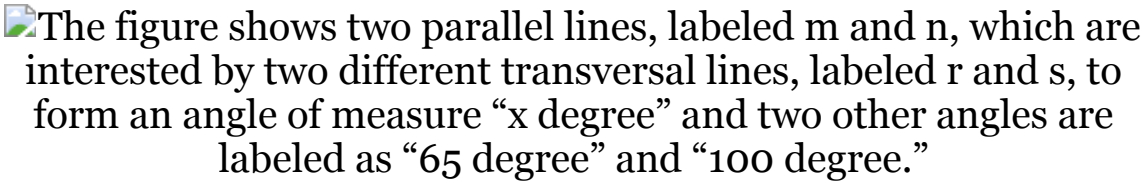
B. 20

C. 30

D. 40

E. 50

121. In the figure below, lines m and n are parallel, transversals r and s intersect to form an angle of measure $x°$, and 2 other angle measures are as marked. What is the value of x?


The figure shows two parallel lines, labeled m and n, which are interested by two different transversal lines, labeled r and s, to form an angle of measure “x degree” and two other angles are labeled as “65 degree” and “100 degree.”

A. 15

B. 25

C. 35

D. 65

E. 80

122. What is the slope of any line parallel to the line $7x + 9y = 6$?

A. -7

B. inline

C. inline

D. 6

E. 7

123. A ramp for wheelchair access to the gym has a slope of 5% (that is, the ramp rises 5 feet vertically for every 100 feet of horizontal distance). The entire ramp is built on level ground, and the entrance to the gym is 2 feet above the ground. What is the *horizontal* distance, in feet, between the ends of the ramp?

A. 4

B. 10

C. 40

D. 100

E. 400

124. inline is equivalent to:

F. inline

G. inline

H. inline

J. inline

K. inline

125. Which of the following sets of 3 numbers could be the side lengths, in meters, of a 30°–60°–90° triangle?

A. 1, 1, 1

B. 1, 1, inline

C. 1, inline

D. 1, inline

E. 1, inline, 2

126. Which of the following statements is NOT true about the arithmetic sequence 17, 12, 7, 2, … ?

A. The fifth term is –3.

B. The sum of the first 5 terms is 35.

C. The eighth term is –18.

D. The common difference of consecutive terms is –5.

E. The common ratio of consecutive terms is –5.

127. The normal amount of lead in a certain water supply is inline milligrams per liter. Today, when the water was tested, the lead level found was exactly 100 times as great as the normal level, still well below the Environmental Protection Agency's action level. What concentration of lead, in milligrams per liter, was in the water tested today?

A. inline

B. inline

C. inline

D. inline

E. inline

128. inline ?

A. inline

B. inline

C. inline

D. inline

E. inline

129. How many prime numbers are there between 30 and 50 ?

F. 4

G. 5

H. 6

J. 7

K. 8

130. The lengths, in feet, of the sides of right triangle ΔABC are as shown in the diagram below, with $x > 0$. What is the cotangent of $\angle A$, in terms of x ?


The figure shows a right-angled triangle ABC, where the base is labeled as “x,” height is labeled as “start root 4 minus x superscript 2 end root” and hypotenuse is labeled as “2.”

A. inline

B. inline

C. inline

D. inline

E. inline

131. The trapezoid below is divided into 2 triangles and 1 rectangle. Lengths are given in inches. What is the combined area, in square inches, of the 2 shaded triangles?


The figure shows a trapezoid divided into 2 triangles and 1 rectangle. The height of the rectangle is labeled as “3” and the base as “4.”

A. 4

B. 6

C. 9

D. 12

E. 18

132. In the figure below, *ABCD* is a square and *E*, *F*, *G*, and *H* are the midpoints of its sides. If inline inches, what is the perimeter of *EFGH*, in inches?


The figure shows a square EFGH inscribed in a square ABCD.

F. 24

G. inline

H. inline

J. inline

K. 72

133. If the value, to the nearest thousandth, of cos θ is −0.385, which of the following could be true about θ?

A. inline

B. inline

C. inline

D. inline

E. inline

134. Which of the following quadratic equations has solutions $x = 6a$ and $x = -3b$?

F. inline

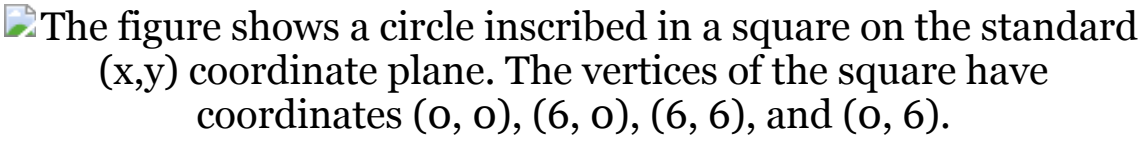
G. inline

H. inline

J. inline

K. inline

135. In the standard (x,y) coordinate plane below, the vertices of the square have coordinates (0,0), (6,0), (6,6), and (0,6). Which of the following is an equation of the circle that is inscribed in the square?


The figure shows a circle inscribed in a square on the standard (x,y) coordinate plane. The vertices of the square have coordinates (0, 0), (6, 0), (6, 6), and (0, 6).

A. inline

B. inline

C. inline

D. inline

E. inline

136. What is the value of the expression $g \cdot (g + 1)^2$ for $g = 2$?

F. 10

G. 12

H. 18

J. 20

K. 36

137. Company A sells 60 pens for $15.00. Company B sells the same type of pens in packs of 40 for $8.00. Which company's price per pen is cheaper, and what is that price?

A. Company A, at $0.20

B. Company A, at $0.23

C. Company A, at $0.25

D. Company B, at $0.20

E. Company B, at $0.25

138. A ladder is 10 ft long and reaches 8 ft up a wall, as shown below. How many feet is the bottom of the ladder from the base of the wall?


The figure shows ladder which is 10 feet long and it reaches 8 feet up a wall.

F. 2

G. 3

H. 6

J. inline

K. inline

139. A city utility department charges residential customers $2.50 per 1,000 gallons of water and $16.00 per month for trash pickup. Which of the following expressions gives a residential customer's total monthly charges, in dollars, for use of g thousand gallons of water and trash pickup?

F. inline

G. inline

H. inline

J. 18.50 *g*

K. inline

140. What is the value of *x* that satisfies the equation inline?

A. −1

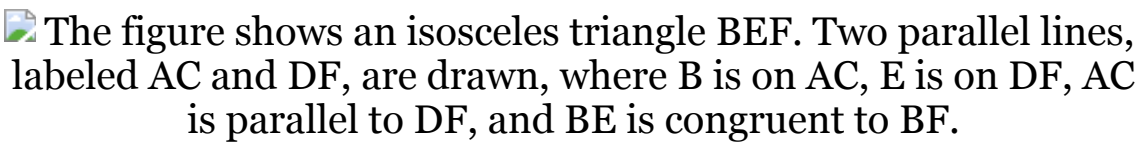
B. inline

C. inline

D. 5

E. inline

141. In the figure below, *B* is on inline, *E* is on inline, inline is parallel to inline, and inline is congruent to inline. What is the measure of inline?


The figure shows an isosceles triangle BEF. Two parallel lines, labeled AC and DF, are drawn, where B is on AC, E is on DF, AC is parallel to DF, and BE is congruent to BF.

F. 35°

G. 135°

H. 145°

J. 155°

K. 215°

142. What is the least common denominator when adding the fractions inline, and inline?

A. 45

B. 90

C. 135

D. 270

E. 810

143. Which of the following expressions is equivalent to $3x(x^2y + 2xy^2)$?

F. $3x^2y + 6xy^2$

G. $3x^3y + 2xy^2$

H. $3x^3y + 6x^2y^2$

J. $5x^4y^3$

K. $9x^4y^3$

144. A certain type of notebook costs $2.50 before sales tax is added. When you buy 9 of these notebooks you receive 1 additional notebook free. What is the average cost per notebook for the 10 notebooks before sales tax is added?

A. $2.78

B. $2.50

C. $2.30

D. $2.25

E. $2.15

145. For all x, $(3x + 1)^2 = ?$

F. $6x + 2$

G. $6x^2 + 2$

H. $9x^2 + 1$

J. $9x^2 + 3x + 1$

K. $9x^2 + 6x + 1$

146. On the real number line, what is the midpoint of -5 and 17?

A. -11

B. 6

C. 11

D. 12

E. 22

147. If inline

F. inline

G. inline

H. inline

J. inline

K. inline

148. A system of linear equations is shown below.

image

Which of the following describes the graph of this system of linear equations in the standard (x,y) coordinate plane?

A. Two distinct intersecting lines

B. Two parallel lines with positive slope

C. Two parallel lines with negative slope

D. A single line with positive slope

E. A single line with negative slope

149. Which real number satisfies $(2^x)(4) = 8^3$?

F. 2

G. 3

H. 4

J. 4.5

K. 7

150. The graph shown in the standard (x,y) coordinate plane below is to be rotated in the plane 180° about the origin.


A graph is shown in the standard (x,y) coordinate plane. The x axis represents “values” ranges from negative 4 to positive 4. The y-axis represents “values” ranges from negative 2 to positive 4. The standard (x,y) coordinate plane is rotated in the plane 180 degree about the origin. A line is drawn from the origin point and passes through the point “2” and “4” on the x-axis.

One of the following graphs is the result of this rotation. Which one is it?

A. 
A graph is shown in the standard (x,y) coordinate plane. The x axis represents “values” ranges from negative 4 to positive 4. The y-axis represents “values” ranges from negative 2 to positive 4. The standard (x,y) coordinate plane is rotated in the plane 180 degree about the origin. A line is drawn from the origin point and passes through the point “1.5” and “2.5” below the x-axis.

B. 
A graph is shown in the standard (x,y) coordinate plane. The x axis represents “values” ranges from negative 4 to positive 4. The y-axis represents “values” ranges from negative 2 to positive 4. The standard (x,y) coordinate plane is rotated in the plane 180 degree about the origin. A line is drawn from the origin point and passes through the point “negative 2” and “negative 4” on the negative x-axis.

C. 
A graph is shown in the standard (x,y) coordinate plane. The x axis represents “values” ranges from negative 4 to positive 4. The y-axis represents “values” ranges from negative 2 to positive 4. The standard (x,y) coordinate plane is rotated in the plane 180 degree about the origin. A line is drawn from the origin point and passes through the point “negative 2” and “negative 4” below the negative x-axis.

D. 
A graph is shown in the standard (x,y) coordinate plane. The x axis represents “values” ranges from negative 4 to positive 4. The y-axis represents “values” ranges from negative 2 to positive 4. The standard (x,y) coordinate plane is rotated in the plane 180 degree about the origin. A line is drawn from the origin point and passes through the point “negative 2” and “negative 4” below the negative x-axis.


A graph is shown in the standard (x,y) coordinate plane. The x axis represents “values” ranges from negative 4 to positive 4. The y-axis represents “values” ranges from

E. negative 2 to positive 4. The standard (x,y) coordinate plane is rotated in the plane 180 degree about the origin. A line is drawn from the origin point and passes through the point “2” and “4” below the x-axis.

151. On the real number line below, with coordinates as labeled, an object moves according to the following set of instructions: from point *P* the object moves right to *Q*, then left to *R*, then right to *S*, and finally left until it returns to its original position at *P*. What is the closest estimate of the total length, in coordinate units, of the movements this object makes?


The diagram shows a horizontal line that is divided into four parts with markings P, R, Q, and S, where P (on the left-hand side) and S (on the right-hand side) represent the endpoints of the segment. The line represents real numbers from negative 4 to positive 8.

A. 0

B. 4

C. 12

D. 22

E. 36

152. By definition, the determinant 
equals $ad - bc$. What is the value of
when $x = -3$ and $y = 2$?

F. −138

G. −42

H. 12

J. 42

K. 138

153. When Angela was cleaning her refrigerator, she found 2 bottles of catsup. Looking at the labels, she noticed that the capacity of

the larger bottle was twice the capacity of the smaller bottle. She estimated that the smaller bottle was about inline full of catsup and the larger bottle was about inline full of catsup. She poured all the catsup from the smaller bottle into the larger bottle. Then, about how full was the larger bottle?

A. inline

B. inline

C. inline

D. Completely full

E. Overflowing

154. When Jeff starts a math assignment, he spends 5 minutes getting out his book and a sheet of paper, sharpening his pencil, looking up the assignment in his assignment notebook, and turning to the correct page in his book. The equation $t = 10p + 5$ models the time, t minutes, Jeff budgets for a math assignment with p problems. Which of the following statements is necessarily true according to Jeff's model?

F. He budgets 15 minutes per problem.

G. He budgets 10 minutes per problem.

H. He budgets 5 minutes per problem.

J. He budgets 10 minutes per problem for the hard problems and 5 minutes per problem for the easy problems.

K. He budgets a 5-minute break after each problem.

155. Kaya drove 200 miles in 5 hours of actual driving time. By driving an average of 10 miles per hour faster, Kaya could have saved how many hours of actual driving time?

A. inline

B. inline

C. inline

D. 1

E. 4

156. If the inequality $|a| > |b|$ is true, then which of the following *must* be true?

A. $a = b$

B. $a \neq b$

C. $a < b$

D. $a > b$

E. $a > o$

157. What is the slope of the line given by the equation $14x - 11y + 16 = 0$?

F. -11

G. inline

H. inline

J. inline

K. 14

158. In ΔABC below, *D, E,* and *F* are points on inline, inline, and inline, respectively, and inline is congruent to inline. What is the *sum* of the measures of the angles marked *x* and *y*?

The figure shows an isosceles triangle ABC, where D, E, and F are points on AB, BC, and AC, respectively. DF is congruent to EF.

F. 40°

G. 80°

H. 90°

J. 100°

K. 130°

159. Which of the following expressions is equivalent to $(-2x^5y^2)^4$?

A. inline

B. inline

C. inline

D. $16x^9y^6$

E. $16x^{20}y^8$

160. If $a^2 = 49$ and $b^2 = 64$, which of the following CANNOT be a value of $a + b$?

A. -15

B. -1

C. 1

D. 15

E. 113

Chapter 8:
Answers and Explanations

Check your answers with the following answer key. If you missed a question or would like tips on solving it faster, review the answer explanations on the following pages.

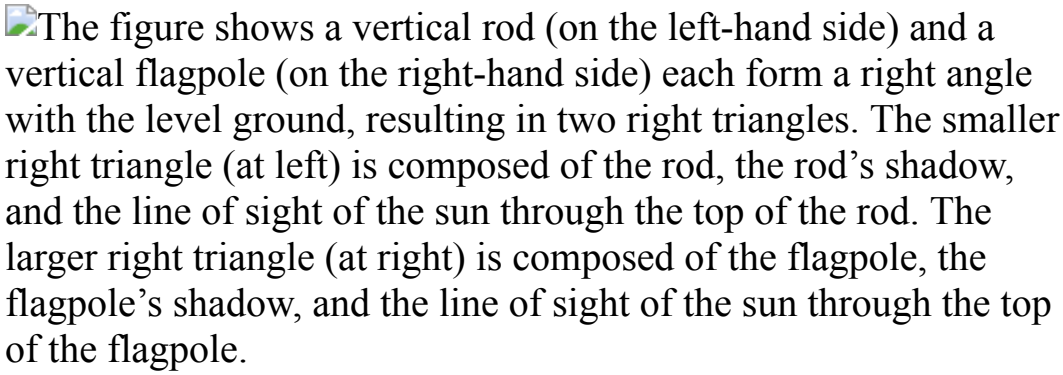
Answer Key

1. E	41. B	81. H	121. C
2. K	42. F	82. G	122. B
3. A	43. C	83. D	123. C
4. K	44. F	84. H	124. J
5. E	45. E	85. B	125. E
6. G	46. G	86. G	126. E
7. B	47. E	87. B	127. D
8. J	48. H	88. B	128. B
9. E	49. D	89. J	129. G
10. J	50. F	90. B	130. E
11. C	51. E	91. K	131. B
12. J	52. H	92. J	132. G
13. E	53. B	93. C	133. D
14. G	54. G	94. B	134. J
15. A	55. B	95. K	135. A
16. J	56. H	96. E	136. H
17. E	57. E	97. F	137. D
18. K	58. K	98. E	138. H
19. A	59. B	99. G	139. F
20. J	60. K	100. D	140. D
21. E	61. H	101. D	141. H
22. G	62. A	102 B	142. B

23. C	63. J	103. J	143. H
24. H	64. D	104. A	144. D
25. B	65. J	105. K	145. K
26. H	66. A	106. A	146. B
27. C	67. F	107. F	147. G
28. J	68. C	108. A	148. A
29. A	69. H	109. K	149. K
30. G	70. C	110. J	150. D
31. B	71. K	111. B	151. E
32. F	72. D	112. H	152. J
33. D	73. H	113. A	153. C
34. J	74. C	114. K	154. G
35. B	75. H	115. B	155. D
36. H	76. E	116. K	156. B
37. E	77. J	117. C	157. J
38. G	78. A	118. C	158. H
39. E	79. H	119. H	159. E
40 H	80 A	120. C	160. E

Explanatory Answers

Question 1. The correct answer is E. You may want to make a sketch of this situation in your mind or, better yet, in the space in your test booklet. A sample sketch is shown in the following:


The figure shows a vertical rod (on the left-hand side) and a vertical flagpole (on the right-hand side) each form a right angle with the level ground, resulting in two right triangles. The smaller right triangle (at left) is composed of the rod, the rod's shadow, and the line of sight of the sun through the top of the rod. The larger right triangle (at right) is composed of the flagpole, the flagpole's shadow, and the line of sight of the sun through the top of the flagpole.

The vertical rod and the vertical flagpole each form a right angle with the level ground, resulting in two right triangles. The smaller right triangle (at left) is composed of the rod, the rod's shadow, and the line of sight of the sun through the top of the rod. The larger right triangle (at right) is composed of the flagpole, the flagpole's shadow, and the line of sight of the sun through the top of the flagpole. Because the angle of elevation of the sun is the same for each triangle, the two triangles are similar by the angle-angle similarity property. Using the ratios of corresponding sides of the similar triangles, the proportion inline is solved to find the height of the flagpole, $h = 36$ feet.

Common errors in this problem result from relying on an incorrect mental image or labeling the dimensions on the sketch incorrectly. If you chose **A**, you might have set up and solved the proportion inline.

Question 2 . **The correct answer is K.** If you knew the unknown score, you could check to see that it was correct by adding up all five scores, dividing by 5 to get the average, and checking to see that the result was 90. Let x be the unknown score. Then the sum of all the scores is $85 + 95 + 93 + 80 + x$, and the average is inline. For the average to be 90, that means inline. To solve that equation, you can multiply both sides by 5 to get $85 + 95 + 93 + 80 + x = 450$ and then subtract 353 from both sides to get $x = 97$.

G is closest to the average of the four given scores, inline. To raise an average of 88.25 up to 90 would take an increase of about 2 points, but a single new score of 92 (answer choice **J**) would not raise the average much. You can check your answer to see that it is too low: inline. You can check any answer choice to see whether it is correct.

Question 3. The correct answer is A. Substituting –5 for x produces a numerator equal to $(–5)^2 – 1 = 25 – 1 = 24$ and a denominator equal to $–5 + 1 = –4$. Therefore, inline The most common wrong answer is **C**, which comes from forgetting the negative sign in the given x-value: inline

Question 4. The correct answer is K. To find the total distance, in miles, Kaya ran, you need the sum of 1inline and 2inline. To add mixed

numbers together, each fraction must have a common denominator. Because 3 and 5 do not have any common factors besides 1, the least common denominator is 3(5), or 15. To convert inline, you multiply by inline. The result is inline. To convert inline, you multiply by inline . The result is inline. To add 1inline and 2inline, you first add 1 and 2 and then inline + inline. The result is 3inline, or 3inline.

If you chose **F**, you probably added the whole number parts and multiplied the fractions. If you chose **G**, you probably added the whole number parts and added the numerators and the denominators separately: inline. If you chose **J**, you probably converted inline to inline incorrectly and then added 1 + 2 and inline + inline.

Question 5. The correct answer is E. Although you could try out various combinations of the given three statements and try to make a conclusion, it might be more straightforward to look at each of the answer choices to see whether it contradicts one of the given three statements or whether it could be deduced from the given three statements.

A and **B** each say that Insect I is an ant. This directly contradicts the second given statement, so **A** and **B** are false.

Consider **C**: if it is true (Insect I is attracted to honey), then the first given statement implies that Insect I is an ant. This contradicts the second given statement, so **C** is false.

D directly contradicts the third given statement, so **D** is false.

For **E**, consider Insect J. The third given statement tells you that Insect J is attracted to honey, and the first given statement tells you that, because all insects attracted to honey are ants, Insect J must be an ant. So **E** must be true.

Question 6. The correct answer is G. You can find the value of this expression by substituting the given values of *x* and *m* into the expression and then simplifying: inline = inline.

You may have gotten **F** if you did inline.

You may have gotten **H** if you did inline You may have gotten **J** if you did inline.

You may have gotten **K** if you did inline.

Question 7. The correct answer is B. The amount collected from the sale of 142 tickets bought in advance is equal to ($6 per ticket)(142 tickets) = $852. The amount collected from the sale of *d* tickets bought at the door is equal to ($8 per ticket)(*d* tickets) = $8*d*. The total amount collected from all ticket sales is 852 + 8*d*. To determine the minimum number of tickets to produce $2,000 in ticket sales, you can set up an inequality: $852 + 8d \geq 2{,}000$. Subtracting 852 from both sides and then dividing by 8 produces the equivalent inequality $d \geq 143.5$. Keep in mind, however, that *d* must be a whole number of tickets, so you must select the whole number *d* to satisfy the inequality. This means you must round 143.5 *up* to obtain the correct answer. If you chose **A**, you probably rounded *down* to 143. If you chose **D**, you might have divided 2,000 by 8 without thinking carefully about what the numbers represent. If you chose **C** or **E**, you probably set up the inequality incorrectly.

Question 8. The correct answer is J. For each kind of bread, there are 5 kinds of meat, so that is $3 \cdot 5$ combinations of bread and meat. For each of these 15 combinations of bread and meat, there are 3 kinds of cheese. That makes $15 \cdot 3 = 45$ combinations of bread, meat, and cheese.

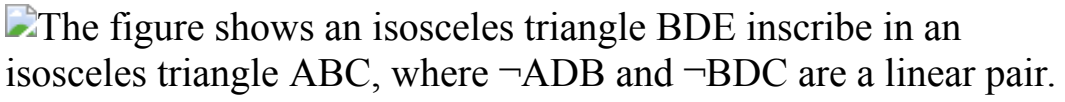
The tree diagram on page 200 shows all 45 combinations. It would take a lot of time to list all these cases, but you can imagine what the tree looks like without having to write it all out. You can see that parts of the tree repeat many times, and so you can use multiplication to help you count.

> The figure shows a tree diagram illustrating 45 combinations of bread (wheat, rye and oats); meat (Chicken, Ham, Beef, Turkey and Salami); and cheese (Cheddar, Swiss and American).

Question 9. The correct answer is E. Use of the distributive property gives the equivalent equation $12x - 132 = -15$. Adding 132 to both sides of the equation results in the equation $12x = 117$, implying that the solution is $x =$ inline, or inline when reduced to lowest terms. If you distributed 12

to *only* the first term, x, but forgot to distribute 12 to the second term, you probably got an answer of –inline.

Question 10. The correct answer is J. The following figure illustrates the progression of angle measures found in determining the measure of inline *BCE*.


The figure shows an isosceles triangle BDE inscribe in an isosceles triangle ABC, where ¬ADB and ¬BDC are a linear pair.

Because inline is isosceles, so its base angles are congruent. Therefore, $m$inline = $m$inline = 25°. Because the sum of the angle measures in inline must equal 180°, $m$inline = 180° − (25° + 25°) = 130°. Because inline and inline are a linear pair, $m$inline = 180° − 130° = 50°. Because inline ≅ inline, inline is isosceles, so *its* base angles are congruent: $m$inline = $m$inline = 50°. Finally, inline and inline are a linear pair, so $m$inline = 180° − 50° = 130°.

Question 11. The correct answer is C. When you substitute –2 for x, you get $9(-2)^2 + 5(-2) - 8 = 9(4) + (-10) - 8 = 18$. If you chose **A**, you probably evaluated $9(-2)^2$ as –36. If you chose **E**, you probably evaluated 5(–2) as 10.

Question 12. The correct answer is J. One efficient way to solve this problem numerically is by listing the multiples of the largest of the 3 numbers (70) as a sequence and determining whether or not each succeeding term in the sequence is a multiple of *both* 20 and 30.

70 (multiple of neither)

140 (multiple of 20 only)

210 (multiple of 30 only)

280 (multiple of 20 only)

350 (multiple of neither)

420 (multiple of both 20 and 30)

The first term in the sequence that is a multiple of both 20 and 30 is 420, which is the least common multiple of 20, 30, and 70. You can also find the least common multiple by expressing each of the three numbers as a product of primes (with exponents), listing all bases of exponential expressions shown, and choosing for each base listed the highest-valued exponent shown.

$30 = 2^1 \times 3^1 \times 5^1$

$20 = 2^2 \times 5^1$

$70 = 2^1 \times 5^1 \times 7^1$

The least common multiple is $2^2 \times 3^1 \times 5^1 \times 7^1 = 420$.

Question 13. The correct answer is E. You may want to choose an even integer as Tom's initial number, follow his steps in obtaining the *incorrect* answer, and then determine *what* operation using *what* number is needed to obtain the desired number. For example:

1. Choose the integer 6 as the initial number.
2. When Tom "accidentally multiplies the number by 2," he obtains an incorrect answer of 12.
3. Had Tom correctly divided the initial number by 2, he would have obtained 3 as the answer.
4. To convert his incorrect answer of 12 to the desired answer of 3, he must divide by 4.

You may want to confirm that **E** is the correct answer by choosing a different initial number and repeating the steps.

Question 14. The correct answer is G. The 8-sided figure in the problem consists of 5 congruent squares whose areas total 125 square inches.

Therefore, each congruent square has an area of 125 ÷ 5 = 25 square inches, so each side of each square is inline inches long. The perimeter of the 8-sided figure is composed of 12 of these sides, each of length 5 inches, as shown in the following figure. Therefore, the 8-sided figure has a perimeter of 12 × 5 = 60 inches.

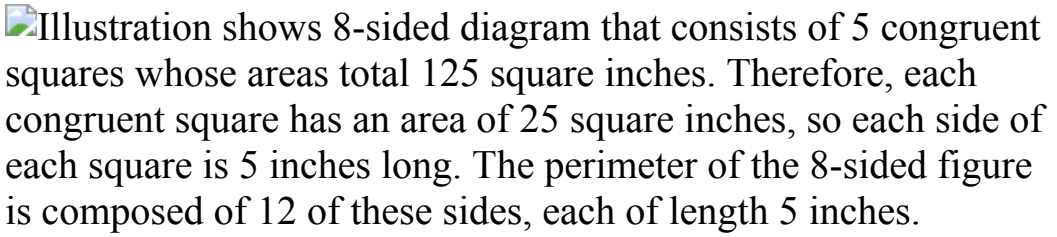


Illustration shows 8-sided diagram that consists of 5 congruent squares whose areas total 125 square inches. Therefore, each congruent square has an area of 25 square inches, so each side of each square is 5 inches long. The perimeter of the 8-sided figure is composed of 12 of these sides, each of length 5 inches.

Question 15. The correct answer is A. You can find the total number of USBs Hai can buy by dividing the total cost of $100 by the cost of 1 USB plus tax for 1 USB, $8 × (1 + 0.07): inline You cannot buy a partial USB; therefore, you must round down to 11 whole USBs. You may have gotten **B** if you did inline and rounded up. You may have gotten **C** if you did inline and rounded down. You may have gotten **D** if you did inline and rounded up. You may have gotten **E** if you did inline and rounded down.

Question 16. The correct answer is J. Because the item gives a unit rate, you can set up a proportional relationship and solve for x: inline.

You divide the coefficients and subtract the exponents because the bases are the same in each expression: $4.0 \times 10^{16-8} = 4.0 \times 10^{8}$.

You may have gotten **F** if you did inline $= 0.25 \times 10^{-8}$. You may have gotten **G** if you did 6.0(1.5) × 10. You may have gotten **H** if you incorrectly simplified inline to be 4.0 × inline. You may have gotten **K** if you did $(1.5 \times 10^{8})(6.0 \times 10^{16})$.

Question 17. The correct answer is E. Each answer choice is a linear equation in *slope-intercept form*; that is, $y = mx + b$, where the value of m gives the slope of the line and the value of b gives the y-intercept of the line. Only the equation shown in **E** represents a line having a y-intercept

($b = 5$), that matches the value of the y-intercept indicated by the given graph.

Question 18. The correct answer is K. To find the area of a square circumscribed about a circle with a radius of 7 feet, you would need to find the side length of the square. Because the diameter of the circle is the distance between 2 opposite sides of the square, the side length of the square is twice the radius, 2(7), or 14 feet. To find the area of the square, you square the side length, 14^2, to get 196 square feet.

If you chose **F**, you probably thought 7 feet was the side length and squared 7 to get 49 square feet. If you chose **J**, you probably used the formula for the area of a circle,  , where r is the radius, to get  , or  square feet.

Question 19. The correct answer is A. For x years of full years' employment after being hired, Worker A's starting salary ($20,000) increases by $800 per year and Worker B's starting salary ($15,200) increases by $2,000 per year. After x years, Worker A's salary has increased by 800x$ and Worker B's salary has increased by $2,000$x$. So, for x years of full years' employment after being hired, Worker A's yearly salary is represented by the expression $20,000 + 800x$ and Worker's B's salary is represented by the expression $15,200 + 2,000x$. These 2 yearly salaries are equal at the value of x for which the equation $20,000 + 800x = 15,200 + 2,000x$ is true.

Question 20. The correct answer is J. The figure shows a right triangle with two given side measures. To find the length of the third side, use the Pythagorean theorem:

$$(\text{length of the hypotenuse})^2 = (\text{length of one side of the triangle})^2 + (\text{length of the other side of the triangle})^2$$

In this problem, the 13-foot measure represents the length of the hypotenuse. So the formula gives the equation $13^2 = 12^2 + x^2$, where x feet is the length of the missing side. To find x^2, subtract 12^2 from each side of

the equation. The subtraction results in the equivalent equation $25 = x^2$, resulting in the solution $x = \pm 5$. Because the side length of a triangle must be positive, you can ignore the negative solution. If you chose **F**, you probably took the length of the hypotenuse and subtracted the length of the given leg, without applying the Pythagorean theorem at all. If you chose **G**, you probably *doubled* the lengths, rather than *squaring* them.

Question 21. The correct answer is E. To simplify this expression, use the distributive property: $7(x + 3) - 3(2x - 2) = 7x + (7)(3) + (-3)(2x) + (-3)(-2) = 7x + 21 + (-6x) + 6$. Then combine like terms to obtain $x + 27$. If you chose **B**, perhaps you forgot that $a - b = a + (-b)$, and so you distributed 3 rather than –3 to the –2 term in $(2x - 2)$. If you chose **A**, perhaps you forgot to distribute the 7 and the –3 to the second term in each set of grouping symbols, setting $7(x + 3)$ equal to $7x + 3$ and $-3(2x - 2)$ equal to $-6x - 2$.

Question 22. The correct answer is G. To find 75% of a number, n, for which 115% of n is 460, you first set up an equation to find n using the fact that 115% of n is 460, or $1.15n = 460$. After dividing by 1.15, you find $n = 400$. Then, 75% of 400 is 400(0.75), or 300.

If you chose **J**, you probably found 75% of 460 as 460(0.75), or 345. If you chose **K**, you probably found n using the equation $1.15n = 460$ by dividing by 1.15 and getting 400.

Question 23. The correct answer is C. This problem tests your knowledge of how to square a binomial. The expression $(2x - 3)^2$ can be expanded into the $ax^2 + bx + c$ form using the distributive property as shown in the following:

$$(2x - 3)^2 = (2x - 3)(2x - 3) = (2x)(2x) + (2x)(-3) + (-3)(2x) + (-3)(-3)$$

$$= 4x^2 - 6x - 6x + 9 = 4x^2 - 12x + 9$$

When the coefficients of $4x^2 - 12x + 9$ are matched with the coefficients of $ax^2 + bx + c$, you can see that $a = 4$, $b = -12$, and $c = 9$, and that $a + b + c = 4 + (-12) + 9 = 1$.

When you square a binomial, you must multiply two binomial expressions using the distributive property. Common errors result from reasoning that $(2x - 3)^2$ is equivalent to $(2x)^2 + (-3)^2$ or $(2x)^2 - (3)^2$, resulting in **B** or **D**.

Question 24. The correct answer is H. Two common approaches are often used in solving this problem.

In the first approach, the polygon can be divided into a 15 ft by 15 ft square and a 10 ft by 5 ft rectangle (see the following Figure 1). The area of the polygon is equal to the sum of the areas of the rectangle and the square: (15)(15) + (10)(5) = 275 square feet.

In the second approach, you take the rectangle formed by the 15 ft and 25 ft sides of the polygon and "cut away" a 10 ft by 10 ft square (see the following Figure 2). In this case, the area of the polygon is equal to the difference of the areas of the rectangle and the square: (15)(25) − (10)(10) = 275 square feet.

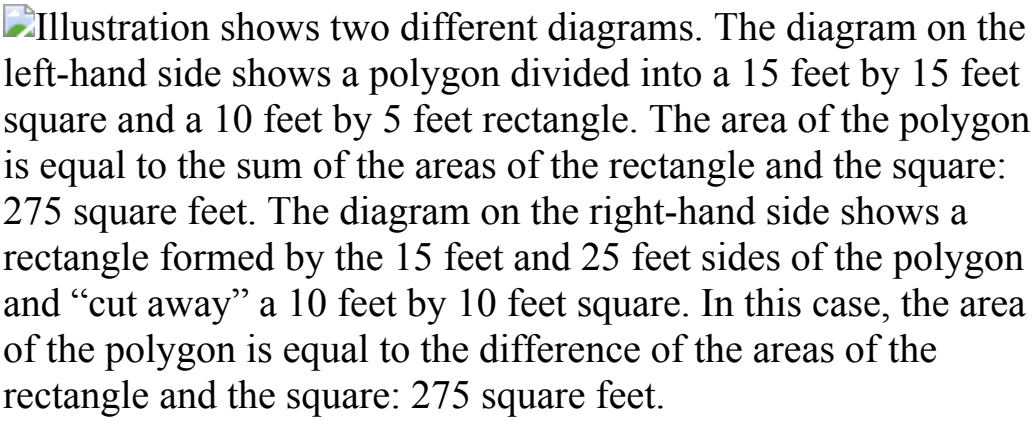
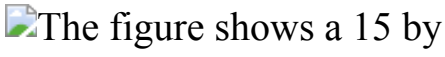


Illustration shows two different diagrams. The diagram on the left-hand side shows a polygon divided into a 15 feet by 15 feet square and a 10 feet by 5 feet rectangle. The area of the polygon is equal to the sum of the areas of the rectangle and the square: 275 square feet. The diagram on the right-hand side shows a rectangle formed by the 15 feet and 25 feet sides of the polygon and "cut away" a 10 feet by 10 feet square. In this case, the area of the polygon is equal to the difference of the areas of the rectangle and the square: 275 square feet.

Question 25. The correct answer is B. The way to use the minimum number of blocks is to have a side of the block with the largest area face upward. That is the side that is 4″ by 8″. The 4″ width of the blocks will fit 3 to each foot. The 8″ length of the blocks will fit 3 to each 2 feet. The blocks could be arranged as shown, with 24 block widths in one direction and 15 block lengths in the other direction. That makes $15 \cdot 24 = 360$ blocks. The blocks could be arranged in different patterns, but the top area of all the blocks has to equal the (8)(10) = 80-square-foot area Barb is covering.



An alternate way to work this problem is to calculate the total area, 80 square feet, and divide by the largest area a single block can cover, 32 square inches. The area to be covered is (8)(12) = 96 inches in one direction and (10)(12) = 120 inches in the other direction, which makes the total area (96)(120) = 11,520 square inches. If you divide this by the area of a single block, 32 square inches, you will get inline = 360 blocks.

If you got answer **A**, you may have calculated the area of the patio in square feet.

Question 26. The correct answer is H. To find the slope, you can manipulate the equation $6y - 14x = 5$ algebraically in order to find its equivalent equation expressed in *slope-intercept form*, which is $y = mx + b$, where m is the slope and b is the y-intercept. The manipulations are shown in the following:

The slope of the line equals inline, or inline when reduced to lowest terms.

Question 27. The correct answer is C. First we will show that $m <$ inline:

Because m and n are positive integers such that $m < n$, $n = m + k$ where k is a positive integer and $m \geq 1$. For this reason, we know inline = inline = inline Because the square root function increases as its input increases and inline > inline, inline > inline, and inline. Thus, inline.

Then, we will show that inline by a similar argument:

Start by solving $n = m + k$ for m: $n - k = m$. Then write inline = inline= inline. Because inline < inline, inline < inline, and inline. Thus, inline.

We have shown $m <$ inline and inline $< n$; thus, $m <$ inline.

If you chose **A** or **B**, you might have not realized that inline must be larger than m. Notice that for $m = 1$ and $n = 4$, inline because $2 > 1$. If you chose **D**, you might have not considered cases such as when $m = 1$ and $n = 2$. Notice that for this case inline = inline; thus, inline< inline is false. If you chose **E**, you might have not considered cases such as when $m = 3$ and $n = 4$. Notice that for this case inline > inline because inline > inline; thus, inline < inline is false.

Question 28. The correct answer is J. Similar triangles are triangles whose corresponding sides are proportional. The solution, x, is found by setting up the following proportion:

image

To solve inline = inline for x, cross-multiply to obtain the equivalent equation $3x = 75$, and divide by 3 to obtain $x = 25$.

Question 29. The correct answer is A. The area of the whole board is inline $= \pi \cdot 10^2 =$ inline square inches. The radius of the outside of the 20 ring is $10 - 2 = 8$ inches. The radius of the outside of the 30 ring is $8 - 2 = 6$ inches, so the area of the circle that includes 30, 40, and 50 points is inline $\cdot 6^2 =$ inline square inches. If a dart hits at a random spot on the board, the chance of it hitting in a certain region is proportional to the area of that region. So, the percent chance of hitting inside a region that is worth at least 30 points is inline $\cdot$ 100% = 36%.

The figure shows a dart board. The board represents five concentric circles. The radius of the outside circle is 10 inches, and each of the other circles has a radius 2 inches smaller than the next larger circle.

C is the percent chance of getting *more* than 30 points, using the 4-inch radius of the inside of the 30 ring.

Question 30. The correct answer is G. You can use translation skills to set up an algebraic equation that, when solved, yields the solution to the

problem.

1. Let the variable *a* represent the teacher's age.
2. "If you *square* my age" translates into "a^2."
3. "23 *times* my age" translates into "23*a*."
4. "then *subtract* 23 times my age" translates into "$a^2 - 23a$." Because the words *than* and *from* do not appear in the sentence, the order of the terms "a^2" and "23*a*" is NOT reversed when translated into mathematical language.
5. "the result is 50" translates into "= 50."

Therefore, the translation gives the equation $a^2 - 23a = 50$, which you may solve by subtracting 50 from both sides and factoring.

The equation $a^2 - 23a - 50 = 0$ is true, provided that $(a + 2)(a - 25) = 0$, which happens if $a = -2$ (but age cannot be negative) or $a = 25$.

Question 31. The correct answer is B. "If a car accelerates from a stop at the *rate* of 20 meters per second per second" implies that $a = 20$, and "travels a *distance* of 80 meters" implies that $d = 80$. Substituting these values into the equation $d =$ inline gives the equation $80 =$ inline $(20)t^2$ or, equivalently, $80 = 10t^2$, or $8 = t^2$. Therefore, $t =$ inline ≈ 2.8 seconds.

Question 32. The correct answer is F. To find the real numbers x such that $x + 3 > x + 5$, you would subtract x and 3 from both sides. The result is $0 > 2$, but that inequality is never true, so there is no solution for x. It is the empty set.

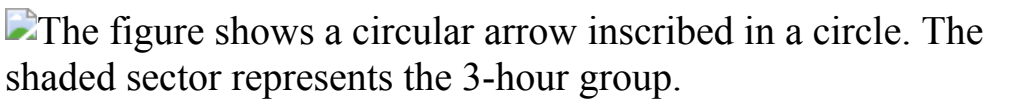
If you chose **G**, you probably switched the inequality and got $0 < 2$ after you subtracted x and 3 from both sides. If you chose **H**, you probably got $0 > 2$ and then thought that a negative value for x would change the inequality.

Question 33. The correct answer is D. You must first determine from the frequency bar graph the number of students in the class responding that they spent 0, 1, or 2 hours studying on the previous evening. The bars in the graph indicate that 2 students studied 0 hours, 5 students studied 1 hour,

and 6 students studied 2 hours. Therefore, the fraction of students in the class that responded they had spent less than 3 hours studying is equal to inline = inline = inline.

If you chose **B** (the most common incorrect answer), perhaps you overlooked the phrase "less than" and selected the number of students studying exactly 3 hours as the numerator, obtaining the fraction inline, or inline.

Question 34. The correct answer is J. In the following figure, the shaded sector represents the 3-hour group:


The figure shows a circular arrow inscribed in a circle. The shaded sector represents the 3-hour group.

For this circle graph, the ratio inline is equivalent to the ratio inline. These ratios, in turn, are equivalent to the ratio inline. Letting the numerator of this last ratio equal $x°$, and using the fact that the denominator of this ratio is 360°, you obtain the proportions inline= inline = inline, so $x = 72$.

Question 35. The correct answer is B. Because this frequency bar graph gives the number of times each response was given, the frequency bar graph was constructed from the following 20 data values:

0, 0, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 5

The average number of hours for the 20 responses is equal to the average of the data values, which is defined to be inline.

This is equal to inline, or, equivalently, inline = inline = 2.1.

Question 36. The correct answer is H. To find the number of diagonals the octagon has, you would label the 8 vertices as endpoints. Those segments (which are the diagonals) are inline, inline, inline, inline, inline, inline, inline, inline, inline, inline, inline, inline, inline,

inline, inline, inline, inline, inline, inline, inline, and inline. There are 20 diagonals.

If you chose **F**, you probably just found the number of vertices in an octagon. If you chose **G**, you probably just multiplied the number of vertices by the number of endpoints of a diagonal, 8(s), or 16.

Question 37. The correct answer is E. As shown in the following figure, the tether line, the level ground, and a line segment representing the altitude of the bottom of the basket form a right triangle.

> The figure shows a right-angled triangle, where the angle is labeled as "72 degree" and hypotenuse is labeled as "144 feet." A hot-air balloon is placed on the top side of the right-angled triangle.

Let the length of the altitude equal h feet. You can use a trigonometric ratio to find h. The tether line forms the hypotenuse of the right triangle, and the line segment representing the altitude is the side opposite the 72° angle. Therefore, the trigonometric ratio to be used with respect to the 72° angle is the sine ratio, which gives the following equation:

sin 72° = inline = inline

Multiplying both sides of the equation inline = sin 72° by 144 yields the value of h, which is 144 sin 72°.

Question 38. The correct answer is G. The midpoint of a line segment is the point halfway between the two endpoints of the line segment. A formula for finding the midpoint (x_m,y_m) of two points (x_1,y_1) and (x_2,y_2) in the standard (x,y) coordinate plane is (x_m,y_m) = inline, inline The x-coordinate of the midpoint, x_m, is the *average* of the x-coordinates of the endpoints of the line segment. In the case of inline, x_m = inline = inline = –3.

Question 39. The correct answer is E. To determine the value of $8x + 9y$, find the solution (x,y) to the system of two equations given in the problem.

This system can be solved using an elimination method. First, determine which of the two variables would be easiest to eliminate. For this system, eliminating the y-variable would be easier. In order to eliminate the y-variable, the y-coefficients in each equation (or equivalent equation) must be opposite numbers (–6 and 6 would be the best choice). Therefore, all terms in the upper equation should be multiplied by –2 (Step 1 following) to form an equivalent equation with –6 as the y-coefficient. Now the two equations have y-coefficients of –6 and 6, respectively, so adding the equations will eliminate the y-variable (Step 2 following).

Substituting into either of the two initial equations produces an equation that can be solved for y; for example, letting $x = -1$ in $2x + 3y = 4$ gives $2(-1) + 3y = 4$. Solving for y, $-2 + 3y = 4$, so $3y = 6$. Therefore, $y = 2$.

Substituting $x = -1$ and $y = 2$ into $8x + 9y$ yields $8(-1) + 9(2) = -8 + 18 = 10$.

Question 40. The correct answer is H. If we draw  in standard position and its terminal side intersects the unit circle at (x,y), then tan  =  . Because we want to solve $\tan \theta = -1$, we want the ratio = –1. This only happens when =  or =  . Recall that tan < 0 only when the terminal side of θ is in Quadrants II or IV.

You may have gotten **K** if you solved tan = ±1. You may have gotten **F** if you thought the tangent of an angle was negative when its terminal side was in Quadrants I and II. You may have gotten **G** if you thought the tangent of an angle was negative when its terminal side was in Quadrants II and III. You may have gotten **J** if you thought the tangent of an angle was negative when its terminal side was in Quadrants III and IV.

Question 41. The correct answer is B. By definition, $i^2 = -1$, so $i^4 = (i^2)^2 = (-1)^2 = 1$. Therefore, $i^x = 1$ for $x = 4$, so 1 is a possible value of i^x when x is an integer. More generally, when x is an integer, the only values of i^x possible are i, -1, $-i$, and 1, as shown in the following table:

This rules out **A**, **C**, **D**, and **E.**

Question 42. The correct answer is F. Because the diameter of the can is 2 inches and the diameter of the glass is 3 inches, the radius of the can is 1 inch and the radius of the glass is 1.5 inches. Therefore, using $\pi r^2 h$, the volume of the can is $\pi(1^2)(6)$, or 6π, and the volume of the glass is $\pi(1.5^2)h$, or $2.25\pi h$. Because the volume of the soda pop can and the volume of the glass are equal, $6\pi = 2.25\pi h$. Solving for h gives us $h = 2$ inline.

You may have gotten **G** if you thought that height and diameter were directly proportional: $2(6) = 3h$. You may have gotten **H** if you thought that because the radius increased by 1 inch, then the height should decrease by 1 inch. You may have gotten **J** if you added all the values in the stem. You may have gotten **K** if you did inline

Question 43. The correct answer is C. The volume in cubic meters, V, of a right circular cylinder of radius r meters and height h meters is given by the formula $V =$ inline. For the cylinder in this problem, $r = 5$ and $h = 6$. Therefore, $V = \pi(5^2)(6) = 150\pi$ cubic meters.

Question 44. The correct answer is F. The three triangles in the given figure (inlineABC, inlineADE, and inlineAFG) can be shown to be similar by use of the angle-angle similarity property.

Because inlineABC inline inlineAFG, the statement "The ratio of the perimeter of inlineABC to the perimeter of inlineAFG is 1:3" implies that the ratio of AC to AG is 1:3. So if $AC = 1$ unit, then $AG = 3$ units (see the following Figure 1).

Because inlineADE inline inlineAFG, the statement "The ratio of DE to FG is 2:3" implies that the ratio of AE to AG is 2:3. So if $AG = 3$ units, then $AE = 2$ units (see the following Figure 2).

This means that $AE = 2$ units when $AC = 1$ unit, implying that $CE = 1$ unit (see the following Figure 3). Therefore, the ratio of AC to CE is 1:1.

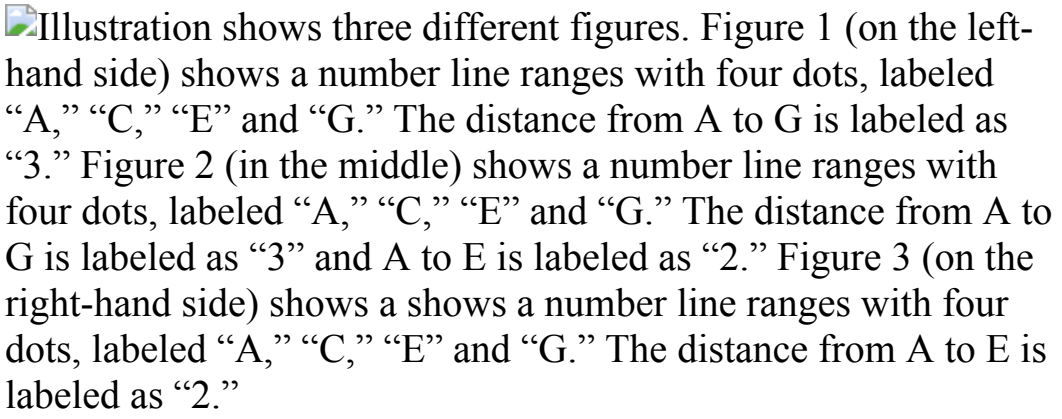


Illustration shows three different figures. Figure 1 (on the left-hand side) shows a number line ranges with four dots, labeled "A," "C," "E" and "G." The distance from A to G is labeled as "3." Figure 2 (in the middle) shows a number line ranges with four dots, labeled "A," "C," "E" and "G." The distance from A to G is labeled as "3" and A to E is labeled as "2." Figure 3 (on the right-hand side) shows a shows a number line ranges with four dots, labeled "A," "C," "E" and "G." The distance from A to E is labeled as "2."

Question 45. The correct answer is E. The following Figure 1 shows the first phase, when the rocket traveled vertically for 30 kilometers.

Figure 2 shows the second phase, when the rocket traveled 40 km at 30° from the vertical. The three distances shown in Figure 2 are in the ratio 1: inline:2, a characteristic of 30°−60°−90° triangles. The vertical distance covered in the second phase is 20inline km.

Figure 3 shows the third phase, when the rocket traveled 100 km at 45° from the vertical. The three distances shown in Figure 3 are in the ratio 1:1: inline, a characteristic of 45°−45°−90°
triangles. The vertical distance covered in the third phase is 50inline km.

Taking the sum of the vertical distances covered by each of the three phases gives the vertical distance of the rocket above the launch pad after the third phase.

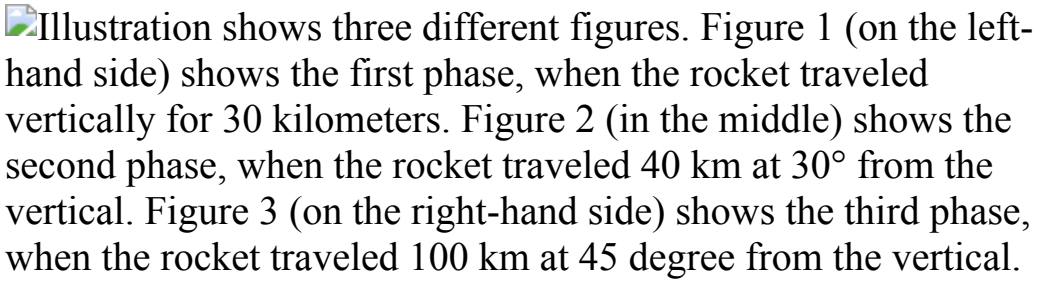


Illustration shows three different figures. Figure 1 (on the left-hand side) shows the first phase, when the rocket traveled vertically for 30 kilometers. Figure 2 (in the middle) shows the second phase, when the rocket traveled 40 km at 30° from the vertical. Figure 3 (on the right-hand side) shows the third phase, when the rocket traveled 100 km at 45 degree from the vertical.

Question 46. The correct answer is G. Let X be a random variable that can take on all and only the values of $x_1, x_2, x_3,$ inline x_n. The expected value of X is defined by $EV(X) = x_1p_1 + x_2p_2 +$ inline $+ x_np_n$, where, for $k = 1, 2, 3,$ inline n, X takes the value of x_k with a probability of p_k. Using this formula, we see $EV(n) = 0(0.70) + 1(0.20) + 2(0.05) + 3(0.05) = 0.45$, **G**. If you chose either **F** or **H**, you probably did not recall the definition of the expected value of a random variable. If you chose **J**, you added the probabilities of the four possible values of n. If you chose **K**, you found the mean of the four possible values of n. In all four of these incorrect cases, please see the previously given definition of expected value.

Question 47. The correct answer is E. We can find the factored form of the equation by first factoring out a GCF of –2, $h = –2(t^2 − 5t − 24)$, and then further factoring the quadratic to $h = –2(t + 3)(t − 8)$. The object reaches the ground when $h = 0$ and time is positive. By the zero product property, $0 = –2(t + 3)(t − 8)$ when $t = –3$ or $t = 8$. Because 8 is the positive value of t that is the solution to $h = 0$, the object will hit the ground at 8 seconds.

If you chose **D**, you might have picked the negative value of t in the solution to $0 = –2(t + 3)(t − 8)$ and then taken the absolute value. If you chose **B** or **C**, you might have confused the factored form of a quadratic equation with the vertex form. This equation can be written as $h = –2(x − 2.5)^2 + 60.5$, so the maximum height is 60.5 units. If you chose **A**, you might have not realized that the starting point is the value of h when $t = 0$, which is 48.

Question 48. The correct answer is H. Rewrite the values in the radicand as exponents, and then simplify.

image

You probably got **F** if you did $g^2 +$ inline $\cdot h^2 +$ inline. You probably got **G** if you did $g^{\text{inline}} \cdot h^{\text{inline}}$. You probably got **J** if you did $g^2 +$ inline $\cdot h^2 +$ inline. You probably got **K** if you did $g^{\text{inline}} \cdot h^{\text{inline}}$.

Question 49. The correct answer is D. The value of $\log_5($inline$)$ is found by solving the equation $\log_5($inline$) = x$. By definition of logarithm to the base 5, this equation is equivalent to the equation $5^x =$ inline. The equation $5^x =$ inline is equivalent to the equation $x =$ inline, whose value is between 6 and 7.

Question 50. The correct answer is F. We need to find the percent decrease, which is found by using the formula for percent change.

percent change = inline

The Size 3 unit is \$100 per month, and the special rate is \$1 for the first month. The original cost would have been 12(100), but because there is a special rate of \$1 for the first month, Daria only needs to pay the original monthly fee for 11 months, 11(100) + 1. Use the formula to find the percent decrease.

inline × 100 = inline × 100

You may have gotten **G** if you did inline × 100. You may have gotten **H** if you did inline × 100.
You may have gotten **J** if you did inline × 100. You may have gotten **K** if you did inline × 100.

Question 51. The correct answer is E. The Size 5 unit is 8 × 16, and the Size 1 unit is 2 × 4. One way to solve is to divide the width by width and length by length, if width × length. We can see that the Size 1 unit's width of 2 can fit into the Size 5 unit's width of 8 four times, or inline.
Similarly, we see that Size 1 unit's length of 4 can fit into Size 5 unit's length of 16 four times, or inline. Because we are dealing with area, we multiply inline(inline), or 4 × 4, to get the maximum number of Size 1 units into Size 5 units. A drawing would also help you see this answer. A possible drawing:

image

You may have gotten **A** if you divided length from the Size 1 unit and width from the Size 5 unit as  . You may have gotten **B** if you divided only the widths as  . You may have gotten **C** if you did  +  . You may have gotten **D** if you did  +  .

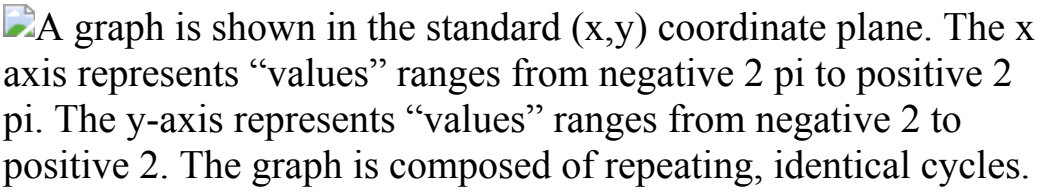
Question 52. The correct answer is H. x = unit size number. The relationship between the unit size number and the area of each unit is as follows:

x	Area	Pattern
1	8	8×2^0
2	16	8×2^1
3	32	8×2^2
4	64	8×2^3
5	128	8×2^4

Notice as the unit size number increases, the area increases exponentially. The pattern is shown in the table.

The expression that summarizes this pattern is $8 \times 2^{(x-1)}$ or $2^3 \times 2^{(x-1)}$. Using properties of exponents, the simplest form is $2^{(2+x)}$. You may have gotten **F** if you thought the pattern was 8 × unit size number. You may have gotten **G**, **J**, or **K** if you did not understand the pattern.

Question 53. The correct answer is B. The graph of any trigonometric function of the form $y = a\sin(bx + c)$ is cyclical. That is, the graph is composed of repeating, identical *cycles*. The shaded region in the following graph shows one such cycle. The *period* of the function is the width of the smallest interval of x on which one of these cycles appears. In the following graph, the period is the width of the shaded region, given by  − $\pi = \pi$.


A graph is shown in the standard (x,y) coordinate plane. The x axis represents “values” ranges from negative 2 pi to positive 2 pi. The y-axis represents “values” ranges from negative 2 to positive 2. The graph is composed of repeating, identical cycles.

The period of $y = \sin x$ is inline, which is **C**, the most common incorrect answer. If you chose **E**, you may have confused *period* with *amplitude.*

Question 54. The correct answer is G. The component form of w can be found by adding the opposite of the scalar horizontal components together and the opposite of the scalar vertical components together:

$-2\langle 5,3\rangle + 3\langle 2,-7\rangle = \langle -10+6,-6-21\rangle$.

If you got **F**, you may have done

$-2\langle 5,3\rangle - 3\langle 2,-7\rangle = \langle -10-6,-6+21\rangle$.

If you got **H**, you may have done $\langle 5,3\rangle - \langle 2,-7\rangle = \langle 5-2,3+7\rangle$.

If you got **J**, you may have done

$2\langle 5,3\rangle - 3\langle 2,-7\rangle = \langle 10-6,6+21\rangle$.

If you got **K**, you may have done

$2\langle 5,3\rangle + 3\langle 2,-7\rangle = \langle 10+6,6-21\rangle$.

Question 55. The correct answer is B. We can rewrite the bases as powers of a common base. The common base here is 3, because $3^1 = 3$, and $3^2 = 9$.

$3^{x+1} = 3^{2(x-2)}$

$3^{x+1} = 3^{2x-4}$

$x + 1 = 2x - 4$

$x = 5$

This is the one and only solution that makes the equation true.

If you chose **A**, you may have set the exponents equal to each other before finding a common base and found there was no solution, OR you may have

not found a common base and tried instead to solve for x. If you chose **C**, you may have rewritten 9 as 3^2 and misinterpreted the squared for two solutions. If you chose **D**, you may have done inline. If you chose **E**, you thought the exponential equation had an infinite number of solutions.

Question 56. The correct answer is H. The area of the original triangle is inline(20)(15)sininline.
The area of the resulting triangle is inline(20)(15 − 2*t*)sininline. To find the time, t, when the resulting triangle has an area that is inline the area of the original triangle, you need to set the area of the resulting triangle equal to inline the area of the original triangle and solve for t:

image

If you chose **F**, you might have incorrectly set the area of the original triangle equal to the area of the resulting triangle and solved for t: inline (20)(15)sininline = inline(20)(15 − 2*t*)sininline. If you chose **G**, **J**, or **K**, you might have set up the problem correctly but made a mistake in your algebra. For **G**, you might not have distributed the four correctly:

image

For **J**, you might have dropped the inline in the area of the original triangle:

image

For **K**, you might have thought inline · inline canceled out, incorrectly distributed the inline in the area of the resulting triangle, and then dropped the negative:

image

Question 57. The correct answer is E. Distinct permutations are permutations without repetition. We want to find how many unique orderings there are of the letters PEOPLE. The number of ways to order 6 different letters would be 6!. Because the P and the E are repeated twice,

we must divide by 2! to account for the repeated P and again by 2! to account for the repeated E. Thus we get inline.

If you got **A** you probably didn't account for letters P and E that cannot be repeated. If you got **B** you probably didn't understand distinct permutations. If you got **C** you probably considered the 6 letters in the word and 4 different letters in the word. If you got **D** you probably considered the 6 different letters in the word and that only 1 letter repeated.

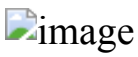
Question 58. The correct answer is K. Complex conjugate pairs can be written in the form $(a + bi)(a - bi)$, which when simplified equals $a^2 - b^2i^2 = a^2 + b^2$.

The value of $i^2 = -1$; therefore, when $(7x - 9i)(7x + 9i)$ is simplified, it equals $49x^2 - 81i^2 = 49x^2 - 81(-1) = 49x^2 + 81$. You may have gotten **F** if you took the square root of $49x^2$ and 81. Notice that $(7x + 9)^2 = 49x^2 + 126x + 81$. You may have gotten **G** or **H** if you forgot that a complex conjugate pair consists of one expression with an addition and its conjugate pair has subtraction. You may have gotten **J** if you forgot the i in the conjugate pairs.

Question 59. The correct answer is B. Given an input x_i, the absolute value of the residual gives the vertical distance between the observed y_i-value and the $y(x_i)$-value predicted by the line of best fit. By inspecting the graph, the ordered pair (32, 135) appears to have the greatest vertical distance from the line of best fit. The absolute value of this residual is given by $|135 - y(32)| = |135 - (1.1(32) + 93)| = |135 - 128.2| = 6.8$. The remaining residuals are $|120 - y(27)| = |127 - (1.1(27) + 93)| = 2.7$, $|140 - y(42)| = |140 - (1.1(42) + 93)| = 0.8$, and $|130 - y(37)| = |130 - (1.1(37) + 93)| = 3.7$. If you chose **A**, you may have chosen the correct ordered pair (32, 135) but incorrectly computed the absolute value by adding 32 and 93 before multiplying by 1.1. If you chose **C**, you probably computed the difference in the 2 y_i-values 135 and 120. If you chose **D**, you may have incorrectly thought that the residual is the absolute value of the difference in the largest and smallest y_i-values, 140 and 120. If you chose

E, you may have correctly identified the point farthest from the line but computed the vertical distance between 135 and the *y*-intercept of the line, 93.

Question 60. The correct answer is K. Making at least 1 error on any given day and making no errors are complementary events. Hence,



If you chose **F**, you might have thought that the probability of *at least* 1 error is the same as the probability of *exactly* 1 error,  . If you chose **G** or **J**, you might have thought that the table shows the probabilities for making any number of errors possible on a given day and not realized that it only shows the probabilities for making 0–4 errors. For **G**, you might have thought that the probability of at least 1 error is the same as the sum of the probabilities of 2–4 errors,  . For **J**, you might have thought that the probability of at least 1 error is the same as the sum of the probabilities of 1–4 errors,  . If you chose **H**, you might have thought that the probability of making at least 1 error is equal to the complement of the sum of the probability of 1 error and the probability of no errors, 

Question 61. The correct answer is H. To find an equivalent expression, you can multiply the constants ($3 \cdot 2 \cdot 4$), combine the *x* terms ($x^3x^2x^2 \Rightarrow x^{3+2+2} \Rightarrow x^7$, because when you have a common base you use the base and add the exponents), and combine the *y* terms ($y \cdot y \Rightarrow y^1y^1 \Rightarrow y^{1+1} \Rightarrow y^2$). The result is $24x^7y^2$.

K is the most common incorrect answer and comes from multiplying the exponents on the *x* terms instead of adding. If you chose **F**, you probably added the constants instead of multiplying. If you chose **G**, you could have added the constants and multiplied the exponents on the *x* terms instead of adding. If you chose **J**, you possibly multiplied the exponents on the *x* terms and *y* terms instead of adding.

Question 62. The correct answer is A. To find Mr. Dietz's pay per day, you can divide his salary, $22,570, by the number of days he works, 185. His pay per day is  , or $122. When Mr. Dietz takes a day off

without pay and the school pays a substitute $80, the school district saves the difference in these amounts, 122 − 80, or $42.

If you chose **B**, you probably just picked a number from the problem. If you got **E**, you probably found Mr. Dietz's pay per day and stopped.

Question 63. The correct answer is J. To find what the student needs to score on the fifth 100-point test to average a score of 80, you need to find the point total for the student so far by adding 65, 73, 81, and 82. That sum is 301. Averaging 80 points on 5 tests means the student must earn 400 points (80 • 5). The score needed on the last test is the difference, 400 − 301, or 99.

F is the average of the 4 scores, rounded to the nearest whole point. If you chose **H**, you probably took the average of 65, 73, 81, and 82, averaged that average with 80, and rounded to the nearest whole point. If you chose **K**, you possibly thought you needed 5(100), or 500, points total, and this total is not possible when adding a number 100 or less to 301.

Question 64. The correct answer is D. To find the oxygen saturation loss, you divide the current number of milligrams of dissolved oxygen per liter of water by the dissolved oxygen capacity in milligrams per liter of water, or  . Then, you approximate that fraction as a decimal, 0.7449, then convert to a percent, 74.49%, and round to 74%.

If you chose **A**, you probably divided 9.8 by 7.3, subtracted 1, converted to a percent, and rounded to the nearest whole percent. If you chose **B**, you probably rounded to the nearest 10%, that is, 74.49% to 70%. If you chose **C**, you probably just used numbers from the problem.

Question 65. The correct answer is J. To find the length of fence needed to fence a rectangular lot 150 ft by 200 ft, you need to find the perimeter. The formula for the perimeter of a rectangle is 2 times the sum of the length and width, or $P = 2(l + w) = 2(150 + 200) = 2(350) = 700$.

If you chose **G**, you probably added the dimensions, but didn't double the sum. If you chose **F** or **H**, you possibly used only one dimension and doubled it.

Question 66. The correct answer is A. To find an equivalent expression, multiply a by $b + c - d$. This results in $a(b) + a(c) + a(-d)$, or $ab + ac - ad$.

If you chose **E**, you probably forgot to distribute the a to c and d.

Question 67. The correct answer is F. To solve for x in the equation $4x + 3 = 9x - 4$, you could subtract $4x$ and add 4 to both sides. That results in the equation $7 = 5x$. Then, dividing both sides by 5, the result is  .

If you chose **G**, you probably got to $7 = 5x$ and then divided 5 by 7. If you chose **H**, you probably added $4x$ to $9x$, resulting in $7 = 13x$, and then divided by 13. If you chose **J**, you might have combined the 3 and -4 and somehow got 1, then got to $1 = 5x$ and divided both sides by 5.

Question 68. The correct answer is C. These 4 numbers will be an arithmetic sequence. In an arithmetic sequence, each pair of successive terms differs by the same amount. To find the difference, you can define d as that difference and let 17 be the first term and 41 be the fourth term. By definition, the second term is $17 + d$ and the third term is $(17 + d) + d$. The fourth term, 41, can also be written as $(17 + d + d) + d$. Using that expression you obtain the equation $41 = 17 + d + d + d$, or $41 = 17 + 3d$. After subtracting 17 from both sides, you can then divide by 3, resulting in $8 = d$. The difference is 8. Then, the second term is $17 + 8$, or 25. The third term is $17 + 8 + 8$, or 33.

If you chose **A**, you probably reasoned that because 41 is the fourth term, the relationship is $4d = 24$ (rather than $3d = 24$) and so the difference is 6. If you chose **B**, you probably added 7 to the first term and subtracted 7 from the fourth term. If you chose **E**, you probably added 10 to the first term and subtracted 10 from the fourth term.

Question 69. The correct answer is H. To find what  equals, you need to solve $x^3 = 64$ for x. The solution is  , which is 4. Then, substituting into the original expression, you get $4^2 +$  . This expression simplifies to $16 + 2$, or 18.

If you chose **F**, you probably solved $x^3 = 64$ for x and stopped. If you chose **G**, you could have gotten $x = 4$, used 4(2) for 4^2, and added 4(2) and 2 to get 10. If you chose **J**, possibly you got $x = 4$ and then simplified inlineto be 4.

Question 70. The correct answer is C. To find the volume, you substitute inlinefor r in the equation inline. This yields inline, orinline. This expression is about 8.18, or 8 to the nearest cubic inch.

If you chose **A**, you might have substituted to getinline, yieldinginline, which is about 5. If you chose **B**, you probably substituted to get inline, yielding inline, or about 7. If you chose **D**, you probably substituted to getinline, yielding 5π, or about 16.

Question 71. The correct answer is K. The probability that the marble chosen will not be white when 8 marbles are red, 6 are blue, and 6 are white is the number of favorable outcomes divided by the total number of possible outcomes. The number of *favorable* outcomes is 14 because there are 8 red marbles and 6 blue marbles—a total of 14 marbles. The total number of *possible* outcomes is 8 + 6 + 6 = 20, the total number of marbles. Thus, the probability of the marble NOT being white is inline.

If you chose **G**, you probably added the number of blue marbles and the number of white marbles and divided by the total number of marbles: inline. If you chose **H**, you probably found inline. If you chose **J**, you probably found the probability of choosing a white marble:inline.

Question 72. The correct answer is D. To find the number of sports awards earned, the number of participants in each sport is multiplied by the ratio for that sport and then the 4 products are added. This is a matrix multiplication.

inline

If you chose **B**, you probably reversed the order on the first matrix to get 80(0.3) + 80(0.4) + 60(0.2) + 40(0.5) = 24 + 32 + 12 + 20 = 88. If you chose **C**, you probably totaled the number of athletes and multiplied it by the average of the ratios, 260(0.35), which is 91.

Question 73. The correct answer is H. To find the average number of students enrolled per section of Algebra I, you add up the students in all the sections and divide by the number of sections. Thus, you add 24 + 25 + 29 and get 78, then divide by 3. This results in an average of 26 students enrolled per section in Algebra I.

If you chose **G**, you could have found the median (or middle number) of 24, 25, and 29. Sometimes, *average* can mean the median or the mode. For this test, the directions say that, unless otherwise stated, "The word *average* indicates arithmetic mean." If you chose **J**, you likely found the average of 25 and 29.

Question 74. The correct answer is C. The total number of calculators available is 30 − 2 + 30 − 6 = 52. To find the class periods for which there are not enough school calculators, find the total needed for each period, as given in the table below.

Period	1	2	3	4	6
Calculators needed	21	46	48	57	19

The only entry in the table more than 52 is 57 for Period 4.

If you chose **D**, you possibly looked at the Algebra I rows in the table and saw that Section B and Section C could not both be covered by the available calculators, and these sections are in Period 3 and Period 4. If you chose **E**, you probably used 60 for the available number of calculators and did not take into account the 8 calculators that are being repaired and are unavailable.

Question 75. The correct answer is H. Because the sum of each row is equivalent, the sum of Row 1 is the same as the sum of Row 2.

The question mark must represent 2x. You could have done this with other rows, columns, or diagonals.

If you chose **G**, you probably just added the first and last entries in either Row 2, Column 2, or one of the diagonals. If you chose **K**, you may have thought that each sum must be 0 and found that −4x would make the sums of Row 2, of Column 2, and of both diagonals 0.

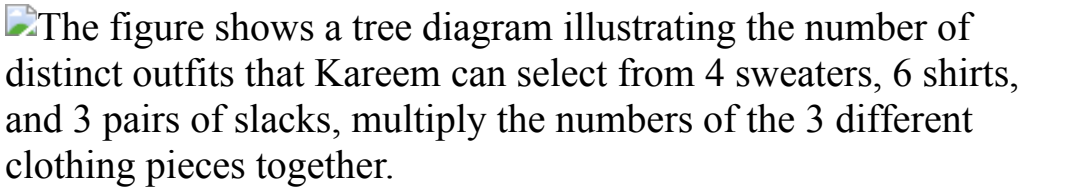
Question 76. The correct answer is E. The *x*-coordinate is positive if *A* is to the right of the *y*-axis. The *y*-coordinate is positive if *A* is above the *x*-axis. The table below shows the sign of *x* and the sign of *y* in the four quadrants.

	Sign of:	
Quadrant	***x***	***y***
I	+	+
II	−	+
III	−	−
IV	+	−

Thus, the signs are opposite in Quadrants II and IV only.

If you chose **C** or **D,** you probably got confused about where *x* and *y* are positive and negative or about the order of the quadrants.

Question 77. The correct answer is J. To find the number of distinct outfits that Kareem can select from 4 sweaters, 6 shirts, and 3 pairs of slacks, multiply the numbers of the 3 different clothing pieces together. Thus, there are 4(6)(3), or 72, distinct outfits that Kareem can select. The figure below shows that for each sweater, there are 6 shirts, and for each shirt, there are 3 pairs of slacks.


The figure shows a tree diagram illustrating the number of distinct outfits that Kareem can select from 4 sweaters, 6 shirts, and 3 pairs of slacks, multiply the numbers of the 3 different clothing pieces together.

If you chose **F**, you probably added the 3 numbers together, getting 4 + 6 + 3 = 13.

Question 78. The correct answer is A. To find the number of tons of sand needed to produce 3,000 barrels of a tarry material that requires 100,000 tons of sand for 60,000 barrels, you can set up a proportion with ratios of tons of sand to barrels of tarry material, such as inline, which results in 5,000 tons of sand.

If you chose **B**, you probably calculated inline.

Question 79. The correct answer is H. The following figure shows the rectangle and a diagonal. To find the length of the diagonal, you could use the Pythagorean theorem because the sides of the rectangle are the legs of a right triangle and the diagonal of the rectangle is the hypotenuse of the right triangle. Then $h^2 = 72^2 + 54^2 \Rightarrow h = 90$.

> The figure shows a rectangle with a diagonal. The length of the diagonal is labeled as "h." The base of the rectangle is labeled as "72 meters" and the height as "54 meters."

G is the average of 54 and 72. If you chose **J**, you probably added 54 and 72.

Question 80. The correct answer is A. To find an equivalent expression for inline, you must either multiply or divide both the numerator and the denominator by the same value. Multiplying inline by inline yields inline.

If you chose **B**, you probably thought you could multiply by the expression and obtain an equivalent expression, but if inline, theninline. If you chose C, you probably thought you could multiply by the reciprocal and obtain an equivalent expression, but if inline, then inline. If you chose **E**, you probably thought you could add the same number to both the numerator and the denominator and obtain an equivalent expression, but if inline, and $z = 2$ then inline.

Question 81. The correct answer is H. To find the slope-intercept form of the equation $8x - y - 6 = 0$, you could first add 6 and subtract $8x$ from both sides of the equation to get $-y = -8x + 6$. Then, multiply by -1 to get $y = 8x - 6$.

If you chose **F**, you probably forgot to switch the sign on $8x$ when you multiplied by -1. If you chose **G**, you probably just dropped the sign on $-y$. If you chose **J**, you probably forgot to multiply 6 by -1 in the last step.

Question 82. The correct answer is G. To find tan R in ΔRST, take the ratio of the length of the opposite leg to the length of the adjacent leg, or ST to RS, or r to t, or inline.

F is sin R, **H** is cot R, **J** is cos R, and **K** is sec R. If you did not get the correct answer, it would be wise to review trigonometric ratios in a right triangle.

Question 83. The correct answer is D. To find the radius, you can use the right triangle shown on the diagram. Half the length of the chord is 12 inches, which is the length of one leg. The other leg is 5 inches long, and the hypotenuse is r inches long. (This is a right triangle because the distance between a point and a line must be measured perpendicular to the line.) Using the Pythagorean theorem $r^2 = 12^2 + 5^2 \Rightarrow r^2 = 169 \Rightarrow r = 13$ inches.

A is 24 + 5, which is clearly much longer than the radius. If you chose **B**, you probably used 24 and 5 for the leg lengths and got inline, which is about $r = 24.5$ inches. Choice **C** is closest to 5 + 12. Going along the radius line must be shorter than going along the 2 legs of the triangle.

Question 84. The correct answer is H. To find the force F (in newtons) corresponding to a spring length, L, of 0.18 meters when the relationship is given by the equation inline, you would substitute 0.18 for L to get inline. After subtracting 0.03 from both sides, you'd get inline. Then, after multiplying by inline, you'd get $0.225 = F$.

G is the result of replacing F by 0.18 and solving for L. If you chose **J**, possibly you got $0.225 = F$ and added 0.03.

Question 85. The correct answer is B. To find the uniform depth the 10,000 cubic yards of snow would be on the rectangular football field with dimensions 120 yards by 53.5 yards, you would substitute in the formula for volume, V, of a rectangular prism with the height h, length l, and width

w, which is $V = lwh$. After substituting you should have 10,000 = 120(53.5)(h), or 10,000 = 6,420h. Thus, inline, or about 1.558. And 1.558 is between 1 and 2.

If you chose **A**, you probably took inline and got 0.642, which is less than 1. If you chose **C** or **D**, you probably used the wrong dimensions or made a mistake in calculations.

Question 86. The correct answer is G. To find the length of inline in ΔPQR, where inline is 16 feet long and sin inline, use the definition of sine: the ratio of the length of the opposite side to the length of the hypotenuse. In ΔPQR, inline. After substituting for sin $\angle P$, and PR, the length of the hypotenuse, you obtain inline feet.

F is inline of PR. If you chose **H**, you probably found cos $\angle P = 0.8$ and then multiplied 16(0.8) to get 12.4.

Question 87. The correct answer is B. To find the fraction of cars assembled in Coupeville, you would divide the number assembled in Coupeville by the total number assembled. The following table shows the conversion of car symbols to numbers for the 4 cities and the total.

City	**Number of Cars Assembled**
Car Town	40,000
Coupeville	25,000
Truck City	20,000
Sedan Falls	15,000
All	100,000

The fraction assembled in Coupeville is inline, or inline.

If you chose **A**, you probably found the fraction for Truck City, inline, or inline. If you chose **C**, you may have thought a half car represented 10,000, so your fraction was inline, or inline. If you chose **D**, you probably used the fraction inline, or inline. If you chose **E**, you

probably used the number in Coupeville divided by the total number from the other 3 cities, inline, or inline.

Question 88. The correct answer is B. To find the x-coordinate where the 2 lines $y = 2x + 6$ and $y = 3x + 4$ intersect, you could substitute $y = 2x + 6$ into $y = 3x + 4$ to get $2x + 6 = 3x + 4$. Subtracting $2x$ and 4 from both sides results in the equation $2 = x$.

Another strategy is to graph the equations and estimate the coordinates of the intersection point.

If you chose **C**, you probably used the constant from the second equation. If you chose **D**, you probably used the constant from the first equation. If you chose **E**, you probably found the y-coordinate instead of the x-coordinate.

Question 89. The correct answer is J. To solve the equation $M = 3V + 6$ for V, you could subtract 6 from both sides to get $M - 6 = 3V$, and then divide by 3 on both sides to get inline.

If you chose **F**, you did not divide the 6 by 3. If you chose **G**, you might have moved the 6 from the right side to the left and a1so forgotten to divide it by 3. If you chose **H**, you possibly transferred the 3 from the V to the M. If you chose **K**, you probably made a sign error.

Question 90. The correct answer is B. The area is bh for a parallelogram with base b and corresponding height h. For parallelogram $ABCD$, base inline is 3 + 6, or 9 inches long, and the corresponding height is 4 inches. So the area is 9(4), or 36 square inches.

The most common wrong answer is **D,** which comes from multiplying the two side lengths: (3 + 6)(5) = 9(5) = 45.

Question 91. The correct answer is K. To find $(b - a)^4$ given $a = b + 2$, you could solve the equation for $b - a$. By subtracting a and 2 from both sides, you get $-2 = b - a$. Substituting -2 for $b - a$ in $(b - a)^4$ yields $(-2)^4$, or 16.

If you got stuck working this one, you could try choosing a specific value *for b,* say $b = 3$. Then a must be $3 + 2 = 5$. And $(b - a)^4 = (3 - 5)^4 = (-2)^4 = 16$.

If you chose **F,** you probably got -2 for $b - a$ but then replaced $(-2)^4$ by -2^4 or -16. Be careful $(-2)^4 = (-2)(-2)(-2)(-2) = 16$, but $-2^4 = -(2 \cdot 2 \cdot 2 \cdot 2) = -16$.

If you chose **H,** you probably got $b - a = 1$ or $b - a = -1$, and either $(1)^4$ or $(-1)^4$ is 1. Choices **G** and **J** come from calculating 2^4 as $2 \cdot 4$ and, for **G**, making a minus sign mistake.

Because $(b - a)^4$ is an even power of the number $(b - a)$, you can eliminate any negative numbers (**F** and **G**). This kind of observation can help you catch mistakes even when your problem is not multiple-choice.

Question 92. The correct answer is J. One strategy for solving this problem is to find equations.

You can let y be the larger number and obtain the equation $y = 2x + 8$ from the first sentence.

The second sentence says that $2y + 3x = 65 \Rightarrow 2(2x + 8) + 3x \Rightarrow 65$.

If you chose **F**, you probably took 3 times the larger number and added it to twice the smaller number to get 65, rather than the other way around. If you chose **G**, you probably defined y as $y = 2x - 8$ and then also made the same error as in **F.** Choice **H** can come from distributing the 2 in $2(2x + 8)$ as $2(2x) + 8$ and doing everything else correctly.

Question 93. The correct answer is C. To find out how far a 30-foot ladder 10 feet away from the base of a building reaches tip the building, you can use the Pythagorean theorem. Let the length of the ladder be the hypotenuse, and let the legs be the distances away from the base of the building and from the ground to the top of the ladder along the building (see the figure below).

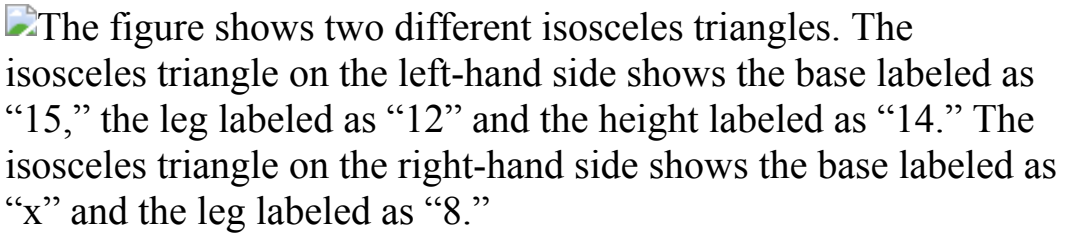
This gives the equation $30^2 = 10^2 + d^2$, where d is the distance the ladder reaches up the building.

Simplifying, you get $900 = 100 + d^2 \Rightarrow 800 = d^2 \Rightarrow d$ is about 28 feet.


The figure shows a 30-foot ladder 10 feet away from the base of a building reaches tip the building.

B comes from subtracting 10 from 30 or "simplifying" inline to inline.

Question 94. The correct answer is B. It might be good to sketch a picture, something like the diagram below. To find the length of the longest side of the second triangle, you can use the ratios of corresponding sides of each triangle. For example, inline, where x is the length of the longest side of the second triangle. After cross multiplying, you get $12x = 120$. Then, you divide by 12 to get $x = 10$ inches.


The figure shows two different isosceles triangles. The isosceles triangle on the left-hand side shows the base labeled as "15," the leg labeled as "12" and the height labeled as "14." The isosceles triangle on the right-hand side shows the base labeled as "x" and the leg labeled as "8."

If you chose **A**, you probably noticed that the first triangle's longest side is 3 units longer than its shortest side. If this same relation held in the second triangle, its longest side would be 8 + 3 = 11. This additive relation does not hold. If you chose **E**, you may have thought you needed the length of the middle side of the second triangle to solve the problem.

Question 95. The correct answer is K. To find the measure of $\angle BDC$ in the figure below, it is helpful to recognize that inline and inline are parallel and are connected by transversal inline. Then $\angle CBD$ and $\angle ADB$ are alternate interior angles and so each measures 30°. (Go ahead and write in "30°" for $\angle ADB$ on the figure.)


The figure shows a trapezoid ABCD, where E lies on AD, and angle are labeled as "60 degree" for A and "30 degree" for B.

By definition, $\angle ADE$ is a straight angle and has a measure of 180°. Because $\angle ADE$ is made up of $\angle ADB$, $\angle BDC$, and $\angle CDE$, you know that the measures of those 3 angles add up to 180°. You might write this, using m to represent *measure,* as $m\angle ADB + m\angle BDC + m\angle CDE = 180°$.

Substituting the measures you know gives $30° + m\angle BDC + 105° = 180° = > m\angle BDC + 135° = 180° = > m\angle BDC = 45°$.

If you chose **H,** you might have thought $\angle BDC$ is isosceles. If you chose **J,** you possibly estimated the measure of $\angle BDC$ or made a subtraction error.

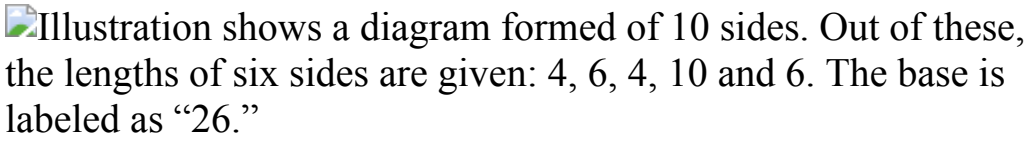
Question 96. The correct answer is E. This figure has 10 sides, but lengths are given for only 6 of the sides. Those lengths add up to 4 + 6 + 4 + 10 + 6 + 26 = 56 inches. The perimeter is longer than this because of the missing 4 sides.

Then you should find the lengths of the missing sides, right? The following figure focuses on the vertical sides. The vertical sides that face left have lengths 4, 6, and 4. The lengths of the sides that face right are unknown. But, the vertical distance that the left-facing sides cover is the same as the vertical distance that the right-facing sides cover.


Illustration shows a diagram formed of 10 sides. Out of these, the lengths of six sides are given: 4, 6, 4, 10 and 6. The base is labeled as “26.”

So, since the total length of the left-facing sides is 4 + 6 + 4 = 14 inches, the total length of the right-facing sides is also 14 inches.

Finding the lengths of the horizontal sides (see the figure below) is a similar process. The horizontal distance covered by the top-facing sides must be 26 inches because that's what's covered by the bottom-facing sides.



This makes the perimeter the sum of the lengths of the left-facing, right-facing, top-facing, and bottom-facing sides, which is 14 + 14 + 26 + 26 = 80 inches. You can't know the length of each side, but you can find the perimeter.

If you chose **C**, you probably just found the sum of the side lengths shown: 4 + 6 + 4 + 10 + 6 + 26 = 56. If you chose **D**, you may have left out the right-facing sides, or you may have estimated the lengths of the 4 missing sides and been too low. Estimation is a reasonable strategy for this question.

Question 97. The correct answer is F. To find out how many of the 804 seniors in a certain high school are going to a state university when approximately inline of the seniors are going to college, and when inline of those going to college are going to a state university, you could first find how many of the 804 seniors are going to college. This is inline, or almost 322 seniors. Then, find the number of those 322 seniors going to college who are going to a state university, which is inline, or about 80 seniors that are going to a state university.

J is closest to inline of 804.

Question 98. The correct answer is E. To find the distance between 2 points in the standard (x,y) coordinate plane, you can use the distance formula, inline. So the distance is inline coordinate units.

A can come from mixing x and y coordinates: inline. If you chose **B,** you probably added 1 + 5 or simplified the radical expression incorrectly.

Question 99. The correct answer is G. To find the ratio of the circumferences of 2 circles for which the ratio of their radii is 4:9, you would recognize that both circumference and radius are one-dimensional attributes of a circle. Because of that, the ratios should be the same, 4:9. Another way is to use the ratio of the radii and let $4x$ be the radius of the

first circle and $9x$ be the radius of the second circle. Then, the circumferences would be $2\pi(4x)$ and $2\pi(9x)$, respectively. Setting them in a ratio, you get $8\pi x{:}18\pi x$, which simplifies to 4:9.

If you chose **F**, you probably thought that you should take the ratio of the square roots, inline:inline, or 2:3. If you chose **H,** you probably thought that you should take the ratio of the squares, $4^2{:}9^2$, or 16:81 (which is the ratio of the circles' areas).

Question 100. The correct answer is D. You may want to have a picture of this situation in your mind, or even sketch it out in the space in your test booklet. Your picture might look something like this.

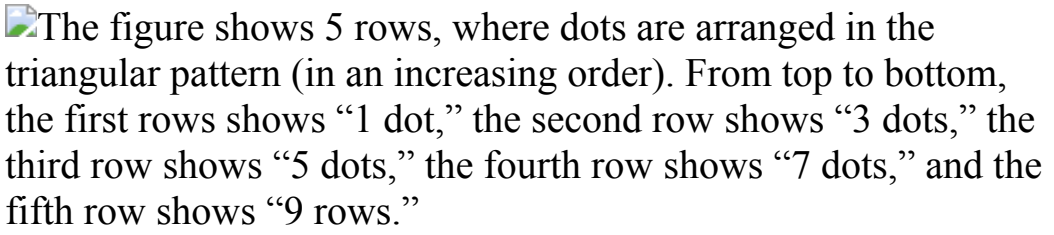
> A graph is shown in the standard (x,y) coordinate plane. The x axis represents “values” ranges from 5 to 10. The y-axis represents “values” ranges from 5 to 10. A circle is drawn, which lies on the x-axis and touches the coordinate (5, 5).

One way to find an equation for a circle is to know the coordinates of the center, (h,k), and the radius, r. Then, an equation is $(x-h)^2+(y-k)^2=r^2$. For this circle, the center is at (5,5) and the radius is 5. (It's fairly easy to see that. If you needed to prove those are exactly right, you could use symmetry or you could use the fact that a tangent line is perpendicular to the radius that goes through the point of tangency.) Given center (5,5) and radius 5, the circle has equation $(x-5)^2+(y-5)^2=5^2$.

Another way to solve this problem is to find the coordinates of points on the circle and see which equation(s) each point satisfies. The points (0,5), (5,0), (5,10), and (10,5) are all on the circle. Testing these points in all the equations would probably take longer than the first method, but testing the points in the equation you think is correct would be a good check of your answer.

B is a circle centered at (0,0) instead of (5,5). If you chose **C**, you probably forgot to square the radius on the right side of the equation. If you chose **E,** you likely used $(x+h)$ and $(y+k)$ in the equation. Testing (10,5) would have helped you eliminate these incorrect answers.

Question 101. The correct answer is D. You want to find which statement describes the total number of dots in the first *n* rows of the figure below.


The figure shows 5 rows, where dots are arranged in the triangular pattern (in an increasing order). From top to bottom, the first rows shows "1 dot," the second row shows "3 dots," the third row shows "5 dots," the fourth row shows "7 dots," and the fifth row shows "9 rows."

You could make a table like the one below, showing the number of rows and the total number of dots.

Number of rows	1	2	3	4	5
Total number of dots	1	4	9	16	25

You would probably recognize that the total number of dots is the square of the number of rows. This seems like a consistent relationship (it works for all 5 columns in your table). You can rule out **A** because the total is not always 25. For **B**, the total would have to go 2, 4, 6, 8, 10. For **C**, the total would need to be 5, 10, 15, 20, 25.

B works for the total of the first 2 rows. If you chose **C**, that works for the total of the first 5 rows. If you chose **E**, you might have seen that the relationship was not linear and viewed this as inconsistent.

Question 102. The correct answer is B. Douglas will count any color other than red, blue, green, and purple in the Other sector. The table below gives percentages of friends who picked red, blue, green, and purple.

Color	Red	Blue	Green	Purple	Other
Percentage	25%	30%	20%		10%

The 4 known percentages add up to 85%. That leaves 15% for the Other sector. That means 15% of the 360° in the circle belong in the Other sector. This is (0.15)(360°) = 54°.

C is 15% of 180° rather than of 360°. If you chose **D**, you probably found the correct percent for the Other sector and then just labeled it degrees.

Question 103. The correct answer is J. One way to find tan θ, given that inline and inline, is to first find cos θ, then find inline (which is equivalent to tan θ). To find cos θ, use the facts that cos θ < 0 in Quadrant III and that $\sin^2 \theta + \cos^2 \theta = 1$. Substituting, you get inline or inline. After subtracting inline, you get inline. After taking the square root of both sides, you get inline. Because inline. Substituting into inline gives you inline, which simplifies to inline.

Another way you could do this problem is to construct an angle in Quadrant III with inline. (Recall that sine is the ratio of opposite to hypotenuse.) Such an angle is shown below.

> A graph is shown in the standard (x,y) coordinate plane. A right-angled triangle is drawn on negative side of the x-axis. The base of the triangle rests on the x-axis, and the height of the triangle is a segment perpendicular to the x-axis. The base is labeled as “theta,” height is labeled as “negative 3” and hypotenuse is labeled as “5.”

By the Pythagorean theorem, the missing side of the right triangle is 4 coordinate units long, and the directed distance along the side is −4. The figure below shows this.

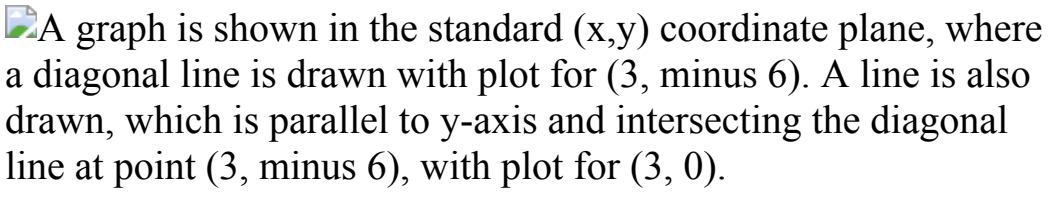
> A graph is shown in the standard (x,y) coordinate plane. A right-angled triangle is drawn on negative side of the x-axis. The base of the triangle rests on the x-axis, and the height of the triangle is a segment perpendicular to the x-axis. The base is labeled as “negative 4,” height is labeled as “negative 3” and hypotenuse is labeled as “5.”

From this right triangle, knowing that tangent is inline you can get tan θ inline.

G comes from using inline for cos θ instead of inline If you chose **H**, you might have mixed up the definition of sine or tangent in the right

triangle.

Question 104. The correct answer is A. To find the system of inequalities represented by the shaded region of the graph below,



you could first find the equations of the line through (0,0) and (3,−6) and the line through (3,0) and (3,−6). Those are $y = -2x$ and $x = 3$. It is clear from the graph that the inequality that represents the shaded side of $x = 3$ is $x \geq 3$. For the other line, if you test (3,0), you find it satisfies $y > -2x$. Because (3,0) is on the wrong side (the unshaded side) of $y = -2x$, the correct inequality is $y \leq -2x$.

The graphs of the incorrect answer choices are shown below.

Choice **C** is the most common incorrect answer (about as many people choose this as choose the correct answer). The inequality sign is backwards for the line $y = -2x$.

Choice **B** differs from the correct answer only in the “or” connector. The graph of **B** includes points that satisfy one of the inequalities but not necessarily the other inequality, while the “and” connector means the graph can only include points that satisfy both inequalities.

 Illustration shows four different graphs, labeled (from left-to right) “Graph of B,” “Graph of C,” “Graph of D” and “Graph of E.” Each graph consists of different shaded region. (1) Graph of B: A graph is shown in the standard (x,y) coordinate plane, where a diagonal line is drawn with plot for (3, minus 6). A line is also drawn, which is parallel to y-axis and intersecting the diagonal line at point (3, minus 6), with plot for (3, 0). (2) Graph of C: A graph is shown in the standard (x,y) coordinate plane, where a diagonal line is drawn with plot for (3, minus 6). A line is also drawn, which is parallel to y-axis and intersecting the diagonal line at point (3, minus 6), with plot for (3, 0). (3) Graph of D: A graph is shown in the standard (x,y) coordinate plane, where a diagonal line is drawn with plot for (3, minus 6). A line is also drawn, which is parallel to y-axis and intersecting the diagonal line at point (3, minus 6), with plot for (3, 0). (4) Graph of E: A graph is shown in the standard (x,y) coordinate plane, where a diagonal line is drawn with plot for (3, minus 6). A line is also drawn, which is parallel to y-axis and intersecting the diagonal line at point (3, minus 6), with plot for (3, 0).

Question 105. The correct answer is K. To find $f(x + h)$ when $f(x) = x^2 - 2$, you would substitute $(x + h)$ for x in $f(x) = x^2 - 2$. The result is $(x + h)^2 - 2$. Multiplying out $(x + h)^2$ yields $x^2 + xh + xh + h^2$, or $x^2 + 2xh + h^2$. Then add -2 to the result.

If you chose **G**, you interpreted $f(x + h)$ as $f(x) + h$. If you chose **H,** you replaced $(x + h)^2$ with $x^2 + h^2$. If you chose **J,** you found $(x + h)^2$.

Question 106. The correct answer is A. It might be surprising to see that the graph of this complicated function looks almost like a line. The equation  inline can be written as  inline. This is equivalent to $y = 2x + 1$ except when $x = 0$. When $x = 0$, the original equation is undefined. So the correct graph is $y = 2x + 1$ with a point removed where $x = 0$.

If you noticed that the function was undefined when $x = 0$, you may have thought the open dot belonged at (0,0). That leaves **B** as the only answer choice that also goes through (1,3).

Choice **C** is the only one that involves (0,2), and you may have gotten this by substituting $x = 0$ to get inline, and decided all the zeros could be dropped to yield $y = 2$.

If you chose **D,** you may have "cancelled" x's as inline to get $y = 2x^2 + 1$. You could have eliminated this answer by testing $(-1,3)$ in the original equation, but testing (1,3) would not have been enough.

Choice **E** can come from "cancelling" x's as inline to get $y = 2x^2$. You could have eliminated this answer by testing (1,2) in the original equation.

Question 107. The correct answer is F. To find the coordinates of vertex A after it is reflected across the x-axis, notice that a reflection across the x-axis does not change the x-coordinate but does change the sign of the y-coordinate. You might sketch or imagine a figure like the one below. Thus, the reflection of $A(c,d)$ across the x-axis is $A'(c, -d)$.

> A graph is shown in the standard (x,y) coordinate plane. The top side of the x-axis shows an isosceles triangle ABC. The bottom side shows the reflection of the isosceles triangle ABC, drawn above.

G gives A reflected across the y-axis. **H** gives A reflected across (0,0). **J** gives A reflected over the line $y = x$ and is the most common answer.

Question 108. The correct answer is A. To obtain an expression for y in terms of x when $x = 2t - 9$ and $y = 5 - t$, you can first solve $x = 2t - 9$ for t by adding 9 to both sides to get $x + 9 = 2t$. Then, divide both sides by 2 to get inline. Substitute that expression for t into $y = 5 - t$ to get inline. To simplify the right side, rewrite 5 as inline and then combine the 2 fractions together to get inline. You can then distribute and combine like terms to get inline.

If you chose **B,** you probably got inline and simplified it to inline. If you chose **C**, you may have substituted $2x - 9$ for t in $y = 5 - t$, which results in $y = 5 - (2x - 9)$. After distributing, this would be $y = 5 - 2x + 9$, or $y = 14 - 2x$.

Question 109. The correct answer is K. To find inline using $\sin(\alpha - \beta) = (\sin \alpha)(\cos \beta) - (\cos \alpha)(\sin \beta)$ given that inline, you can first substitute inline for α, and inline for β and get inline. Using the table of values to substitute in that equation, you get inline.

H comes from calculating inline, which is inline. If you chose **J**, you probably just used inline.

Question 110. The correct answer is J. The 12 vases cost \$18, so each vase inline.

If you chose **F**, you probably divided 12 by 18 rather than 18 by 12. If vases cost \$0.67 each, then 12 vases would cost less than \$12.

Question 111. The correct answer is B. The longer side of the apartment is 30 feet long, and it is 6 inches long on the scale drawing. So, the length of the room, in feet, is 5 times the length on the drawing, in inches. Using this relationship, the length of the shorter side of the apartment is 5 times the 4 inches from the scale drawing. This is 20 feet.

Alternately, you could notice that the length of the shorter side is inline the length of the longer side on the drawing and so the length of the shorter side of the room is inline of 30 feet, which is 20 feet.

These solutions are equivalent to using a proportion such as inline and solving inline feet.

Question 112. The correct answer is H. The total profit for the 5 years was, in millions, \$8 + \$8 + \$8 + \$9 + \$9 = \$42. Then the average profit, in millions, was inline.

The most common wrong answer was **J**, which is the average of 8 and 9. Because there were more years with a profit of \$8 million than with \$9 million, the average for the 5 years must be closer to \$8 million than to \$9 million.

Question 113. The correct answer is A. If the van were driven for 20 miles, the cost for those miles would be \$0.30 • 20 = \$6. Then the daily

charge of $25 would have to be added in, for a total of $31. Similarly, if the van were driven for m miles, the cost for those miles would be $0.30m$ dollars, and the daily charge would make the total $0.30m + 25$ dollars.

C comes from treating the 30 cents like it was 30 dollars.

Question 114. The correct answer is K. This problem can be solved by substituting the Celsius temperature ($C = 38$) into the formula and solving for F. The substitution step gives inline, which can be solved as follows: inline. It is appropriate to round this to the nearest degree Fahrenheit because the precision of the Celsius temperature was only to the nearest degree Celsius.

If you chose **J**, you may have added 38 + 32 and missed the inline. An answer of H might come from calculating inline correctly and forgetting to add 32.

Question 115. The correct answer is B. Nick can only order whole cases, which contain 24 boxes of pens with 10 pens per box, for a total of 24 • 10 = 240 pens per case. An order of 2 cases would be 480 pens, which falls short of the desired 500 pens. To get 500 pens from his supplier, Nick needs to order 3 cases, and he will get 720 pens.

If you got answer **A**, you may have correctly divided 500 by 240 to get approximately 2.08 cases, but you may have rounded that to the nearest integer, which does not give the correct answer in this context. Answer **E** represents the number of boxes (not cases) of pens needed if Nick could order any whole number of boxes.

Question 116. The correct answer is K. When $a + b = 6$, then inline becomes inline, which simplifies to $12 + 1 + 36 - 2 = 47$.

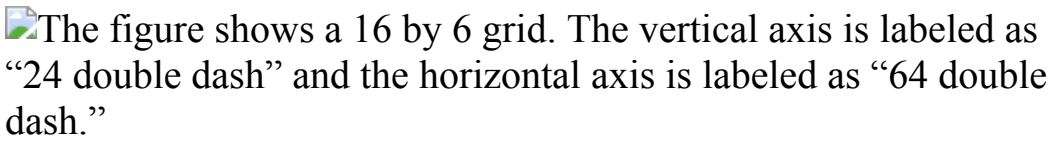
Question 117. The correct answer is C. If you bought 1 hamburger and 1 soft drink, it would cost $2.10. If you bought 1 hamburger more, your order would cost $3.50. So, the cost of the additional hamburger was $3.50 − $2.10 = $1.40. Because 1 hamburger and 1 soft drink cost $2.10, a soft drink must cost $2.10 − $1.40 = $0.70.

Alternatively, you could set up two equations with two unknowns. Let h dollars be the cost of each hamburger and s dollars be the cost of each soft drink. Then $h + s = 2.10$ and $2h + s = 3.50$. Subtraction gives:

And then, substituting 1.40 for h in $h + s = 2.10$ gives $s = 2.10 - 1.40 = 0.70$.

The most common wrong answer is **E**, which is the correct cost of a hamburger. However, the question asks for the cost of a soft drink. Answer choice D is half of $2.10, which would only be correct if a soft drink cost the same as a hamburger.

Question 118. The correct answer is C. There would be 6 tiles along the 24" side (6 • 4" = 24"). There would be 16 tiles along the 64" side (16 • 4" = 64"). Then, 6 • 16 = 96 tiles are needed to completely cover the rectangular countertop:



An alternate solution is to figure the area of the countertop in square inches, 24 • 64, which is 1,536 square inches. Then, divide that by the area of a tile, which is 16 square inches. The result is 96, which is the number of tiles needed. The tiles cover the area without being cut because the side lengths of the countertop are divisible by the side length of a tile.

The most common wrong answer is **E**, which comes from correctly calculating the area of the countertop (1,536 square inches), but dividing by the length of the side of the square (4 inches) rather than by 4 • 4 square inches. If you chose **A** or **B**, you may have confused perimeter and area.

Question 119. The correct answer is H. If the answer choices give 2 of the 3 interior angle measures in a triangle, then the third angle measure is 180° minus the sum of the given angle measures. The chart below shows this calculation.

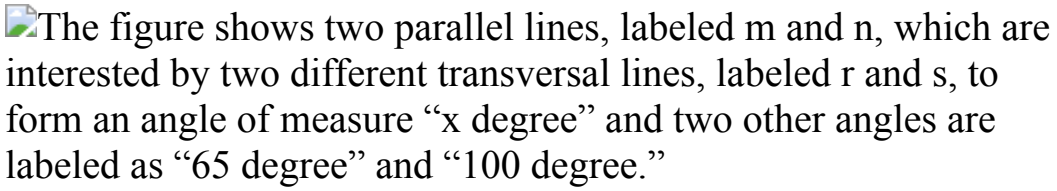
	1st angle	2nd angle	Sum of 1st + 2nd angles	3rd angle (180° − Sum of 1st + 2nd angles)
F.	20°	40°	60°	120°
G.	30°	60°	90°	90°
H.	40°	100°	140°	40°
J.	45°	120°	165°	15°
K.	50°	60°	110°	70°

The only place where the third angle has measure equal to one of the first two angles is in **H**, where there are two 40° angles.

Question 120. The correct answer is C. The perimeter of the triangle, 66 inches, is the length of the three sides added together. Because one side is 16 inches long, the lengths of the other two sides added together must be 66 – 16 = 50 inches. The ratio of the lengths of these two sides is 2:3. This ratio denotes 2 parts for the first side, 3 parts for the second side, and therefore 5 parts altogether. Because there are 50 inches altogether, and this must make up the 5 parts, each part is 10 inches long. That makes one side 2 parts, or 20 inches long, and the other side 3 parts, or 30 inches long. So the longest side of the triangle is 30 inches long.

B is the length of another side of the triangle, but not the longest side.

Question 121. The correct answer is C. In the following figure, the angles with measure $a°$ and 100° form a straight angle along line m. This means $a° + 100° = 180°$, or $a° = 80°$. Now, you know two of the three angles in the larger triangle. The sum of all three must be 180°. So, $80° + b° + 65° = 180°$, which means that $b° = 35°$. The angle measure $b°$ is equal to angle measure $x°$ because vertical angles have the same measure. So, $x° = 35°$.


The figure shows two parallel lines, labeled m and n, which are interested by two different transversal lines, labeled r and s, to form an angle of measure “x degree” and two other angles are labeled as “65 degree” and “100 degree.”

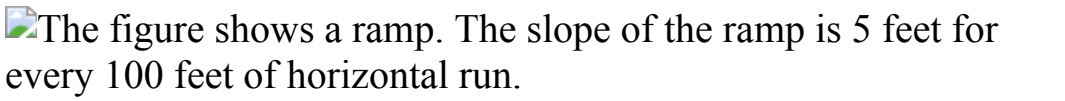
If you chose **A**, you may have calculated 180° – (65° + 100°). These three given angle measures, $x°$, 65°, and 100°, do not need to add to 180°. They

are not the measures of the three interior angles in the *same* triangle.

Question 122. The correct answer is B. Parallel lines have the same slope, so any line parallel to this line has the same slope as this line. To find the slope of this line, you could put it into slope-intercept form, and then the slope is the coefficient of x. inline, and so the slope is inline.

If you chose **E**, you probably knew that the slope is the coefficient of x, but that only is true when the equation is in slope-intercept form. If you chose **A**, you may have gotten the equation into the form $9y = -7x + 6$ and then read off the coefficient of x. You didn't quite have it in slope-intercept form. If you chose **C**, you may have made a mistake putting the equation in slope-intercept form. Answer choice **D** could be reading off the constant 6 from the original equation.

Question 123. The correct answer is C. The figure below shows the ramp.



The slope is given as rising 5 feet for every 100 feet of horizontal run. The ramp's rise is 2 feet, and the horizontal run is unknown. Let the horizontal run be represented as d. Then there is a proportion inline. Its solution is inline feet.

If you chose **A**, you might have simplified 5% to 0.5. Then the proportion inline has the solution 4 feet. The most common incorrect choice was **B**, which happens when the run is 5 times the rise. This gives a reasonable-looking ramp. However, the slope of such a ramp is inline. The required slope is inline.

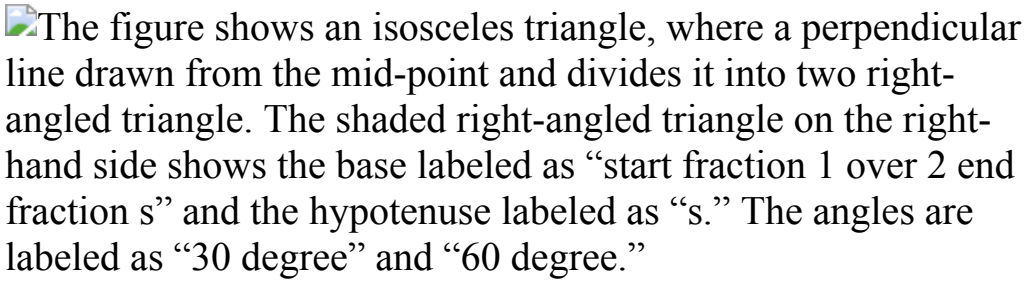
Question 124. The correct answer is J. A quick scan of the answer choices should give you a clue that combining like terms is in order.

image

If you chose **K**, you probably subtracted the $3x^2$ but *added* the $-4x$ and the -3. The solution above took care of this explicitly on the second line. The

other incorrect answers result from various combinations of errors with minus signs.

Question 125. The correct answer is E. The side lengths of a 30°−60°−90° triangle are in the ratio 1:inline: 2. If you didn't remember this, you could view a 30°−60°−90° triangle as half of an equilateral triangle, as shown below.


The figure shows an isosceles triangle, where a perpendicular line drawn from the mid-point and divides it into two right-angled triangle. The shaded right-angled triangle on the right-hand side shows the base labeled as “start fraction 1 over 2 end fraction s” and the hypotenuse labeled as “s.” The angles are labeled as “30 degree” and “60 degree.”

For the shaded triangle, you know that the base is half as long as the hypotenuse, because the base of the equilateral triangle is the same length as the other sides of the equilateral triangle. If the hypotenuse is *s* units long and the base is inline, then the Pythagorean theorem gives the height as inline

This shows that ratios are inline: 1, which are equivalent to those given above, 1 : inline : 2. (Obviously, it's quicker to know the ratios than to try to derive them each time you need them, but don't give up if you can't remember something; try to find it a different way.) You could also use trigonometry to derive the side length ratios in a 30°−60°−90° triangle.

If you chose **B**, you might have been thinking of a 45°−45°−90° triangle, which has this ratio of side lengths. If you chose **D**, you may have reasoned that because the angle measures are in the ratio 1:2:3, maybe the side lengths are in the ratio inline. This is a right triangle (it satisfies the Pythagorean theorem), but the angles are closer to 35°−55°−90°. The triangles in **A** and **C** are not right triangles. You could have eliminated them because they do not satisfy the Pythagorean theorem. For **A**, inline. For **C**, inline.

Question 126. The correct answer is E. This sequence decreases by 17 – 12 = 5 each term. Here are some more terms:

Term#	1	2	3	4	5	6	7	8	9
Value	17	12	7	2	−3	−8	−13	−18	−23

Each of **A–D** can be verified from the chart. The common difference for **D** is defined so that it is positive if the sequence is increasing and negative if the sequence is decreasing. In symbols, if the sequence is represented by terms a_1, a_2, a_3,…, then the difference between two terms is , for all i. If this difference is constant for all i, then it is called the *common difference* and the sequence is called *arithmetic*.

That leaves only **E** that could be false. The ratio of consecutive terms is defined by  for all i.

For the first two terms, the ratio is  , which is a bit more than 0.70. For the second and third terms, the ratio is  , which is a bit less than 0.59. That means the ratios are not equal for all terms, and so there is no common ratio. That means **E** is false.

The most common incorrect answer is **B**. If you chose this, perhaps you reasoned that, because the sum of the first 4 terms is 38, the sum of the first 5 terms cannot be anything less. Writing out more terms makes it clear that this can be so. Or, you may have made an arithmetic mistake finding the fifth term and so arrived at the wrong sum. If you chose **C** and had the correct sum for the first 5 terms, then perhaps you made an arithmetic mistake subtracting 5 for each term, or you miscounted terms. If you have a different sum for **B** and also have a different eighth term than **C**, you know that there is something amiss in your work and you should go back and try to find your mistake if there is time.

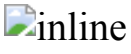
If you chose **D**, perhaps you did not understand the concepts of common difference and common ratio.

Question 127. The correct answer is D. The normal amount of lead is 1.5×10^{-5} milligrams per liter. This can be written as 0.000015 milligrams per liter. Today's level, 100 times the normal amount, is $100(1.5 \times 10^{-5}) = 1.5 \times 10^{-5} \times 10^{2} = 1.5 \times 10^{-5+2} = 1.5 \times 10^{-3}$ milligrams per liter. This can be

written as 0.0015 milligrams per liter. This is larger than the normal amount.

If you chose **A**, you likely added −100 to the exponent. This answer is 0.00000000…0000000015 milligrams per liter, where there are 104 zeros between the decimal point and the "15." This is smaller than the normal amount. If you chose **B**, you possibly multiplied the exponent 2 from 10^2 by the exponent −5 from the normal amount. This is 0.00000000015 milligrams per liter, which is smaller than the normal amount. If you chose **C**, you could have subtracted the exponent 2 from the exponent −5 in the normal amount. This gives 0.00000015 milligrams per liter, which is smaller than the normal amount.

Question 128. The correct answer is B. This problem is in the general form $(a + b)^2$, which is equivalent to $a^2 + 2ab + b^2$ by the following derivation.



If  and $b = -y$, then  . This is equivalent to  .

If you chose **A**, you probably squared the first term and squared the second term. This is the stereotypical error, and it's something that college math teachers (and high school math teachers) want you to know not to do. Something that might help you remember is to have a concrete example: $(1 + 3)^2 = 4^2 = 16$, but $1^2 + 3^2 = 1 + 9 = 10$.

If you chose **C**, you probably did everything correctly except remembering to square the  to get  .

Question 129. The correct answer is G. One way to solve this problem is to list out all the numbers in this range that you think might be prime, then check to see if any of them factor. You probably know that you don't have to check the even numbers. That leaves the following list:

31 33 35 37 39 41 43 45 47 49

If one of these does factor, it will have a prime factor of at most inline, which is 7. You have already eliminated all multiples of 2. If you eliminate all multiples of 3, 5, and 7, anything left on the list is a prime number. The multiples of 3 on the list are 33, 39, and 45. You can eliminate 35 because it is a multiple of 5. You can eliminate 49 because it is a multiple of 7. Then all the numbers remaining, namely 31, 37, 41, 43, and 47, are prime numbers.

If you chose **H**, you must have counted a number as prime that really isn't prime. You might want to figure out which one that was. People who miss this problem tend to count extra numbers as primes.

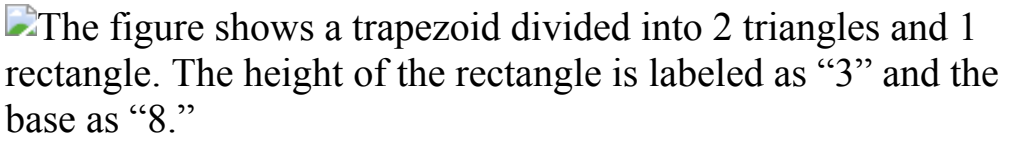
Question 130. The correct answer is E. The cotangent of $\angle A$ in this right triangle is the length of the leg adjacent to the angle divided by the length of the leg opposite the angle. That ratio is inline.

If you chose **D**, you chose the tangent of the angle. Answer choice **C** represents the sine of the angle. Answer choice **B** is the cosecant of the angle. If you want to get problems like this correct, you need to have a way to keep the trig functions straight.

Question 131. The correct answer is B. The area of the trapezoid is inlinesquare inches. The area of the unshaded rectangle is $4 \cdot 3 = 12$ square inches. The triangles and the rectangle together form the trapezoid, so the area of the trapezoid minus the area of the rectangle is the area of the triangles. And this area is $18 - 12 = 6$ square inches. Another way to solve this problem is to slide the two shaded triangles together and calculate the area of the new triangle. From the original figure, notice that the combined base of the triangles is the amount left over from the trapezoid's bottom base when the width of the rectangle is removed. That is $8 - 4 = 4$ inches. The figure below shows the combined triangles. Their combined area is inline square inches.

> The figure shows an isosceles triangle, drawn on the left-hand side, and a rectangle, drawn on the left-hand side. The sides of the rectangle are labeled as “3” and “4.” The base is labeled as “8.”

The most common incorrect answer is **D**, which could come up in a variety of ways. Many students have trouble finding the area of trapezoids and resort to calculating *bh* as if the trapezoid were a rectangle or parallelogram. The figure below shows the trapezoid inside a rectangle with base 8 inches and height 3 inches. The rectangle has area *bh*, which is clearly larger than the area of the trapezoid.


The figure shows a trapezoid divided into 2 triangles and 1 rectangle. The height of the rectangle is labeled as “3” and the base as “8.”

If you chose **E**, perhaps you calculated the area of the trapezoid and stopped. Perhaps you thought the problem was asking for the area of the triangles combined with the area of the rectangle, but it asks for the combined area of the triangles only. The triangle on the left looks like it is half of a square. If so, it has a base of 3 inches and a height of 3 inches and then area inline square inches. If the other triangle has the same area, the combined area of the triangles is inline square inches. Perhaps you reasoned this way if you chose **C**.

Question 132. The correct answer is G. Each of the corner triangles are right triangles because they share an angle with the square. Both legs of these right triangles are 6 inches long because they are half the length of the side of the square. So, the hypotenuse of each of these triangles isinline inches. The perimeter of *EFGH* is made up of 4 of these hypotenuses, so the perimeter of *EFGH* is inline inches.

You could have used the ratio of sides in an isosceles right triangle, rather than the Pythagorean theorem, to get the hypotenuse of the corner triangles. The basic flow of the solution is the same.

If you chose **F**, perhaps you noticed that the area of *EFGH* is half the area of *ABCD*. That's a good observation. That does not mean, though, that the perimeter of *EFGH* is half the perimeter of *ABCD*. If that were true, then each side of *EFGH* would be 6 inches long, and the corner triangles would have 3 sides of length 6 inches. The triangle must then be equilateral and have a right angle. That can't happen. (The perimeter of a figure that is

geometrically similar to the original and has area in the ratio 2:1 has perimeter in the ratio inline :1.)

Question 133. The correct answer is D. The table below shows the value of the cosine function at values of θ that are the endpoints of the intervals from each of the answer choices.

θ	0	image	inline	inline	inline	π
cos θ	1	image	inline	0	inline	−1
approximation of cos θ	1	0.866	0.5	0	−0.5	−1

The value −0.385 is between 0 and −0.5, which is saying that the value of cos θ is between inlineand inline. Because cos θ is continuous, that means there is a value of θ between inline and inline that satisfies the conditions of the problem.

If you chose **C**, you might have been looking for a place where cos θ = +0.385 rather than −0.385.

If you chose **A** or **E**, you might have been looking for a place where sin θ = 0.385.

Question 134. The correct answer is J. The equation $(x - 6a)(x + 3b) = 0$ has these two solutions. (You can check this by substituting $6a$ in for x and substituting $-3b$ in for x.) Multiplying out this equation gives inline, which is the same as inline.

If you chose **F**, you may have gotten the initial equation right, $(x - 6a)(x + 3b) = 0$, but then multiplied incorrectly to get $(x)(x) + (-6a)(3a) + 0$.

Most of the other incorrect answers could be due to mistakes with negative signs. **G** comes from the initial equation $(x + 6a)(x - 3b) = 0$, where the signs are opposite what they should be. (If you substitute $6a$ in for x, you will not get zero on the left side of the equation.) If you chose **H**, you may have started with the equation $(x - 6a)(x - 3b) = 0$.

Question 135. The correct answer is A. The midpoints of the sides of the square are on the circle. These points have coordinates (0,3), (3,6), (6,3),

and (3,0). The first point, (0,3), satisfies **A**, but none of the other equations. Alternately, because the circle centered at (h,k) with radius r has the equation inline, you can find the equation by finding the center and radius. From the diagram, you can see that the center of the circle is the same as the center of the square, which is (3,3). Also, the radius of the circle is the distance from (0,3) to (3,3), which is 3 coordinate units. So inline is an equation of the circle.

One common mistake is to remember the equation of the circle incorrectly, with plus signs where there should be minus signs (**C**). Another common mistake is to not square the radius on the right side of the equation (**B**). Or, some people do both of these things (**E**). If you chose **D**, you may have used the diameter on the right side of the equation, or you may have used the largest coordinate from the figure (and used plus signs instead of minus).

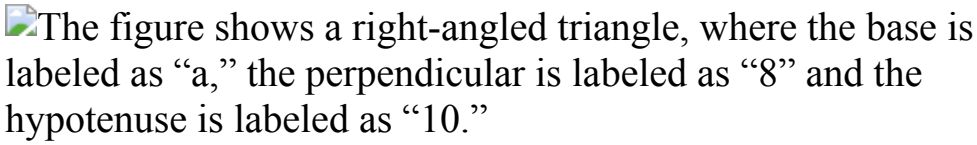
Question 136. The correct answer is H. When $g = 2$, the value of $g \bullet (g + 1)^2$ is $2 \bullet (2 + 1)^2 = 2 \bullet (3)^2 = 2 \bullet 9 = 18$.

If you chose **F**, you might have thought of $(g + 1)^2$ as equivalent to the alternate expression $g^2 + 1^2$. But, when $g = 2$, the original has the value $(2 + 1)^2 = (3)^2 = 9$, while the alternate has the value $2^2 + 1^2 = 5$.

Question 137. The correct answer is D. Company A sells at \$15 for 60 pens, which is inline per pen. That reduces to inline, or \$0.25 per pen. Company B sells at \$8 for 40 pens, which is inline per pen. Company B is a nickel cheaper.

If you chose **A**, you possibly found the minimum cost correctly but identified it with the wrong company. Choice **C** is the average for Company A.

Question 138. The correct answer is H. The Pythagorean theorem applies here, so that inline, where a is the distance from the base of the ladder to the wall, in feet. That means inline and then $a = 6$.



K comes from adding $10^2 + 8^2$, which is 164. If you had noticed that the longest side of the triangle is 10 feet long, and inline is more than 10, you could have eliminated this answer choice. **F** (2 feet) cannot be true. If it were, the path along the two shorter sides of the triangle (ground and wall) would be the same length as a path along the longest side of the triangle (ladder). This is impossible because the shortest path between points must go along a straight line, not over and then up.

Question 139. The correct answer is F. Each 1,000 gallons of water cost \$2.50, so g of these “1,000 gallons of water” cost g • (\$2.50). On top of this, there is a \$16 charge that is added for trash pickup. The result is $\$2.50g + \$16$.

Answer choice **K** represents a cost of \$2.50 per gallon rather than per 1,000 gallons.

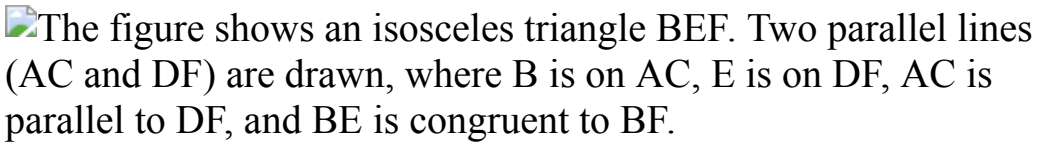
Question 140. The correct answer is D. The left side of the equation, $2(x + 4)$, can be written as $2x + 8$. Then, the equation becomes $2x + 8 = 5x - 7$. One way to solve this is to first add 7 to both sides (resulting in $2x + 15 = 5x$) and then subtracting $2x$ from both sides (resulting in $15 = 3x$). Dividing both sides by 3 gives the result $5 = x$.

It's a good idea to check this answer: does $2(5 + 4)$ equal $5(5) - 7$? The left simplifies to 2 (9), which is 18. The right side simplifies to $25 - 7$, which is also 18. Yes!

Some students will write $2(x + 4)$ as $2x + 4$, but it isn't. That mistake would lead to answer choice **C**. Note that $2(x + 4)$ is two $(x + 4)$s added together, or $x + 4 + x + 4$.

Question 141. The correct answer is H. BF is a transversal between the two parallel lines. Therefore, because $\angle CBF$ and $\angle BFE$ are alternate interior angles, their measures are equal, and so $f°$ in the figure below is 35°. You can also see that ΔBEF is isosceles, which makes the base angles

equal in measure, so $e°$ is also 35°. The angle the problem asks you to find is the supplement of $\angle BEF$, which makes its measure 180° − 35°, or 145°.


The figure shows an isosceles triangle BEF. Two parallel lines (AC and DF) are drawn, where B is on AC, E is on DF, AC is parallel to DF, and BE is congruent to BF.

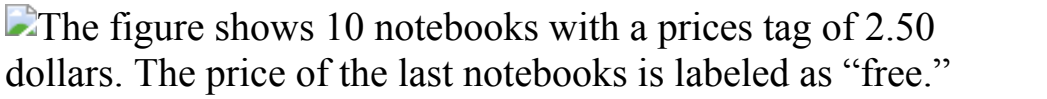
Question 142. The correct answer is B. The least common denominator is the smallest positive multiple of 2, 3, 9, and 15. These four numbers can be written in prime-power form as 2, 3, 3^2, and 3 • 5. The least common denominator must have all of these as factors. The number $2 • 3^2 • 5$ is divisible by 2, by 3, by 3^2, and by 3 • 5, and it has no extra prime factors so it is the smallest of all the common multiples. $2 • 3^2 • 5 = 2 • 9 • 5 = 10 • 9 = 90$.

The most common incorrect answer is **A**, which is divisible by 15, by 9, and by 3, but not by 2.

Question 143. The correct answer is H. $3x(x^2y + 2xy^2) = 3x \cdot x^2y + 3x \cdot 2xy^2 = 3x^3y + 6x^2y^2$.

F is $3(x^2y + 2xy^2)$, dropping the x multiplier in $3x$. If you chose **G**, you probably correctly multiplied $3x$ and x^2y but forgot to multiply $3x$ by $2xy^2$.

Question 144. The correct answer is D. The 10 notebooks would cost 9($2.50), which is $22.50. The average price for one of the 10 notebooks would be inline, which is $2.25.


The figure shows 10 notebooks with a prices tag of 2.50 dollars. The price of the last notebooks is labeled as “free.”

If you chose **B**, you may not have understood that notebook 10 was free, so you only had to pay for 9 notebooks.

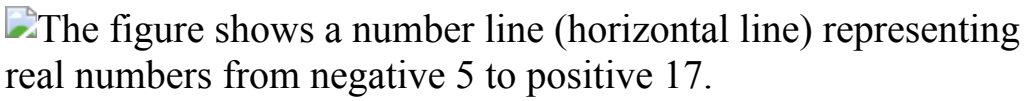
Question 145. The correct answer is K. $(3x + 1)^2 = (3x + 1)(3x + 1)$ because that's what the power 2 means. There are several methods for

continuing from this point, such as FOIL (first, outer, inner, last). The solution shown below uses the distributive property.

inline

It is fairly common for students to think that $(3x + 1)^2$ and $(3x)^2 + (1)^2$ are equivalent, but they're not. If you chose **H**, you may have made this mistake. Understanding why this is a mistake can help you understand other parts of algebra and not make similar mistakes. If you let $x = 1$, then $(3x + 1)^2 = 16$ but $(3x)^2 + (1)^2 = 10$. It makes a difference whether you add first and then square, or whether you square first and then add.

Question 146. The correct answer is B. Here's one where drawing a picture might help you avoid some mistakes.



The points with coordinates −5 and 17 are 22 units apart. If you go up by 11 units from −5, you should get to the same place as if you go down by 11 units from 17. This is 6.

If you chose **C**, you may have calculated inline. This represents the distance from the midpoint to an endpoint. This expression is very similar to the average of the two coordinates, inline (notice the plus sign), which is the coordinate of the midpoint. That's why drawing a picture is a good idea.

Question 147. The correct answer is G.

inline.

The most common wrong answer is **H**, which happens when someone tries to reduce inlineto inline.

Question 148. The correct answer is A. Although you could deduce some things from the way the equations are given, you might be more

comfortable with the equations in slope-intercept form. The system would then be:

image

The slopes of the two lines are different (the first is inline, the second inline). That, by itself, means **A** is correct. The graph below shows what this system looks like. You can see that none of the other answer choices is correct.

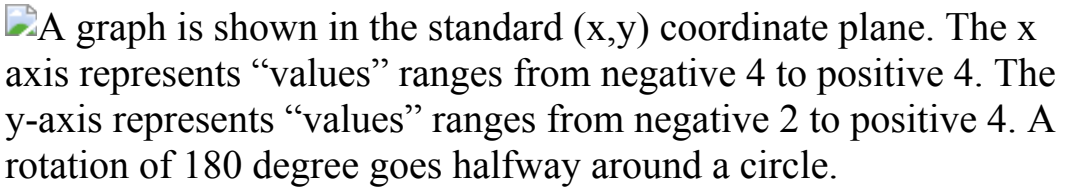
A graph is shown in the standard (x,y) coordinate plane. Two different lines are drawn from two different points on the x-axis and intersect at a point on y-axis. These lines represents two different linear equations: (1) y equals to negative start fraction 2 over 3 end fraction x plus start fraction 8 over 3 end fraction and (2) y equals to start fraction 2 over 3 end fraction x plus start fraction 8 over 3 end fraction.

Question 149. The correct answer is K. Properties of exponents make this straightforward to solve. All of the *bases* (2, 4, and 8) are powers of 2. The equation can be rewritten as $(2^x)(2^2) = (2^3)^3$. Properties of exponents lead to the equation $2^{x+2} = 2^{3 \cdot 3}$, which simplifies to $2^{x+2} = 2^9$. If the two exponents are the same, then the left and right sides are equal. This happens when $x + 2 = 9$, which is when $x = 7$.

You can check this answer (or whatever answer you got). When $x = 7$, the left side of the equation is $(2^7)(4)$, which simplifies to $(128)(4)$, and then to 512. The right side is 8^3, which is also 512.

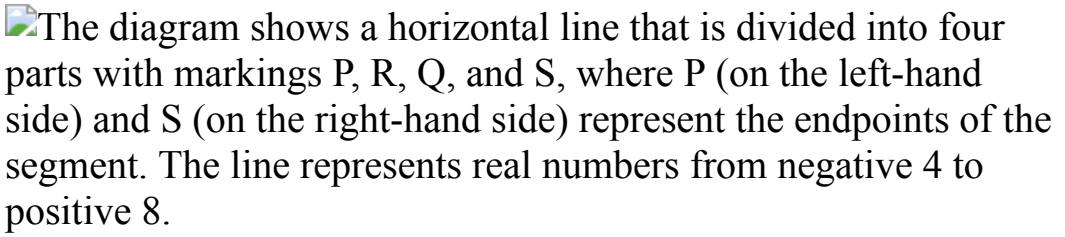
The most common wrong answer for this problem is **G**, which happens when people combine $(2^x)(4)$ and get (8^x). These are not equivalent (check $x = 2$). Another common mistake is to write $(2^3)^3$ as 2^6, which leads to **H**.

Question 150. The correct answer is D. A rotation of 180° goes halfway around a circle. The graph below illustrates this rotation. Because it is halfway, it does not matter whether you rotate clockwise or counterclockwise.



The most common incorrect answer is **B**, which is a reflection across the *y*-axis. Several of the points on this graph are correct for the rotation, but not all of them.

Question 151. The correct answer is E. To go from *P* to *Q* is 9 units. From *Q* to *R* is 6 units. From *R* to *S* is 9 units. From *S* back to *P* is 12 units. The total length is 9 + 6 + 9 + 12 = 36 units.

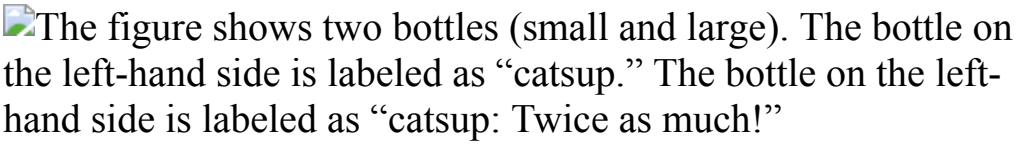


Question 152. The correct answer is J. The determinant inline has the value $(2x)(4y) - (3y)(5x)$, which simplifies to $8xy - 15xy = -7xy$. When $x = -3$ and $y = 2$, the value of the expression is $-7(-3)(2) = 42$.

Another approach is to substitute $x = -3$ and $y = 2$ first, giving the determinant inline, which has the value $(-6)(8) - (6)(-15) = (-48) - (-90) = -48 + 90 = 42$.

If you got answer **G**, you probably made a mistake with a minus sign. If you got answers **F** or **K**, you probably used the second approach and made an error with a minus sign.

Question 153. The correct answer is C. Let the capacity of the larger bottle be *B* ounces. Then, the capacity of the smaller bottle is inline ounces. The larger bottle starts with inline ounces of catsup.The smaller bottle starts with inline ounces of catsup. When this catsup is poured into the larger bottle (when you pour, be sure to get it all out of the smaller bottle), the larger bottle now contains inline ounces of catsup, which is inline ounces of catsup. This means the bottle is inline full.



If you find it hard to follow the steps with the abstract “*B* ounces” for the larger bottle's capacity, try thinking of a particular size of bottle, say a 12-ounce bottle. Then the smaller bottle's capacity is 6 ounces. The larger bottle contains inline ounces of catsup. The smaller bottle contains inline ounces of catsup. When poured together, the larger bottle would now contain 10 ounces of catsup, which is inline of its capacity.

The most common wrong answer is **D**. If one catsup bottle is inline full and another is inline full, then together they would be a whole bottle of catsup—if the bottles were the same size. Because the inline of the smaller bottle is smaller than the missing inline of the larger bottle, the larger bottle won't be full.

Question 154. The correct answer is G. In the given equation, $t = 10p + 5$, the 10 represents the number of minutes that the time changes when the number of problems increases by 1. That's the slope. So, Jeff is budgeting 10 minutes per problem. The 5 minutes for getting set up is not budgeted on a per-problem basis.

For 1 problem, the total time budgeted is $10(1) + 5 = 15$ minutes. This may lead you to believe that **F** is the correct answer, but consider an assignment with 2 problems. Using the given equation, that would be budgeted at $10(2) + 5 = 25$ minutes. Using **F**, it would be budgeted at 30 minutes. Some of the other answer choices could be true, but they do not have to be true. For example, Jeff could budget a 5-minute break after each problem, which would fit into the 10 minutes he budgets per problem. But, he would not have to schedule any breaks between problems, or he could schedule 2-minute breaks.

Question 155. The correct answer is D. Because Kaya drove 200 miles in 5 hours, she was averaging inline miles per hour. If she drove 10 miles per hour faster, that would be 50 miles per hour. To complete the 200 miles at 50 miles per hour would take a driving time of inline hours. That's a savings of 1 hour.

The most common wrong answer was **E**, which is the number of hours the faster trip would take. That isn't what the question asks for.

Question 156. The correct answer is B. If $a = b$, then $|a| = |b|$. (This is a property of a *function*, that if you start with the same number then you will get the same result. Absolute value is a function.) This shows that **A** is false, and it also shows that **B** must be true.

You can eliminate **C** by checking $a = 2$ and $b = 1$. You can eliminate **D** and **E** by checking $a = -2$ and $b = 1$. The most common wrong answer is **D**.

Question 157. The correct answer is J. One way to solve this is to put the equation into slope-intercept form (solve for y). This can be accomplished as follows: inline. The slope is the coefficient of x, which is inline. The most common errors are making a sign mistake (**G**) or getting the fraction upside down (**H**).

Question 158. The correct answer is H. ΔDFB and ΔEFB are congruent, which can be seen by using side-side-side congruence and the following steps. First, $DF \cong EF$ is given. Second, the triangles share a common side, FB, so certainly $FB \cong FB$. And, the third sides are congruent ($DB \cong EB$) because the length of the third side in each triangle is determined from the other two sides by the Pythagorean theorem—the calculation is the same in both cases.

Because ΔDFB and ΔCFB are congruent, $\angle DFB$ is congruent to $\angle EFB$. Then, the three angles within ΔDFB are a right angle, $\angle x$, and an angle congruent to $\angle y$. That means that the measures of the angles add up to 180°, and so the measures of $\angle x$ and $\angle y$ add up to 90°. It really didn't matter that the given angle at A measured 50°. You can't tell the measure of $\angle x$ or $\angle y$, but you can find the sum.

Question 159. The correct answer is E. $(-2x^5y^2)^4 = (-2)^4(x^5)^4(y^2)^4 = 16x^{5 \cdot 4}y^{2 \cdot 4} = 16x^{20}y^8$

If you chose **D**, you likely wrote $(x^5)^4$ as $x^5 + 4$ and $(y^2)^4$ as $y^2 + 4$. If you chose **A**, you likely multiplied exponents properly but wrote $(-2)^4$ as -16 when it is really $(-2)(-2)(-2)(-2) = +16$.

Question 160. The correct answer is E. Because the only real numbers that satisfy $a^2 = 49$ are 7 and -7, and the only real numbers that satisfy $b^2 = 64$ are 8 and -8, the only possibilities for $a + b$ are $7 + 8$, $7 + (-8)$, $-7 + 8$, and $-7 + (-8)$. These possibilities are 15, -1, 1, and -15. The only answer choice left is 113.

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