

**EST II - Individual Subject Test**

**Level 1**

**Student's Name** \_\_\_\_\_  
**National ID** \_\_\_\_\_  
**Test Center:** \_\_\_\_\_

**Subject:** Math

**Duration:** 60 minutes

50 Multiple Choice Questions

**Instructions:**

- Place your answer on the answer sheet. Mark only one answer for each of the multiple choice questions.
- Avoid guessing. Your answers should reflect your overall understanding of the subject matter.
- Calculators are allowed. When a calculator is used, be aware of switching between radian mode and median mode.
- A formula sheet is available at the end of the booklet for your reference.

1. A plane intersects a sphere of radius 4cm in a circle. Which of the following cannot be the area of this circle?
- A.  $4 \pi \text{ cm}^2$
  - B.  $8 \pi \text{ cm}^2$
  - C.  $12 \pi \text{ cm}^2$
  - D.  $16 \pi \text{ cm}^2$
  - E.  $24 \pi \text{ cm}^2$
2. If  $3^n \times 9^m \times 27^p = 1$ , then which of the following is the correct expression of  $n$  in terms of  $m$  and  $p$ ?
- A.  $n = \frac{1}{mp}$
  - B.  $n = -3m - 2p$
  - C.  $n = 1 - 2m - 3p$
  - D.  $n = -m - p$
  - E.  $n = -2m - 3p$
3. The three lines  $l$ ,  $d$ , and  $t$  in the plane are such that they are parallel and  $d$  is equidistant from  $l$  and  $t$ . If the equation of  $l$  is  $y = 2x + 70$  and the equation of  $d$  is  $y = 2x + 10$ , which of the following is the equation of line  $t$ ?
- A.  $2x - y = -40$
  - B.  $2x - y = 50$
  - C.  $2x - y = 40$
  - D.  $x - 2y = 50$
  - E.  $x - 2y = -40$
4. If  $a$  and  $b$  are real numbers,  $i^2 = -1$ , and  $a + bi = (2 + 3i)^2$ , then which of the following is the average of  $a$  and  $b$ ?
- A. 2.5
  - B. 3
  - C. 3.5
  - D. 5
  - E. 12.5
5. The volume of a certain pyramid of rectangular base is 12. We alter its dimensions in such a way that the volume does not change. If the height has been doubled, which of the following could be possible alterations to the length  $L$  and the width  $W$  of the base of the pyramid?
- A. Multiply  $L$  by 12 and divide  $W$  by 6
  - B. Multiply  $L$  by 1 and divide  $W$  by 0.5
  - C. Multiply  $L$  by 9 and divide  $W$  by 18
  - D. Divide both  $L$  and  $W$  by 0.5
  - E. Multiply both  $L$  and  $W$  with 0.5

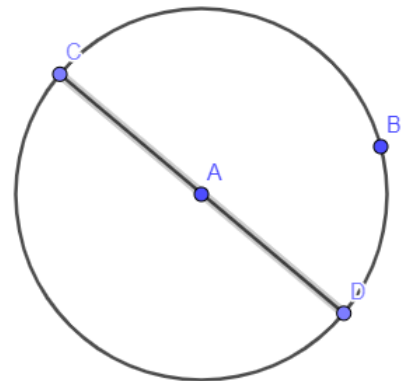
6. If  $f$  is the function defined by  $f(x) = \frac{3x^2 - 4x + 1}{2x^2 + 7x - 9}$ , then which of the following is a root of  $f(x)$ ?

I.  $-\frac{9}{2}$

II.  $\frac{1}{3}$

III. 1

- A. Only I  
 B. Only II  
 C. Only III  
 D. II and III  
 E. I, II, and III
7. In the adjacent figure, B is a point on the circle of center A and diameter DC such that  $BC = 20$  and  $BD = 15$ . What is the perimeter of triangle BDC?



- A. 17.5  
 B. 35  
 C. 60  
 D. 150  
 E.  $60\pi$
8. If  $0^\circ \leq \theta < 90^\circ$ , then  $\frac{1}{1 - \sin\theta} =$

A.  $\frac{1}{\cos^2\theta} + \frac{\tan\theta}{\cos\theta}$

B.  $1 - \frac{1}{\sin\theta}$

C.  $1 + \sin\theta$

D.  $\tan^2\theta + \frac{\tan\theta}{\cos\theta}$

E.  $\frac{1}{\cos^2\theta} + \sin^2\theta$

9. When fidget spinners were first introduced in the market, Leonard decided to sell them in his toy shop. The number of spinners he sold every week is modeled by  $N(w) = -w^2 + 10w$ , where  $w$  is the number of weeks starting the first week of February and  $N(w)$  is the number of spinners sold each week in hundreds. One particular week, Leonard realized that his sales were greater than the sales in any other week. What is this maximum number of sales?

A. -2500

B. 500

C. 1000

D. 2500

E. 5000

10. Celine and Lia are standing on a horizontal track at points A and D respectively. Celine is 1.5m tall and Lia is 1.2m tall (that is  $AC = 1.5\text{m}$  and  $DL = 1.2\text{m}$ ). Celine's shadow occupies the length AB and Lia's shadow occupies the length BD on the ground so the two shadows meet at their heads. The angles between the ground and the ray from the top of the head of each individual to the touching point of the shadows is shown. How far apart are Celine and Lia standing?

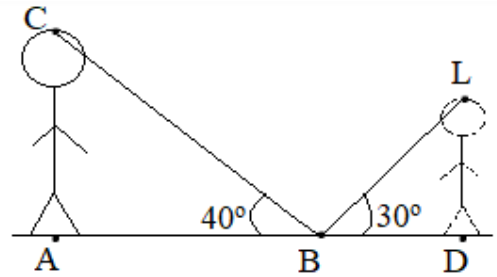


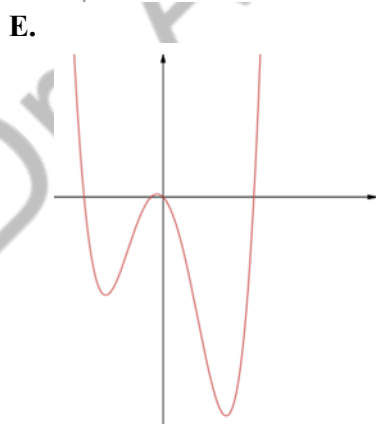
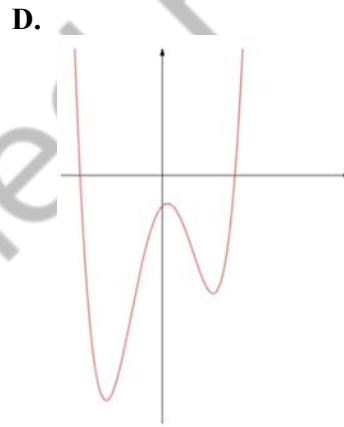
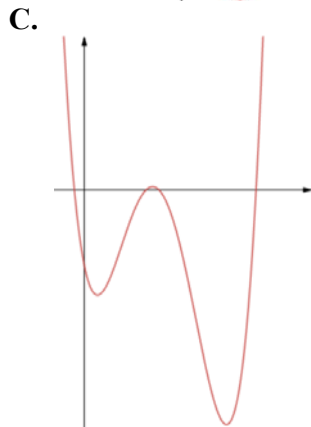
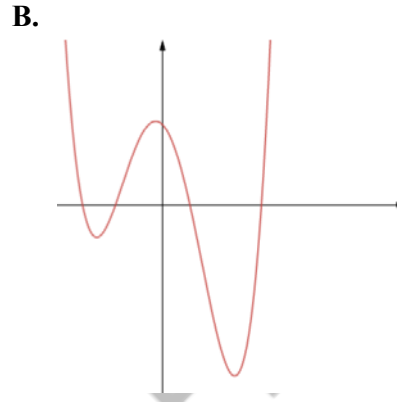
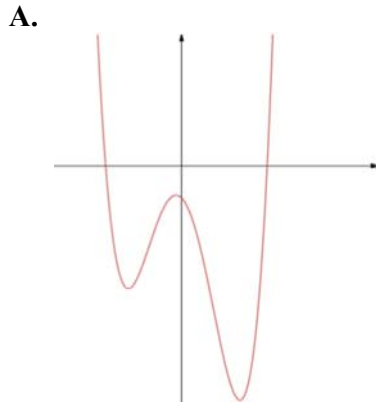
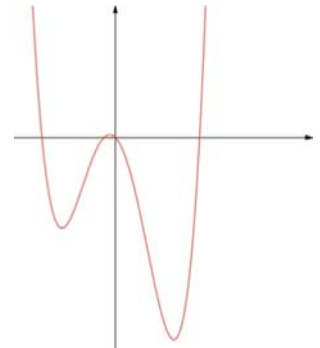
FIGURE IS NOT DRAWN TO SCALE

- A. 1.788 m  
 B. 2.078 m  
 C. 2.7 m  
 D. 3.344 m  
 E. 3.866 m
11. Consider the function  $f$  defined by  $f(x) = (m - 1)x^3 - 3m - 1$ , where  $m$  is a real number. If the graph of  $f$  has a  $y$ -intercept of 2, then  $f(1) =$
- A. -6.4  
 B. -1  
 C. 0  
 D. 1  
 E. 2.2
12. If  $y$  varies directly with  $z$  and inversely with  $v$ , then which of the following could be a correct expression of  $z$  as a function of  $y$  and  $v$ ?
- A.  $z = 3yv$   
 B.  $z = \frac{1y}{6v}$   
 C.  $z = \frac{7v}{6y}$   
 D.  $z = 8\frac{y}{v}$   
 E.  $z = \frac{1}{6}(y + v)$
13. If  $\sqrt{a} + \sqrt{b} = 4.6$  and  $a + b = 13.46$ , then  $ab =$
- A. 1.21  
 B. 3.85  
 C. 7.7  
 D. 12.25  
 E. 14.8225

14. In a certain country, the ratio of male individuals who got infected by the COVID-19 virus to the female individuals who got infected by the same virus is 6 to 4. Moreover, it was observed that only 9% of the males and 5% of the females who get infected by the virus got severe symptoms. If the number of COVID-19 cases in the country is 1200, how many females have severe symptoms?
- A. 24
  - B. 162
  - C. 600
  - D. 1200
  - E. 1800
15. Which of the following is equal to  $\frac{5^{202} + 5^{200}}{25^{100}}$  ?
- A. 25
  - B. 26
  - C.  $2^{202}$
  - D.  $5^{202}$
  - E.  $10^{202}$
16. Each day, Jack's donut shop starts with 100 donuts. Throughout the day, some donuts are sold, some are discarded, and some more is made according to the following principle: for every three donuts sold, one more is made. If on a particular day, Jack sold  $m$  donuts and discarded  $d$  donuts, and at the end of the day had no donuts left, which of the following gives the correct relation between  $m$  and  $d$ ?
- A.  $2m + 3d = 300$
  - B.  $3m + 2d = 300$
  - C.  $3d - 2m = 300$
  - D.  $2d - 3m = 100$
  - E.  $m + d = 100$
17. Consider the set  $\{-3, 0, 7, a, b, 20\}$  whose elements are arranged in increasing order, and  $a$  and  $b$  are real numbers. If the median of this set above is 12, which of the following is a possible value of  $b$ ?
- I. 8
  - II. 17
  - III. 20
- A. Only I
  - B. Only II
  - C. Only III
  - D. I and II
  - E. II and III

18. Let  $f$  be the function defined by  $f(x) = x^4 - 3x^2 - k$  where  $k$  is a real number. For which values of  $k$  does the line  $d$  of equation  $y = 1$  cut the graph of  $f$  in 4 distinct points?
- A.  $-3.7$   
 B.  $-1.9$   
 C.  $-1$   
 D.  $0$   
 E.  $1.2$
19. The product of three consecutive integers is equal to the middle integer. What is the smallest of these integers?
- A.  $-1 - \sqrt{2}$   
 B.  $1 + \sqrt{2}$   
 C.  $-1$   
 D.  $0$   
 E.  $1$
20. A line in the plane passes through the 3 distinct points  $(a, b)$ ,  $(c, d)$ , and  $(e, f)$ . Then all of the following give a correct relation between  $a, b, c, d, e$ , and  $f$  except:
- A.  $\frac{c-a}{d-b} = \frac{c-e}{d-f}$   
 B.  $\frac{d-b}{d-f} \times \frac{c-e}{a-c} = 1$   
 C.  $\frac{d-b}{d-f} \times \frac{c-e}{c-a} = 1$   
 D.  $\frac{c-a}{d-b} + \frac{c-e}{f-d} = 0$   
 E.  $\frac{d-b}{d-f} - \frac{c-a}{c-e} = 0$
21. Consider the parabola of equation  $y = x^2 + 22x - 320$ . What are the coordinates of the midpoint of the segment joining the two  $x$  -intercepts of the parabola?
- A.  $(-11, 0)$   
 B.  $(0, -11)$   
 C.  $(11, 0)$   
 D.  $(0, 11)$   
 E.  $(0, -160)$
22. In a certain company of 400 employees, 70% wear face masks at all times. Out of those who do not wear face masks, 15% got the disease. Out of those who do wear a mask, only 5% got the disease. If an employee is chosen at random from this company, what is the probability that he or she got the disease?
- A.  $0.35$   
 B.  $0.08$   
 C.  $0.09$   
 D.  $0.12$   
 E.  $0.35$

23. The adjacent graph is that of a function  $f(x)$ . The graph of the function  $g$  is obtained by translating the graph of  $f$ , that is  $g(x) = f(x + a) + b$  for some real numbers  $a$  and  $b$ . Which of the graphs below cannot be that of  $g(x)$  for any real values of  $a$  and  $b$ ?

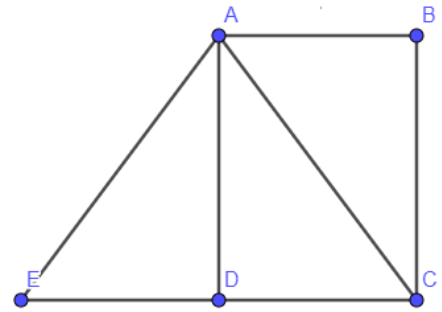


24. If  $f$  is the function defined by  $f(x) = (2x + 1)^3$ , for which of the following values of  $x$  is  $f(f(x)) = f(x)$ ?
- A. -2
  - B. -1
  - C. 0
  - D. 1
  - E. 2
25. The range of the function defined by  $f(x) = |1 - x^2| + 2$  is:
- A.  $f(x) > 2$
  - B.  $f(x) > 3$
  - C.  $f(x) < 2$
  - D.  $f(x) \geq 2$
  - E.  $f(x) \leq 2$
26. A circle inscribed in a square has an area of  $36\pi$ . What is the length of the diagonal of the square?
- A. 8.485
  - B. 12
  - C. 16.97
  - D. 36
  - E. 50.912
27. If  $f(x) = x^2 - 4x + 1$ , which of the following is also equal to  $f(x)$ ?
- A.  $f(x - 4)$
  - B.  $f(2 - x)$
  - C.  $f(4 - x)$
  - D.  $f(4 + x)$
  - E.  $f(4) - f(x)$
28.  $C$  is a circle tangent to the two straight lines of equations  $y = 2$  and  $y = -4$ . Which of the following could be the equation of the circle?
- A.  $(x + 1)^2 + y^2 = 9$
  - B.  $(x - 1)^2 + (y + 1)^2 = 3$
  - C.  $(x + 4)^2 + (y + 2)^2 = 9$
  - D.  $(x + 1)^2 + y^2 = 3$
  - E.  $(x + 2)^2 + (y + 1)^2 = 9$
29. Consider all spheres of nonzero volume having as their center the point  $A$  in space. How many of these spheres can have a surface area whose value is numerically equal to the volume of the sphere?
- A. None
  - B. 1
  - C. 2
  - D. 3
  - E. Further information is required to determine the answer



30. The two lines  $m$  and  $n$  of respective equations  $3x - y = 1$  and  $ax + by = 2$  are parallel, where  $a < 0$  and  $b$  is a real number. Which of the following may be true about  $b$ ?
- I.  $b = -2.31$
  - II.  $b = 1.04$
  - III.  $b = 0$
- A. Only I
  - B. Only II
  - C. I and III
  - D. II and III
  - E. I, II, and III

31. In the adjacent figure,  $ABCD$  is a rectangle such that  $BC = 4$  and  $AB = 3$ ,  $AEC$  is a triangle such that the two angles  $\angle AEC$  and  $\angle ACE$  are equal. What is the perimeter of triangle  $AEC$ ?

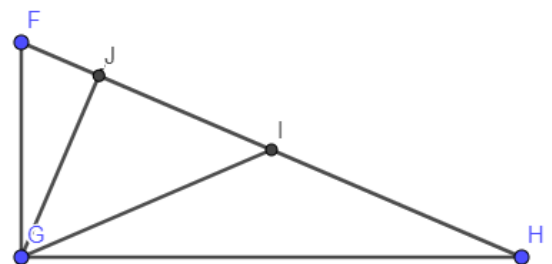


- A. 10
- B. 12
- C. 14
- D. 16
- E. 18

32.  $A$ ,  $C$ ,  $E$ , and  $D$  are four collinear points in this order such that  $AC = a$ ,  $ED = b$ , and  $AE + CD = 12$ . Which of the following is a correct expression of  $CE$  in terms of  $a$  and  $b$ .

- A.  $CE = 6 - \frac{a+b}{2}$
- B.  $CE = \frac{a+b}{2}$
- C.  $CE = 12 - (a + b)$
- D.  $CE = 6 - (a + b)$
- E.  $CE = 12 - \frac{a+b}{2}$

33. The triangle shown in the adjacent figure (not drawn to scale) is right angled at  $G$  with  $GF = 4$  and  $\angle GFH = 60^\circ$ .  $I$  is the midpoint of segment  $FH$  and  $GJ$  is the altitude drawn from  $G$  to  $FH$ . What is the area of triangle  $GJI$ ?

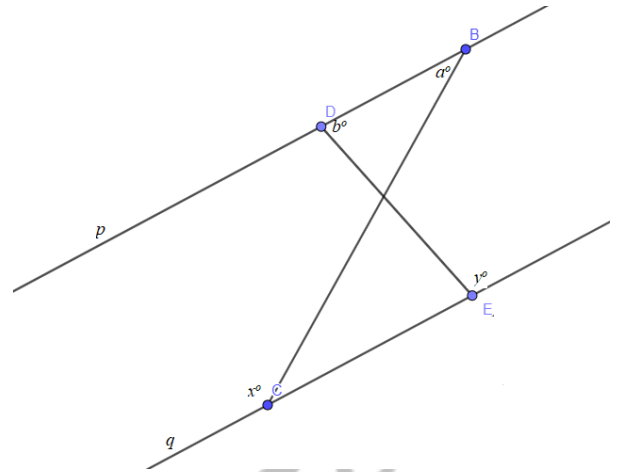


- A. 3.46
- B. 4.62
- C. 6
- D. 6.93
- E. 13.86

34. In the adjacent figure,  $p$  and  $q$  are two parallel lines. B and D are points on  $p$ ; C and E are points on  $q$ . Which of the following statements is true?

- I.  $a^\circ + b^\circ = x^\circ + y^\circ$
- II.  $a^\circ, b^\circ, x^\circ,$  and  $y^\circ$  can be the values of the four angles of a quadrilateral.
- III.  $a^\circ + x^\circ = b^\circ + y^\circ$

- A. Only I
- B. Only II
- C. Only III
- D. I and II
- E. II and III



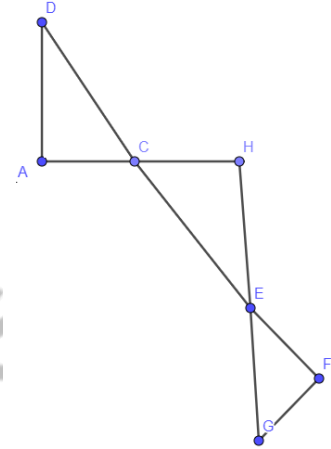
Dr. Ahmed Hass

35. If  $3k - \frac{k}{k+1} = \frac{16}{k+1}$ , what is a possible value of  $k + 1$ ?

- A.  $-\frac{8}{3}$
- B.  $-\frac{5}{3}$
- C. 2
- D. 5
- E. 6

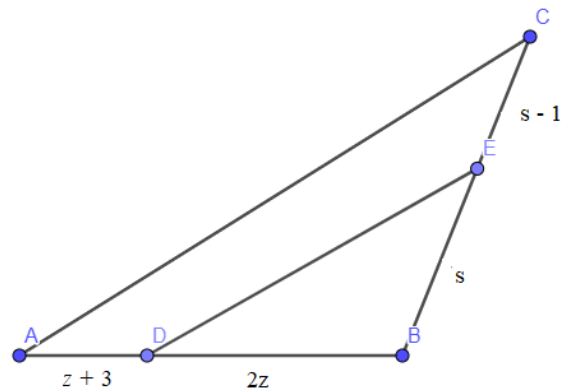
36. In the adjacent figure (not drawn to scale), segments AD and AH are perpendicular. The following degree measures are known in terms of  $b$ :  $\angle ADC = b$ ,  $\angle EGF = 2b + 30$ , and  $\angle EFG = 150 - 3b$ . Which of the following conclusions must be true?

- A. The three triangles ADC, CHE, and EFG are right angled triangles.
- B. Triangle HCE is right angled at H.
- C. Triangles ADC and EFG are similar.
- D. Triangles ADC and HCD are similar.
- E. Only triangle ADC is right angled.



37. In the adjacent figure, lines AC and DE are parallel. The lengths of segments CE, BE, BD, and AD are given in terms of  $s$  and  $z$ . Which of the following is a correct expression of  $s$  in terms of  $z$ ?

- A.  $s = \frac{2z}{z-3}$
- B.  $s = 2 - \frac{3}{2z}$
- C.  $s = \frac{z+3}{3-2z}$
- D.  $s = \frac{3(z+1)}{z+3}$
- E.  $s = \frac{z+3}{z} + 1$



38. The perimeter of a regular hexagon is 24 and the perimeter of a regular octagon is 5. What is the ratio of the length of one side of the hexagon to the length of one side of the octagon?

- A. 0.15625
- B. 0.2083
- C. 1.2
- D. 2.75
- E. 6.4

39. The equation  $\frac{1+|x-1|}{|x-1|} = 5$  has two solutions. What is the sum of these two solutions?
- A. 0.75
  - B. 1.25
  - C. 2
  - D. 2.5
  - E. 3.25
40. Let  $f$  be the function defined by  $f(x) = mx^3 + px^2 + qx - 1$ , where  $m, p$ , and  $q$  are real numbers. If the graph of  $f$  passes through the point  $(1, 7)$ , what is the average of  $m, p$ , and  $q$ ?
- A.  $\frac{7}{3}$
  - B. 2
  - C.  $\frac{8}{3}$
  - D. 3.5
  - E. 4
41. If  $a(b - 1) = 1$  and  $12ab = 4$ , what is the value of  $b$ ?
- A. -0.667
  - B. -0.5
  - C. 0.5
  - D. 0.667
  - E. 2
42. If  $m$  and  $n$  are the roots of the equation  $ax^2 + bx + c = 0$ , where  $a, b$ , and  $c$  are real numbers, which of the following quantities does not change when we vary  $a$ ?
- I.  $m \cdot n$
  - II.  $m + n$
  - III.  $\frac{1}{m} + \frac{1}{n}$
- A. Only I
  - B. Only II
  - C. Only III
  - D. I and II
  - E. I, II, and III
43. If  $f(x) = 1.6^x$  and  $g(x) = 1.4^x$  and  $f(g(0)) = A \times g(f(0))$ , what is the value of  $A$ ?
- A.  $\frac{7}{8}$
  - B. 1
  - C.  $\frac{8}{7}$
  - D.  $\frac{25}{56}$
  - E.  $\frac{56}{25}$

44. Circles C, P, and Q have radii 1, 2, and 3 respectively. If the three circles are tangent to each other, what is the nature of the triangle formed by joining the three centers of the circles?
- A. Isosceles
  - B. Right angled
  - C. Right isosceles
  - D. Equilateral
  - E. Cannot be determined with the information given

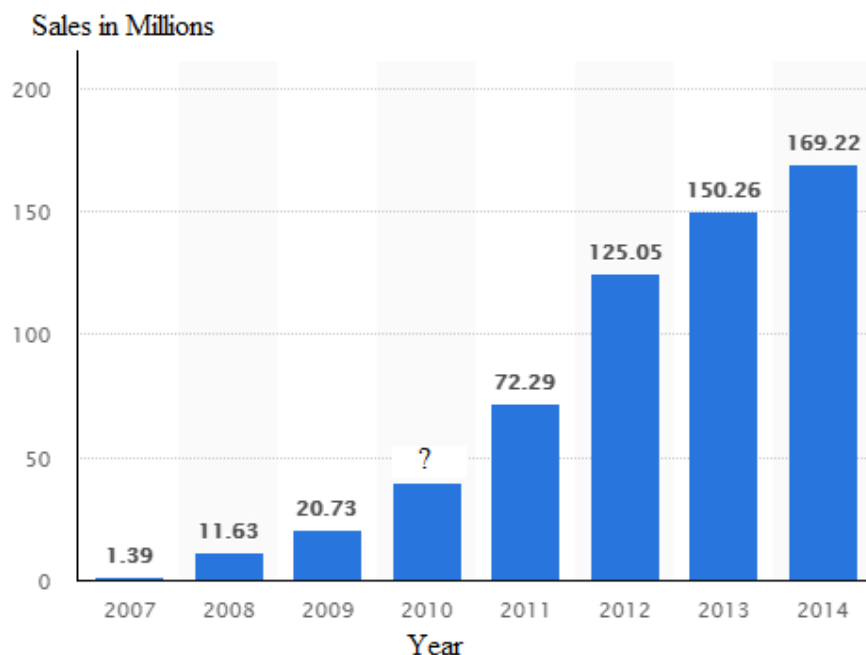
45.

Days d	1	2	3	4	5	6	7	8
Cases	100	350	250	600	723	702	750	790
N								

The table above shows the number of cases of individuals infected with the COVID-19 virus in a certain country every day after Sunday October 18 (so  $d = 1$  is Monday October 19, and so on). The government uses a linear regression model and decides to impose total lockdown when the number of cases reaches 1000. Using the linear model, what is the predicted date at which the lockdown will be imposed?

- A. October 20
- B. October 23
- C. October 29
- D. October 31
- E. November 1

46.



The bar graph given above shows the number of smart phones sold in millions from 2007 when it was first introduced into the market till 2014 except for the year 2010. Knowing that the percent increase from the year 2010 to 2011 for the number of smart phones sold is four times the percent increase from the year 2013 to 2014, what is the approximate number of smart phones sold, in millions, in the year 2010?

- A. 14.458
- B. 32.35
- C. 40.161
- D. 25.73
- E. 71.52

47. For how many strictly positive integer values of  $x$  is the following inequality satisfied?

$$2(3 - x) \geq -14$$

- A. 0
- B. 1
- C. 9
- D. 10
- E. 11

48. If  $f(x) = x^2 + 1$  and  $d(x) = \frac{f(x) - f(2)}{x - 2}$ , then which of the following is true?

- A.  $d(x) = f(x)$  for all values of  $x$
- B.  $d(x) = x - 2$  for all values of  $x$
- C.  $d(x) = x + 1$  for all values of  $x$  except 2
- D.  $d(2) = f(2)$
- E.  $d(x) = x + 2$  for all values of  $x$  except 2

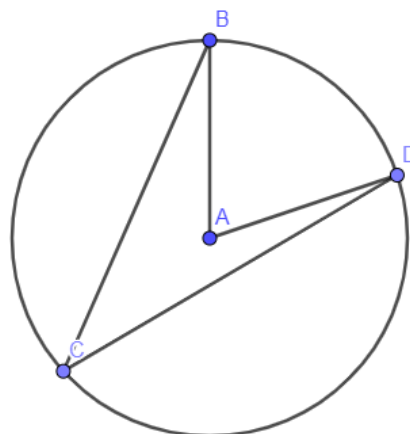
49. 
$$\begin{cases} 12x - 15y = 17 \\ x + 8y = 12 \end{cases}$$

What is the value of  $46y - 22x$ ?

- A. -10
- B. -5
- C. 0
- D. 5
- E. 10

50. In the adjacent figure, the measure of angle  $\angle BAD$  is  $80^\circ$ . Which of the following could be the measure of angle  $\angle BCA$ ?

- A.  $38.96^\circ$
- B.  $40^\circ$
- C.  $45.02^\circ$
- D.  $60.13^\circ$
- E.  $90^\circ$



Dr. Ahmed H

THE FORMULAS BELOW MAY BE USEFUL IN ANSWERING QUESTIONS ON THIS TEST.

$S = 4\pi r^2$  is the formula for the surface area of a sphere with a radius of  $r$ .

$V = \frac{1}{3}\pi r^2 h$  is the formula for a right circular cone with a radius of  $r$  and a height of  $h$ .

$V = \frac{4}{3}\pi r^3$  is the formula for a sphere with a radius of  $r$ .

$V = \frac{1}{3}Bh$  is the formula for a pyramid with a base area of  $B$  and a height of  $h$ .

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**EST II Math Level1- Round 2**

**Answer key**

Question	Answer	Question	Answer
1	E	26	C
2	E	27	C
3	B	28	E
4	C	29	B
5	C	30	B
6	B	31	D
7	C	32	A
8	A	33	A
9	D	34	E
10	E	35	B
11	C	36	B
12	A	37	A
13	E	38	E
14	A	39	C
15	B	40	C
16	A	41	B
17	E	42	C
18	B	43	C
19	C	44	B
20	B	45	C
21	A	46	C
22	B	47	D
23	D	48	E
24	B	49	A
25	D	50	A

**EST II - Individual Subject Test**

**Level 1**

**Student's Name** \_\_\_\_\_  
**National ID** \_\_\_\_\_  
**Test Center:** \_\_\_\_\_

**Subject:** Math

**Duration:** 60 minutes

50 Multiple Choice Questions

**Instructions:**

- Place your answer on the answer sheet. Mark only one answer for each of the multiple choice questions.
- Avoid guessing. Your answers should reflect your overall understanding of the subject matter.
- Calculators are allowed. When a calculator is used, be aware of switching between radian mode and median mode.
- A formula sheet is available at the end of the booklet for your reference.

1. A plane intersects a sphere of radius 4cm in a circle. Which of the following cannot be the area of this circle?

- A.  $4\pi \text{ cm}^2$   
 B.  $8\pi \text{ cm}^2$   
 C.  $12\pi \text{ cm}^2$   
 D.  $16\pi \text{ cm}^2$   
 E.  $24\pi \text{ cm}^2$

$$r \neq 4$$

2. If  $3^n \times 9^m \times 27^p = 1$ , then which of the following is the correct expression of  $n$  in terms of  $m$  and  $p$ ?

- A.  $n = \frac{1}{mp}$   
 B.  $n = -3m - 2p$   
 C.  $n = 1 - 2m - 3p$   
 D.  $n = -m - p$   
 E.  $n = -2m - 3p$

$$3^{(n)} \times 3^{(2m)} \times 3^{(3p)} = 3^0$$

$$n + 2m + 3p = 0$$

$$n = -2m - 3p$$

3. The three lines  $l$ ,  $d$ , and  $t$  in the plane are such that they are parallel and  $d$  is equidistant from  $l$  and  $t$ . If the equation of  $l$  is  $y = 2x + 70$  and the equation of  $d$  is  $y = 2x + 10$ , which of the following is the equation of line  $t$ ?

- A.  $2x - y = -40$   
 B.  $2x - y = 50$   
 C.  $2x - y = 40$   
 D.  $x - 2y = 50$   
 E.  $x - 2y = -40$

$$y = 2x + 70 \xrightarrow{-60} l$$

$$y = 2x + 10 \xrightarrow{-60} d$$

$$y = 2x - 50 \xrightarrow{-2x+y=-50} t$$

4. If  $a$  and  $b$  are real numbers,  $i^2 = -1$ , and  $a + bi = (2 + 3i)^2$ , then which of the following is the average of  $a$  and  $b$ ?

- A. 2.5  
 B. 3  
 C. 3.5  
 D. 5  
 E. 12.5

$$(2 + 3i)^2 = 4 + 12i + 9i^2 = 4 + 12i - 9 = -5 + 12i$$

$$\frac{-5 + 12}{2} = 3.5$$

5. The volume of a certain pyramid of rectangular base is 12. We alter its dimensions in such a way that the volume does not change. If the height has been doubled, which of the following could be possible alterations to the length  $L$  and the width  $W$  of the base of the pyramid?

- A. Multiply  $L$  by 12 and divide  $W$  by 6  
 B. Multiply  $L$  by 1 and divide  $W$  by 0.5  
 C. Multiply  $L$  by 9 and divide  $W$  by 18  
 D. Divide both  $L$  and  $W$  by 0.5  
 E. Multiply both  $L$  and  $W$  with 0.5

$$V = \frac{1}{3} (L \times W \times h) = 12$$

$$V = \frac{1}{3} L \cdot W \cdot (2h) = 12$$

$$2h = 12$$

$$h = 6 \text{ (dec half)}$$

$$\frac{9}{18} = \frac{1}{2}$$

6. If  $f$  is the function defined by  $f(x) = \frac{3x^2 - 4x + 1}{2x^2 + 7x - 9}$ , then which of the following is a root of  $f(x)$ ?

Domain  
Make the f(x) undefined

- I.  $-\frac{9}{2}$   
 II.  $\frac{1}{3}$   
 III. 1

$3x^2 - 4x + 1$   
 Made 5, 3  
 $x = 1, \frac{1}{3}$

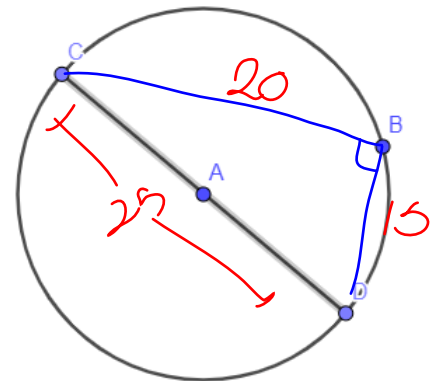
$2x^2 + 7x - 9$   
 $x = 1, -\frac{9}{2}$

- A. Only I  
 B. Only II  
 C. Only III  
 D. II and III  
 E. I, II, and III

7. In the adjacent figure, B is a point on the circle of center A and diameter DC such that  $BC = 20$  and  $BD = 15$ . What is the perimeter of triangle BDC?

- A. 17.5  
 B. 35  
 C. 60  
 D. 150  
 E.  $60\pi$

$25 + 20 + 15 = 60$



8. If  $0^\circ \leq \theta < 90^\circ$ , then  $\frac{1}{1 - \sin\theta} = \frac{1 + \sin\theta}{1 + \sin\theta}$

A.  $\frac{1}{\cos^2\theta} + \frac{\tan\theta}{\cos\theta}$   
 B.  $1 - \frac{1}{\sin\theta}$   
 C.  $1 + \sin\theta$   
 D.  $\tan^2\theta + \frac{\tan\theta}{\cos\theta}$   
 E.  $\frac{1}{\cos^2\theta} + \sin^2\theta$

$$= \frac{1 + \sin\theta}{1 + \sin\theta - \sin\theta - \sin^2\theta} = \frac{1 + \sin\theta}{\cos^2\theta} = \frac{1 + \sin\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} + \frac{\tan\theta}{\cos\theta}$$

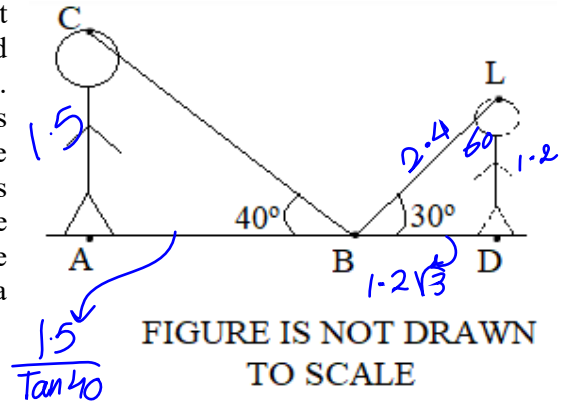
9. When fidget spinners were first introduced in the market, Leonard decided to sell them in his toy shop. The number of spinners he sold every week is modeled by  $N(w) = -w^2 + 10w$ , where  $w$  is the number of weeks starting the first week of February and  $N(w)$  is the number of spinners sold each week in hundreds. One particular week, Leonard realized that his sales were greater than the sales in any other week. What is this maximum number of sales?

- A. -2500  
 B. 500  
 C. 1000  
 D. 2500  
 E. 5000

$$\frac{-b}{2a} = \frac{-10}{-2} = 5$$
  

$$N(5) = 25$$

10. Celine and Lia are standing on a horizontal track at points A and D respectively. Celine is 1.5m tall and Lia is 1.2m tall (that is  $AC = 1.5\text{m}$  and  $DL = 1.2\text{m}$ ). Celine's shadow occupies the length AB and Lia's shadow occupies the length BD on the ground so the two shadows meet at their heads. The angles between the ground and the ray from the top of the head of each individual to the touching point of the shadows is shown. How far apart are Celine and Lia standing?



- A. 1.788 m  
 B. 2.078 m  
 C. 2.7 m  
 D. 3.344 m  
 E. 3.866 m

$$\frac{1.5}{\tan 40} + 1.2 \sqrt{3} = 3.8$$

11. Consider the function  $f$  defined by  $f(x) = (m - 1)x^3 - 3m - 1$ , where  $m$  is a real number. If the graph of  $f$  has a  $y$ -intercept of 2, then  $f(1) =$

- A. -6.4  
 B. -1  
 C. 0  
 D. 1  
 E. 2.2

$(0, 2)$   
 $2 = (m-1)(0)^3 - 3m - 1$   
 $m = -1$   
 $f(1) = (-1-1)(1) - 3(-1) - 1$

12. If  $y$  varies directly with  $z$  and inversely with  $v$ , then which of the following could be a correct expression of  $z$  as a function of  $y$  and  $v$ ?

- A.  $z = 3yv$   
 B.  $z = \frac{1}{6} \frac{y}{v}$   
 C.  $z = \frac{7}{6} \frac{y}{v}$   
 D.  $z = 8 \frac{y}{v}$   
 E.  $z = \frac{1}{6}(y + v)$

$y \propto \frac{z}{v}$   
 $z \propto yv$

13. If  $(\sqrt{a} + \sqrt{b})^2 = 4.6$  and  $a + b = 13.46$ , then  $ab =$

- A. 1.21  
 B. 3.85  
 C. 7.7  
 D. 12.25  
 E. 14.8225

$a + 2\sqrt{ab} + b = 2.16$   
 $2\sqrt{ab} + 13.46 = 2.16$   
 $\sqrt{ab} = \frac{77}{20}$   
 $ab = 14.8225$

14. In a certain country, the ratio of male individuals who got infected by the COVID-19 virus to the female individuals who got infected by the same virus is 6 to 4. Moreover, it was observed that only 9% of the males and 5% of the females who get infected by the virus got severe symptoms. If the number of COVID-19 cases in the country is 1200, how many females have severe symptoms?

- A. 24  
B. 162  
C. 600  
D. 1200  
E. 1800

Male : female : total  
6 : 4 : 10  
20 : 120 : 1200  
480 ← 20  
 $480 \times \frac{5}{100} = 24$

15. Which of the following is equal to  $\frac{5^{202} + 5^{200}}{25^{100}}$ ?

- A. 25  
B. 26  
C.  $2^{202}$   
D.  $5^{202}$   
E.  $10^{202}$

$\frac{5^{202} + 5^{200}}{5^{200}} = \frac{5^{202}}{5^{200}} + \frac{5^{200}}{5^{200}}$   
 $5^2 + 1 = 26$

16. Each day, Jack's donut shop starts with 100 donuts. Throughout the day, some donuts are sold, some are discarded, and some more is made according to the following principle: for every three donuts sold, one more is made. If on a particular day, Jack sold  $m$  donuts and discarded  $d$  donuts, and at the end of the day had no donuts left, which of the following gives the correct relation between  $m$  and  $d$ ?

- A.  $2m + 3d = 300$   
B.  $3m + 2d = 300$   
C.  $3d - 2m = 300$   
D.  $2d - 3m = 100$   
E.  $m + d = 100$

Initial : sold : discarded : Made  
100 :  $-m$  :  $-d$  :  $\frac{1}{3}m$   
 $100 - m - d + \frac{1}{3}m = 0$   
 $3 \times (100 - \frac{2}{3}m - d) \Rightarrow 2m + 3d = 300$

17. Consider the set  $\{-3, 0, 7, a, b, 20\}$  whose elements are arranged in increasing order, and  $a$  and  $b$  are real numbers. If the median of this set above is 12, which of the following is a possible value of  $b$ ?

- I. 8  
II. 17  
III. 20

- A. Only I  
B. Only II  
C. Only III  
D. I and II  
E. II and III

$-3, 0, 7, 12, \boxed{\phantom{00}}, 20$

18. Let  $f$  be the function defined by  $f(x) = x^4 - 3x^2 - k$  where  $k$  is a real number. For which values of  $k$  does the line  $d$  of equation  $y = 1$  cut the graph of  $f$  in 4 distinct points?

- A.  $-3.7$   
 B.  $-1.9$   
 C.  $-1$   
 D.  $0$   
 E.  $1.2$

$$x^2(x^2 - 3)$$

$\downarrow$                        $\downarrow$   
 $x=0$                        $x = \pm\sqrt{3}$

19. The product of three consecutive integers is equal to the middle integer. What is the smallest of these integers?

- A.  $-1 - \sqrt{2}$   
 B.  $1 + \sqrt{2}$   
 C.  $-1$   
 D.  $0$   
 E.  $1$

$$(-1)(0)(1) = 0$$

20. A line in the plane passes through the 3 distinct points  $(a, b)$ ,  $(c, d)$ , and  $(e, f)$ . Then all of the following give a correct relation between  $a, b, c, d, e$ , and  $f$  except:

- A.  $\frac{c-a}{d-b} = \frac{c-e}{d-f}$  ✓  
 B.  $\frac{d-b}{d-f} \times \frac{c-e}{a-c} = 1$  ✗  
 C.  $\frac{d-b}{d-f} \times \frac{c-e}{c-a} = 1$  ✓  
 D.  $\frac{c-a}{d-b} + \frac{c-e}{f-d} = 0$  ✓  
 E.  $\frac{d-b}{d-f} - \frac{c-a}{c-e} = 0$  ✓

$$\frac{d-b}{c-a} = \frac{f-d}{e-c}$$

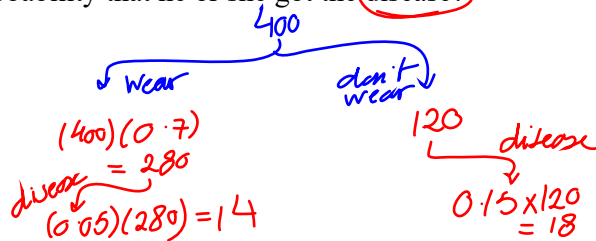
21. Consider the parabola of equation  $y = x^2 + 22x - 320$ . What are the coordinates of the midpoint of the segment joining the two  $x$ -intercepts of the parabola?

- A.  $(-11, 0)$   
 B.  $(0, -11)$   
 C.  $(11, 0)$   
 D.  $(0, 11)$   
 E.  $(0, -160)$

$$\frac{-b}{2a} = \frac{-22}{2(1)} = -11$$

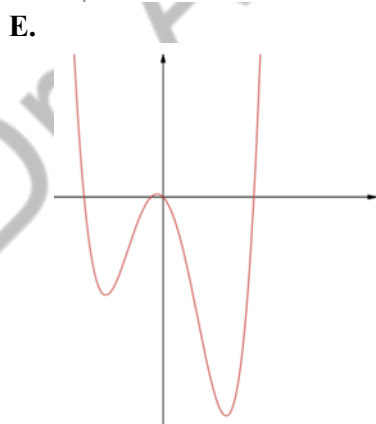
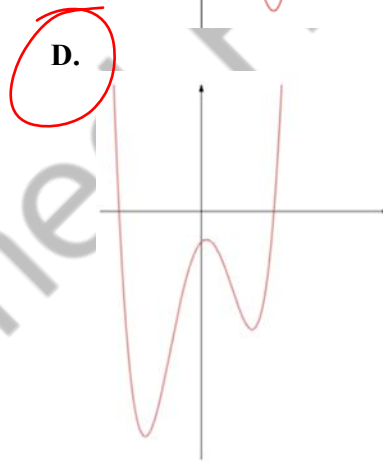
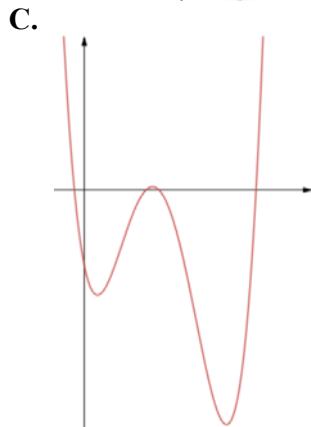
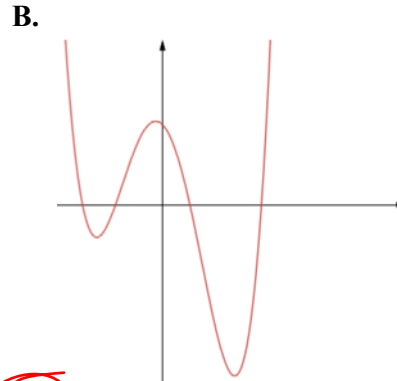
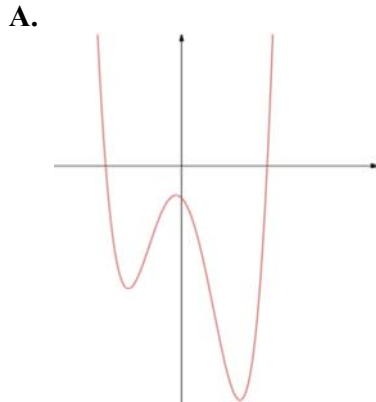
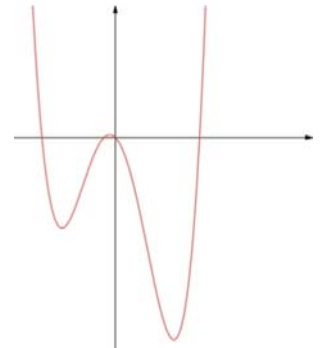
22. In a certain company of 400 employees, 70% wear face masks at all times. Out of those who do not wear face masks, 15% got the disease. Out of those who do wear a mask, only 5% got the disease. If an employee is chosen at random from this company, what is the probability that he or she got the disease?

- A.  $0.35$   
 B.  $0.08$   
 C.  $0.09$   
 D.  $0.12$   
 E.  $0.35$



$$\frac{18 + 14}{400} = 0.08$$

23. The adjacent graph is that of a function  $f(x)$ . The graph of the function  $g$  is obtained by translating the graph of  $f$ , that is  $g(x) = f(x + a) + b$  for some real numbers  $a$  and  $b$ . Which of the graphs below cannot be that of  $g(x)$  for any real values of  $a$  and  $b$ ?





24. If  $f$  is the function defined by  $f(x) = (2x + 1)^3$ , for which of the following values of  $x$  is  $f(f(x)) = f(x)$ ?

A. -2  
**B. -1**  
 C. 0  
 D. 1  
 E. 2

$$\left[ 2(2x+1)^3 + 1 \right]^3 = \left[ 2x+1 \right]^3$$

$x = -1$

25. The range of the function defined by  $f(x) = \sqrt{1 - x^2} + 2$  is:

A.  $f(x) > 2$   
 B.  $f(x) > 3$   
 C.  $f(x) < 2$   
**D.  $f(x) \geq 2$**   
 E.  $f(x) \leq 2$

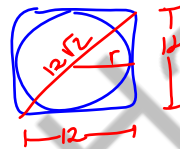
$$y \geq 2$$

26. A circle inscribed in a square has an area of  $36\pi$ . What is the length of the diagonal of the square?

A. 8.485  
 B. 12  
**C. 16.97**  
 D. 36  
 E. 50.912

$$A = \pi r^2 = 36\pi$$

$$r = 6$$



27. If  $f(x) = x^2 - 4x + 1$ , which of the following is also equal to  $f(x)$ ?

A.  $f(x - 4)$   
 B.  $f(2 - x)$   
**C.  $f(4 - x)$**   
 D.  $f(4 + x)$   
 E.  $f(4) - f(x)$

$$f(0) = 1$$

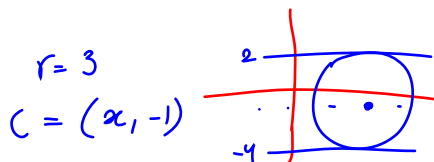
$$f(4) = 16 - 4(4) + 1 = 1$$

$$f(1) = 1 - 4 + 1 = -2$$

$$f(4-1) = f(3) = 9 - 12 + 1 = -2$$

28.  $C$  is a circle tangent to the two straight lines of equations  $y = 2$  and  $y = -4$ . Which of the following could be the equation of the circle?

A.  $(x + 1)^2 + y^2 = 9$   
 B.  $(x - 1)^2 + (y + 1)^2 = 3$   
 C.  $(x + 4)^2 + (y + 2)^2 = 9$   
 D.  $(x + 1)^2 + y^2 = 3$   
**E.  $(x + 2)^2 + (y + 1)^2 = 9$**



29. Consider all spheres of nonzero volume having as their center the point  $A$  in space. How many of these spheres can have a surface area whose value is numerically equal to the volume of the sphere?

A. None  
**B. 1**  
 C. 2  
 D. 3  
 E. Further information is required to determine the answer

$$V = \frac{4}{3}\pi r^3 = 4\pi r^2$$

$$\frac{r}{3} = 1$$

30. The two lines  $m$  and  $n$  of respective equations  $3x - y = 1$  and  $ax + by = 2$  are parallel, where  $a < 0$  and  $b$  is a real number. Which of the following may be true about  $b$ ?

I.  $b = 2.31$

II.  $b = 1.04$

III.  $b = 0$

A. Only I

**B. Only II**

C. I and III

D. II and III

E. I, II, and III

$a$  (-ve)  
 slope =  $-\frac{-(-a)}{b}$   
 must be (+ve)

31. In the adjacent figure, ABCD is a rectangle such that  $BC = 4$  and  $AB = 3$ , AEC is a triangle such that the two angles  $\angle AEC$  and  $\angle ACE$  are equal. What is the perimeter of triangle AEC?

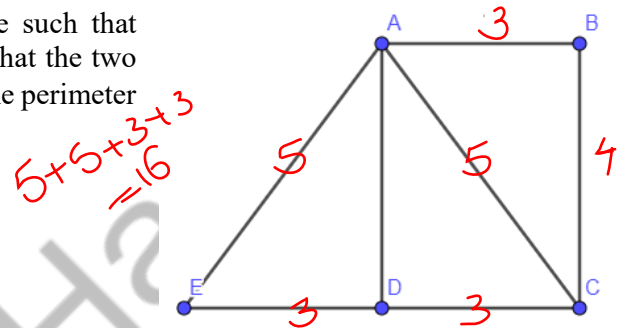
A. 10

B. 12

C. 14

**D. 16**

E. 18



32. A, C, E, and D are four collinear points in this order such that  $AC = a$ ,  $ED = b$ , and  $AE + CD = 12$ . Which of the following is a correct expression of  $CE$  in terms of  $a$  and  $b$ .

**A.  $CE = 6 - \frac{a+b}{2}$**

B.  $CE = \frac{a+b}{2}$

C.  $CE = 12 - (a + b)$

D.  $CE = 6 - (a + b)$

E.  $CE = 12 - \frac{a+b}{2}$

$AE + CD = 12$   
 $a + x + x + b = 12$   
 $2x + a + b = 12$   
 $2x = 12 - (a + b)$

$x = 6 - \frac{(a+b)}{2}$

33. The triangle shown in the adjacent figure (not drawn to scale) is right angled at G with  $GF = 4$  and  $\angle GFH = 60^\circ$ . I is the midpoint of segment FH and GJ is the altitude drawn from G to FH. What is the area of triangle GJI?

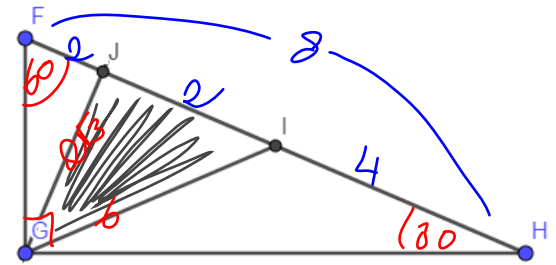
**A. 3.46**

B. 4.62

C. 6

D. 6.93

E. 13.86



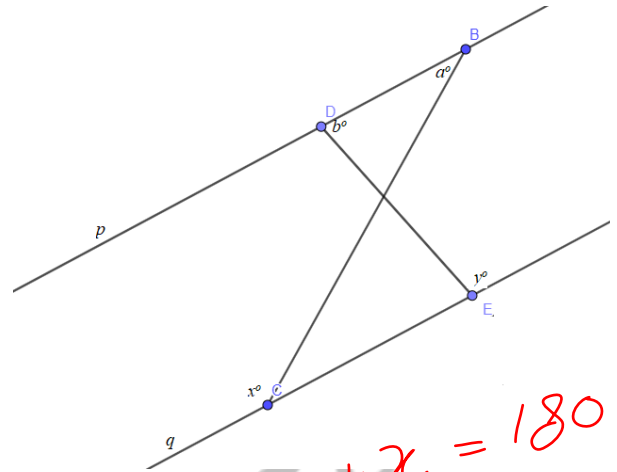
$\frac{1}{2} (2)(2\sqrt{3}) = 3.46$

$2x = 4\sqrt{3}$   
 $x = 2\sqrt{3}$

34. In the adjacent figure,  $p$  and  $q$  are two parallel lines. B and D are points on  $p$ ; C and E are points on  $q$ . Which of the following statements is true?

- I.  $a^\circ + b^\circ = x^\circ + y^\circ$
- II.  $a^\circ, b^\circ, x^\circ,$  and  $y^\circ$  can be the values of the four angles of a quadrilateral.
- III.  $a^\circ + x^\circ = b^\circ + y^\circ$

- A. Only I
- B. Only II
- C. Only III
- D. I and II
- E. II and III



$$a + x = 180$$

$$b + y = 180$$

Dr. Ahmed Hassam

35. If  $\left(3k - \frac{k}{k+1} = \frac{16}{k+1}\right)^{x(k+1)}$  what is a possible value of  $k + 1$ ?

- A.  $-\frac{8}{3}$
- B.  $-\frac{5}{3}$**
- C. 2
- D. 5
- E. 6

$$3k(k+1) - k = 16$$

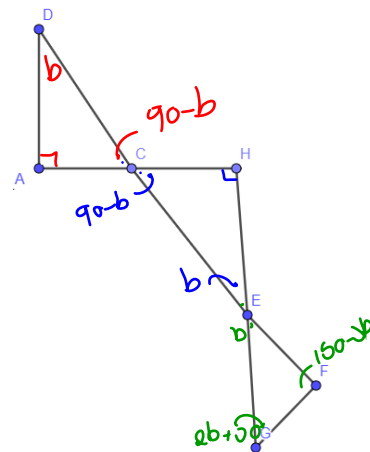
$$3k^2 + 3k - k = 16$$

$$3k^2 + 2k - 16$$

$$k = 2, \frac{8}{3}$$

36. In the adjacent figure (not drawn to scale), segments AD and AH are perpendicular. The following degree measures are known in terms of  $b$ :  $\angle ADC = b$ ,  $\angle EGF = 2b + 30$ , and  $\angle EFG = 150 - 3b$ . Which of the following conclusions must be true?

- A. The three triangles ADC, CHE, and EFG are right angled triangles.
- B. Triangle HCE is right angled at H.**
- C. Triangles ADC and EFG are similar.
- D. Triangles ADC and HCD are similar.
- E. Only triangle ADC is right angled.



37. In the adjacent figure, lines AC and DE are parallel. The lengths of segments CE, BE, BD, and AD are given in terms of  $s$  and  $z$ . Which of the following is a correct expression of  $s$  in terms of  $z$ ?

- A.  $s = \frac{2z}{z-3}$**
- B.  $s = 2 - \frac{3}{2z}$
- C.  $s = \frac{z+3}{3-2z}$
- D.  $s = \frac{3(z+1)}{z+3}$
- E.  $s = \frac{z+3}{z} + 1$

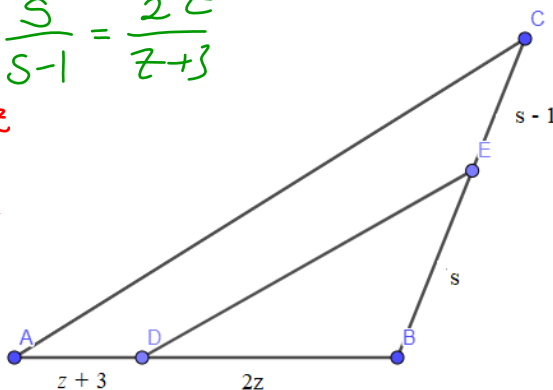
$$\frac{s}{s-1} = \frac{2z}{z+3}$$

$$sz + 3s = 2sz - 2z$$

$$sz + 3s - 2sz = -2z$$

$$s[z+3-2z] = -2z$$

$$s = \frac{2z}{z-3}$$



38. The perimeter of a regular hexagon is 24 and the perimeter of a regular octagon is 5. What is the ratio of the length of one side of the hexagon to the length of one side of the octagon?

- A. 0.15625
- B. 0.2083
- C. 1.2
- D. 2.75
- E. 6.4**

$$\text{Hexagon side} = \frac{24}{6} = 4$$

$$\text{Octagon side} = \frac{5}{8}$$

$$\frac{4}{\frac{5}{8}} = 6.4$$

39. The equation  $\frac{1+|x-1|}{|x-1|} = 5$  has two solutions. What is the sum of these two solutions?

- A. 0.75  
 B. 1.25  
 C. 2  
 D. 2.5  
 E. 3.25
- $\frac{1}{|x-1|} + \frac{|x-1|}{|x-1|} = 5$   
 $x = \frac{1}{4} + 1 = \frac{5}{4}$   
 $x = \frac{-1}{4} + 1 = \frac{3}{4}$   
 $\frac{1}{|x-1|} = 4$   
 $|x-1| = \frac{1}{4}$

40. Let  $f$  be the function defined by  $f(x) = mx^3 + px^2 + qx - 1$ , where  $m$ ,  $p$ , and  $q$  are real numbers. If the graph of  $f$  passes through the point  $(1, 7)$ , what is the average of  $m$ ,  $p$ , and  $q$ ?

- A.  $\frac{7}{3}$   
 B. 2  
 C.  $\frac{8}{3}$   
 D. 3.5  
 E. 4

$f(1) = m + p + q - 1 = 7$   
 $m + p + q = 8$   
 Mean =  $\frac{8}{3}$

41. If  $a(b - 1) = 1$  and  $12ab = 4$ , what is the value of  $b$ ?

- A. -0.667  
 B. -0.5  
 C. 0.5  
 D. 0.667  
 E. 2
- $(ab - a = 1) \times 12$   
 $12ab - 12a = 12$   
 $4 - 12a = 12$   
 $a = \frac{-2}{3}$   
 $12(\frac{-2}{3})b = 4$   
 $b = \frac{1}{2}$

42. If  $m$  and  $n$  are the roots of the equation  $ax^2 + bx + c = 0$ , where  $a$ ,  $b$ , and  $c$  are real numbers, which of the following quantities does not change when we vary  $a$ ?

- I.  $m \cdot n$   
 II.  $m + n$   
 III.  $\frac{1}{m} + \frac{1}{n}$   
 A. Only I  
 B. Only II  
 C. Only III  
 D. I and II  
 E. I, II, and III

Sum	Product
$-\frac{b}{a}$	$\frac{c}{a}$
↑	↑

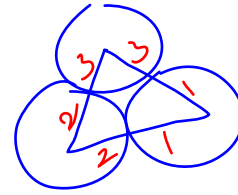
43. If  $f(x) = 1.6^x$  and  $g(x) = 1.4^x$  and  $f(g(0)) = A \times g(f(0))$ , what is the value of  $A$ ?

- A.  $\frac{7}{8}$   
 B. 1  
 C.  $\frac{8}{7}$   
 D.  $\frac{25}{56}$   
 E.  $\frac{56}{25}$

$g(0) = 1$   
 $f(1) = 1.6$   
 $f(0) = 1$   
 $g(1) = 1.4$   
 $1.6 = A(1.4)$   
 $A = \frac{1.6}{1.4}$

44. Circles C, P, and Q have radii 1, 2, and 3 respectively. If the three circles are tangent to each other, what is the nature of the triangle formed by joining the three centers of the circles?

- A. Isosceles
- B. Right angled
- C. Right isosceles
- D. Equilateral
- E. Cannot be determined with the information given



45.

Days d	1	2	3	4	5	6	7	8
Cases	100	350	250	600	723	702	750	790
N								

The table above shows the number of cases of individuals infected with the COVID-19 virus in a certain country every day after Sunday October 18 (so  $d = 1$  is Monday October 19, and so on). The government uses a linear regression model and decides to impose total lockdown when the number of cases reaches 1000. Using the linear model, what is the predicted date at which the lockdown will be imposed?

- A. October 20
- B. October 23
- C. October 29
- D. October 31
- E. November 1

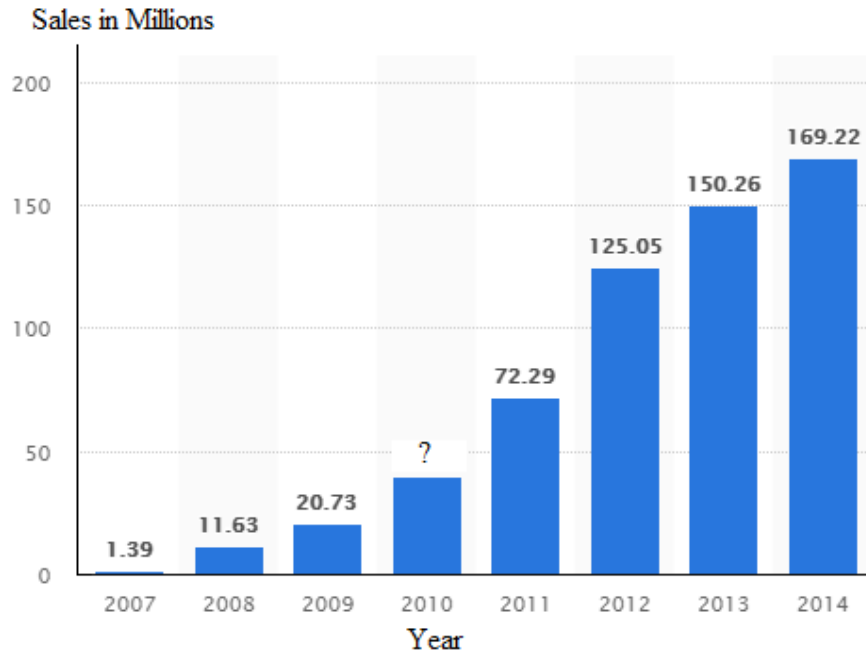
$$y = 88 + 98.9x$$

$y = 1000$

$$1000 = 88 + 98.9167x$$

$$x = 9.2 \Rightarrow 10 \text{ days}$$

46.



The bar graph given above shows the number of smart phones sold in millions from 2007 when it was first introduced into the market till 2014 except for the year 2010. Knowing that the percent increase from the year 2010 to 2011 for the number of smart phones sold is four times the percent increase from the year 2013 to 2014, what is the approximate number of smart phones sold, in millions, in the year 2010?

- A. 14.458
- B. 32.35
- C. 40.161
- D. 25.73
- E. 71.52

$$\frac{72.29 - x}{x} = 4 \left( \frac{169.22 - 150.26}{150.26} \right)$$

$$x = 40.161$$

47. For how many strictly positive integer values of  $x$  is the following inequality satisfied?

$$2(3 - x) \geq -14$$

- A. 0
- B. 1
- C. 9
- D. 10
- E. 11

$$3 - x \geq -7$$

$$10 \geq x \rightarrow x \leq 10$$

48. If  $f(x) = x^2 + 1$  and  $d(x) = \frac{f(x) - f(2)}{x - 2}$ , then which of the following is true?

- A.  $d(x) = f(x)$  for all values of  $x$
- B.  $d(x) = x - 2$  for all values of  $x$
- C.  $d(x) = x + 1$  for all values of  $x$  except 2
- D.  $d(2) = f(2)$
- E.  $d(x) = x + 2$  for all values of  $x$  except 2

$$d(x) = \frac{x^2 + 1 - (2^2 + 1)}{x - 2}$$

$$= \frac{x^2 + 1 - 5}{x - 2}$$

$$= \frac{x^2 - 4}{x - 2} = \frac{(x/2)(x+2)}{x-2} = x+2$$

49.  $\begin{cases} 12x - 15y = 17 \\ x + 8y = 12 \end{cases}$  *Mod 5, ↑*

What is the value of  $46y - 22x$ ?

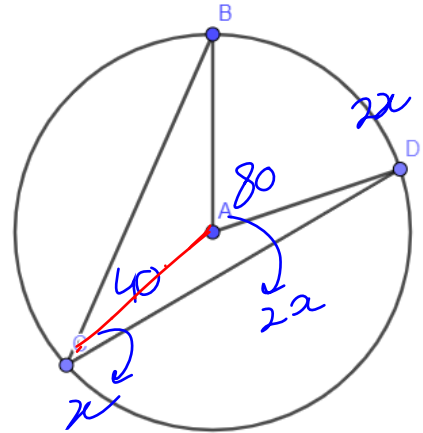
- A. -10
- B. -5
- C. 0
- D. 5
- E. 10

$$46 \left( \frac{127}{111} \right) - 22 \left( \frac{316}{111} \right) = -10$$

50. In the adjacent figure, the measure of angle  $\angle BAD$  is  $80^\circ$ . Which of the following could be the measure of angle  $\angle BCA$ ?

- A.  $38.96^\circ$
- B.  $40^\circ$
- C.  $45.02^\circ$
- D.  $60.13^\circ$
- E.  $90^\circ$

*less than 40*



Dr. Ahmed H



THE FORMULAS BELOW MAY BE USEFUL IN ANSWERING QUESTIONS ON THIS TEST.

$S = 4\pi r^2$  is the formula for the surface area of a sphere with a radius of  $r$ .

$V = \frac{1}{3}\pi r^2 h$  is the formula for a right circular cone with a radius of  $r$  and a height of  $h$ .

$V = \frac{4}{3}\pi r^3$  is the formula for a sphere with a radius of  $r$ .

$V = \frac{1}{3}Bh$  is the formula for a pyramid with a base area of  $B$  and a height of  $h$ .

Dr. Ahmed Hassan

# بسم الله الرحمن الرحيم وبه نستعين دائما

يسعدني تلقي طلباتك و استفساراتكم على هذه اللينكات



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**DR. AHMED HASSAN**

**EST II – Individual Subject Test**

**Level 1**

**Student's Name** \_\_\_\_\_

**National ID** \_\_\_\_\_

**Test Center:** \_\_\_\_\_

**Subject:** Math

**Duration:** 60 minutes

50 Multiple Choice Questions

**Instructions:**

- Place your answer on the answer sheet. Mark only one answer for each of the multiple choice questions.
- Avoid guessing. Your answers should reflect your overall understanding of the subject matter.
- Calculator is allowed. When a calculator is used, be aware of switching between radian mode and median mode.
- Formula sheet is available at the end of the booklet for your reference.

THE FORMULAS BELOW MAY BE USEFUL IN ANSWERING QUESTIONS ON THIS TEST.

$S = 4\pi r^2$  is the formula for the surface area of a sphere with a radius of  $r$ .

$V = \frac{1}{3}\pi r^2 h$  is the formula for a right circular cone with a radius of  $r$  and a height of  $h$ .

$V = \frac{4}{3}\pi r^3$  is the formula for a sphere with a radius of  $r$ .

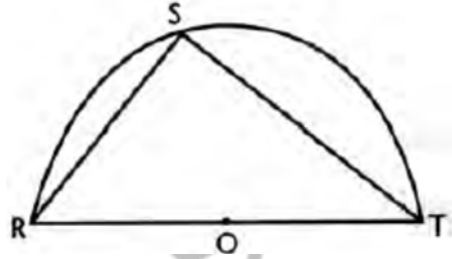
$V = \frac{1}{3}Bh$  is the formula for a pyramid with a base area of  $B$  and a height of  $h$ .

Dr. Ahmed Hassan

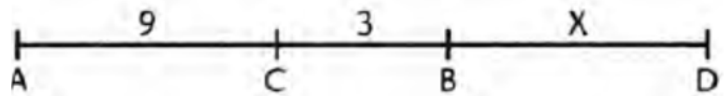
1. The remainder when  $t^{16} + 5$  is divided by  $t + 1$  is
- A. 6
  - B. 0
  - C.  $-1$
  - D. 4
  - E. 1
2. If two roots of the equation  $y^3 + a y^2 + b y + c = 0$  (with  $a$ ,  $b$ , and  $c$  integers) are  $1$  and  $2 - 8i$ , then the value of  $a$  is
- A.  $2 + 8i$
  - B.  $-5$
  - C.  $5$
  - D.  $4 + 8i$
  - E.  $4 - 8i$
3. The root(s) of the equation  $\sqrt{a + 6} = -a$  is / are:
- A.  $3$
  - B.  $-2$
  - C.  $3$  and  $-2$
  - D.  $3$  or  $-2$
  - E. No Roots
4. What are all  $p$  such that  $\frac{p+100}{p} \leq 1$ ?
- A.  $p > 0$
  - B.  $p < 0$
  - C.  $p \leq 0$
  - D.  $-1 < p < 0$
  - E.  $-1 \leq p < 0$
5. The graph of  $t^2 - \sqrt{5} t - 2$  has its minimum value at which the approximate value of  $t$
- A.  $83$
  - B.  $1.12$
  - C.  $1.21$
  - D.  $1.35$
  - E.  $2.47$

6. The diagonals of a parallelogram divide the figure into four triangles that are
- congruent
  - similar
  - equal in area
  - isosceles
  - equal in perimeter

7. In the adjacent figure,  $\overline{RT}$  is a diameter of the semicircle. If  $RS = 2$  and  $ST = 3$ , then the area of the semicircle is



- $\frac{13\pi}{2}$
  - $\frac{13\pi}{4}$
  - $\frac{\pi}{6}$
  - $\frac{13\pi}{8}$
  - $\frac{\pi}{4}$
8. In the adjacent figure,  $AC = 9$ ,  $D$  is three times as far from  $A$  as from  $B$  and  $BC = 3$ . What is  $BD$ ?



- 9
  - 12
  - 18
  - 6
  - 15
9. The graph of the equation  $y = 50 \cos 3x$  has a period, in radians, of
- $3\pi$
  - $\frac{2}{3}\pi$
  - $\frac{2}{5}\pi$
  - $50\pi$
  - $\frac{50}{3}\pi$

10. A right circular cone is cut into two equal parts. The lateral surface area of one of the parts is equal to thrice the lateral surface area of a hemisphere with a radius equal to 8 cm. What is the area of the right circular cone?
- $768\pi \text{ cm}^2$
  - $512\pi \text{ cm}^2$
  - $384\pi \text{ cm}^2$
  - $192\pi \text{ cm}^2$
  - $128\pi \text{ cm}^2$
11. If  $a \neq b$ , then  $x = a \sin t$  and  $y = b \cos t$  represent the parametric equations of a(n):
- Circle
  - Ellipse
  - Straight line
  - Parabola
  - Hyperbola
12.  $2 \cos^3 B \sin B + 2 \sin^3 B \cos B$  is equal to
- $2 \sin B$
  - $\cos^2 B$
  - $\cos 2B$
  - $\sin 2B$
  - $2 \cos B$
13. The irrational number is
- $\frac{3\sqrt{18}}{2\sqrt{6}}$
  - $\sqrt[3]{-27}$
  - $\sqrt{2}(3\sqrt{2} + 2\sqrt{8})$
  - $\frac{2\sqrt{5}}{\sqrt{45}}$
  - $\sqrt{\frac{1}{2}} \cdot \sqrt{\frac{25}{2}}$
14. A car moved a distance of  $x$  meters to a distance of  $y$  meters. The percent of increase was
- $\frac{100(y-x)}{y}$
  - $\frac{100(x-y)}{x}$
  - $\frac{y-x}{x}$
  - $\frac{x-y}{x}$
  - $\frac{100(y-x)}{x}$

15. The solution of  $3x^2 - xy = 3$  and  $6x - y = 10$  is

- A.  $\frac{1}{3}$  and 3
- B. 8 and -8
- C.  $\frac{1}{3}$  and -8
- D. 3 and 8
- E.  $\frac{1}{3}$  and -8; 3 and 8

16. If graphed on the same set of axes and  $0 \leq x \leq 2\pi$ , then  $y = \sin x$  and  $y = \cos x$  will intersect in quadrant(s)

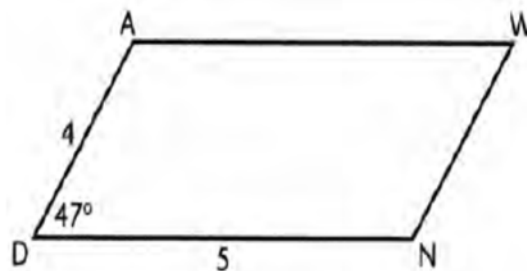
- A. I only
- B. III only
- C. I and III
- D. II only
- E. I and II

17. If A represents the set of rectangles and B the set of rhombi, then  $A \cap B$  represents the set of

- A. Trapezoids
- B. Parallelograms
- C. Squares
- D. Rectangles
- E. Quadrilaterals

18. The approximate area of the parallelogram DAWN, in the adjacent figure, is

- A. 20
- B. 17.25
- C. 14.63
- D. 11.5
- E. 13.64



19. If the line of equation  $5x + 12y - 60 = 0$  is tangent to the circle of equation  $(x - 1)^2 + (y - 3)^2 = r^2$ , then the value of  $r$  is

- A.  $2\sqrt{3}$
- B.  $\sqrt{3}$
- C.  $\sqrt{10}$
- D.  $\frac{19}{13}$
- E.  $\frac{13}{12}$



20. If  $[\sqrt{2} (\cos 30^\circ + i \sin 30^\circ)]^2 = a + bi$ , then  $a = \dots$  and  $b = \dots$
- A.  $a = 2, b = \sqrt{3}$
  - B.  $a = \frac{3}{2}, b = \frac{1}{2}$
  - C.  $a = 1, b = -\sqrt{3}$
  - D.  $a = \frac{3}{2}, b = -\frac{1}{2}$
  - E.  $a = 1, b = \sqrt{3}$
21. The probability of getting 80 % or more of the questions correct on a 10- question true-false exam merely by guessing is
- A.  $\frac{5}{32}$
  - B.  $\frac{1}{16}$
  - C.  $\frac{3}{16}$
  - D.  $\frac{7}{128}$
  - E.  $\frac{7}{32}$
22. The ratio of the diagonal of a cube to the diagonal of a face of this cube is
- A.  $3: \sqrt{6}$
  - B.  $2: \sqrt{6}$
  - C.  $3: \sqrt{2}$
  - D.  $6: \sqrt{2}$
  - E.  $2: \sqrt{3}$
23. The intersection of the three lines of equations  $x = 3y$ ,  $y = 0$  and  $3x + y = 7$  form a(n) ----- triangle
- A. Acute
  - B. Obtuse
  - C. Right
  - D. Isosceles
  - E. Equilateral
24. The distance AB between the points A(-3, 7) and B(6, -5) in a coordinate system in which the y-axis is inclined  $60^\circ$  to the positive x- axis is
- A.  $\sqrt{117}$
  - B.  $\sqrt{120}$
  - C.  $\sqrt{189}$
  - D.  $\sqrt{333}$
  - E.  $\sqrt{108}$

25. The municipality in a certain hospital administered a survey on diabetes. Blood tests for 150 patients gave the following results in millimoles per liter:

Level of glucose in $mmol/L$	$[6 - 8[$	$[8 - 10[$	$[10 - 12[$	$[12 - 14[$	$[14 - 16[$
Frequency	50	10	20	30	40

The Median is

- A. 11  
 B. 10  
 C. 11.5  
 D. 10.5  
 E. 12
26. A bag  $B_1$  contains **two red balls numbered 0** and **three blue balls numbered 1, 2, 3**  
 A bag  $B_2$  contains **three red balls numbered 2, 3, 4** and **one blue ball numbered 0**.  
 One ball is selected from  $B_1$ , then another ball is randomly selected from  $B_2$ . Thus a 2-digit number is formed. The number of possible outcomes is
- A. 25  
 B. 12  
 C. 20  
 D. 16  
 E. 144
27. The following table gives information about a population of smokers and of those having lung diseases due to smoking.

	Men		Men and Women	
	Smokers	Sick	Smokers	Sick
20 - 40 years	2500	30	4500	50
40 - 60 years	3000	50		75
60 - 80 years	2000		4000	
Total		160	13500	265

An individual is selected at random from this population. The probability that this individual is sick knowing that he/she is between 60 and 80 years old is

- A.  $\frac{265}{13500}$   
 B.  $\frac{140}{4000}$   
 C.  $\frac{80}{2000}$   
 D.  $\frac{140}{13500}$   
 E.  $\frac{265}{4000}$

28. The solution of the system  $\begin{cases} 3 - x^2 \leq 0 \\ \frac{x - 11}{x - 4} \geq 2 \end{cases}$  is

- A.  $x \in [-3, -\sqrt{3}] \cup [\sqrt{3}, 4[$
- B.  $x \in ]-3, -\sqrt{3}] \cup [\sqrt{3}, 4[$
- C.  $x \in ]-3, -\sqrt{3}[ \cup [\sqrt{3}, 4[$
- D.  $x \in [-3, -\sqrt{3}] \cup [\sqrt{3}, 4]$
- E.  $x \in [-3, -\sqrt{3}] \cup ]\sqrt{3}, 4[$

29. The real numbers  $m$ ,  $n$  and  $p$  such that  $P(x) = (x^2 - 2)(mx^2 + nx + p)$  will be identical to  $x^4 - x^3 - 3x^2 + 2x + 2$  are:

- A.  $m = 1, n = -1, p = -1$
- B.  $m = -1, n = -1, p = -1$
- C.  $m = 1, n = 1, p = 1$
- D.  $m = 1, n = 1, p = -1$
- E.  $m = -1, n = -1, p = 1$

30. A fair die is tossed and a card is randomly chosen from a deck of 52 cards. The probability that “a multiple of three appears and a king or an ace is selected” is:

- A.  $\frac{8}{312}$
- B.  $\frac{16}{312}$
- C.  $\frac{8}{52}$
- D.  $\frac{16}{52}$
- E.  $\frac{16}{36}$

31. In a given school, the class of grade 12 is divided into three sections: Sociology and economics (SE), Life Sciences (LS) and General Sciences (GS). The students of this class are distributed as follows:

Sections/Students	Boys	Girls	Total
SE	17	21	38
LS	13	14	27
GS	14	5	19
<b>Total</b>	44	40	84

The number of ways that three students can be selected from this class if exactly two of them are girls and if the order is respected is

- A. 205,920
- B. 68,640
- C. 411,840
- D. 70,400
- E. 211,200

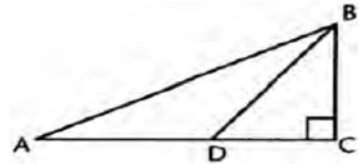
32. A mining company extracts oil from an oilfield since the year 1963. The following table shows the quantity  $y_i$ , in tons, extracted during each year indicated by its rank  $x_i$ .

Year	1963	1968	1973	1978	1983	1988	1993	1998	2003	2008
Rank $x_i$	0	1	2	3	4	5	6	7	8	9
Quantity $y_i$ (in tons)	18.1	15.7	13.3	11	9.3	7.8	7.1	6.1	5.2	4.3

The means  $\bar{x}$  and  $\bar{y}$  of the variables  $x$  and  $y$  respectively are

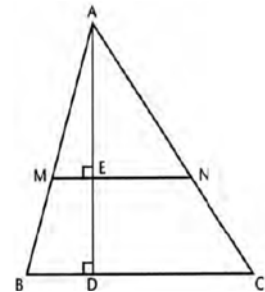
- A.  $\bar{x} = 4.5$   $\bar{y} = 9.79$   
 B.  $\bar{x} = 4$   $\bar{y} = 9.79$   
 C.  $\bar{x} = 4.5$   $\bar{y} = 9$   
 D.  $\bar{x} = 5$   $\bar{y} = 9.79$   
 E.  $\bar{x} = 4.5$   $\bar{y} = 7.9$
33. The length of  $\overline{AB}$  in the adjacent right triangle at C such that  $AD = DB$ ,  $DC = BC = 1$  is

- A.  $\sqrt{10}$   
 B.  $2\sqrt{2}$   
 C.  $2\sqrt{10}$   
 D.  $\sqrt{4 + 2\sqrt{2}}$   
 E.  $4 + 2\sqrt{2}$



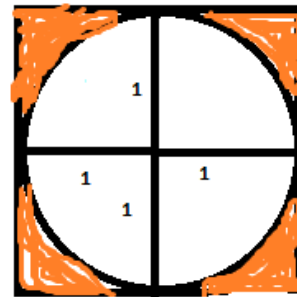
34. In the adjacent  $\triangle ABC$ ,  $AD = 10$ ,  $\overline{MN}$  is parallel to  $\overline{BC}$  and  $\overline{MN}$  bisects the area of  $\triangle ABC$ . The value of  $ED$

- A.  $10 - 5\sqrt{2}$   
 B.  $10 + 5\sqrt{2}$   
 C.  $5\sqrt{10} + 2\sqrt{2}$   
 D.  $5\sqrt{2}$   
 E.  $2\sqrt{10}$



35. A circle is inscribed in the square as shown in the adjacent figure. The area of the shaded region is:

- A.  $4 - \pi$   
 B.  $4 + \pi$   
 C.  $1 - \pi$   
 D.  $1 + \pi$   
 E.  $2 - \pi$



36. The total price of one kilogram of sugar and one kilogram of salt is 20 EGP. During an economic crisis, the price of sugar increased by 12 % and that of salt decreased by 15% such that the total price of 1 Kg of sugar and 1 Kg of salt became 22 EGP.

Reem bought 5 Kg of sugar and 2 Kg of salt during the crisis. How much did she pay, rounded to the nearest integer?

- A. 106 EGP
- B. 100 EGP
- C. 120 EGP
- D. 96 EGP
- E. 48 EGP

37. Consider the polynomial

$$E(x) = (2x - 1)^2 + (x - 2)(1 - 2x)$$

$$E(1 + \sqrt{2}) = c + d\sqrt{2}, \text{ where } c \text{ and } d \text{ are two integers such that}$$

- A.  $c = d = 5$
- B.  $c = 6, d = -5$
- C.  $c = -6, d = -5$
- D.  $c = -6, d = 5$
- E.  $c = 6, d = 5$

38. In the  $xy$ -plane, consider a straight line ( $D$ ) passing through  $A(-\sqrt{3}; -3 - 2\sqrt{3})$  and making an acute angle  $\alpha$  with the  $x$  - axis such that  $\cos \alpha = \frac{\sqrt{3}}{3} \times \sin \alpha$ .

The value of  $\sin \alpha$  and  $\cos \alpha$  are respectively

- A.  $-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$
- B.  $\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$
- C.  $\frac{\sqrt{3}}{2}, \frac{1}{2}$
- D.  $\frac{-\sqrt{3}}{2}, \frac{1}{2}$
- E.  $\frac{\sqrt{3}}{2}, \frac{-1}{2}$

39. Consider the three expressions:

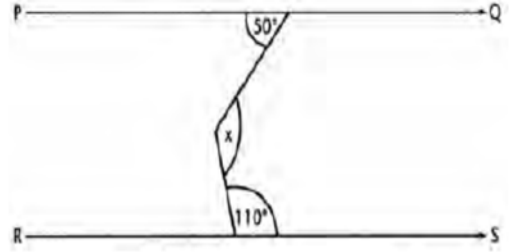
$$A = (x - 1)(x - 2), B = (x - 2)(x - 3) \text{ and } C = (x - 3)(x - 4)$$

$A - 2B + C$  is equal to

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5

40. In the adjacent figure, if the two lines (PQ) and (RS) are parallel, then x is equal to

- A.  $50^\circ$
- B.  $110^\circ$
- C.  $120^\circ$
- D.  $60^\circ$
- E.  $100^\circ$



41. The largest real range of the function  $y = 1 - \frac{1}{x}$  is

- A.  $y > 0$
- B. All Real numbers for y except  $y = 1$
- C.  $y \neq 0$
- D.  $y \geq 0$
- E.  $y \leq 0$

42. A company runs three production lines that together have an output of 54 parts/ hour. Twice the production of the second line is equal to the combined output of the other two lines, and the output of the third line is three parts per hour more than the second line.

The production rate of the first line x, the production rate of the second line y and that of the third one z are:

- A.  $x = 21, y = 18, z = 15$
- B.  $x = 18, y = 21, z = 15$
- C.  $x = 15, y = 21, z = 18$
- D.  $x = 21, y = 15, z = 18$
- E.  $x = 15, y = 18, z = 21$

43. What is the equation of the tangent to the curve of function  $f$  defined by  $f(x) = x^2 + 6x + 8$  at a point of abscissa 1 ?

- A.  $y = -3x + 18$
- B.  $y = 8x + 18$
- C.  $y = 8x + 7$
- D.  $y = -3x + 7$
- E.  $y = 8x - 7$

44. The solution of the system defined by  $\begin{cases} x^2 - x - 2 \geq 0 \\ \frac{x-3}{x+2} \leq 0 \end{cases}$  is

- A.  $] -\infty, -1] \cup [2, +\infty [$
- B.  $] -2, 3]$
- C.  $] -2, -1] \cup [2, 3 [$
- D.  $] -2, -1] \cup [2, 3]$
- E.  $[-2, -1] \cup [2, 3 [$

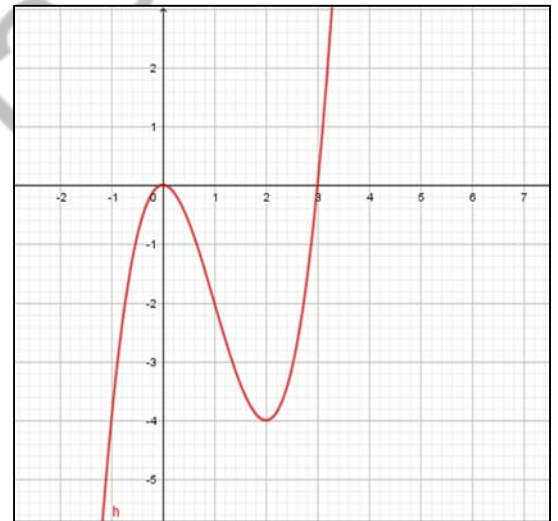
45.  $\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + 4x + 1} - 1}{x}$  is

- A. 2
- B.  $+\infty$
- C. 0
- D. 1
- E.  $-\infty$

46. Suppose that the adjacent curve ( $h$ ) is the curve representing the function  $f$  defined by  $f(x) = ax^3 + bx^2$ .

Graphically,

- A.  $a = 1$  and  $b = -3$
- B.  $a = -1$  and  $b = -3$
- C.  $a = -3$  and  $b = 1$
- D.  $a = 1$  and  $b = 3$
- E.  $a = 3$  and  $b = -1$



47. The function  $f$  defined by  $f(x) = \frac{x^2 - 3x - 3}{x - 4}$  admits the center of symmetry

- A.  $(-4, 5)$
- B.  $(4, -5)$
- C.  $(-4, -5)$
- D.  $(5, 4)$
- E.  $(4, 5)$

48. On a given day, a department store sold 40 shirts. The shirts come in three styles: Casual, sports and dress. A casual shirt costs \$20, a sports shirt costs \$40, and a dress shirt costs \$60. The store sold \$1080 worth of these shirts on that day and the number of casual shirts exceeded by 20 the combined numbers of the sports and dress shirts.

The number of shirts of each type is:

- A. 6 casual shirts, 30 sport and 4 dress shirts
  - B. 30 casual shirts, 7 sport and 3 dress shirts
  - C. 30 casual shirts, 6 sport and 4 dress shirts
  - D. 4 casual shirts, 6 sport and 30 dress shirts
  - E. 6 casual shirts, 4 sport and 30 dress shirts
49. If  $a$ ,  $b$ , and  $c$  are three positive numbers and if  $a > b$ , which of the following is not necessarily true?
- A.  $a + c > b + c$
  - B.  $a - c > b - c$
  - C.  $\frac{a}{c} > \frac{b}{c}$
  - D.  $\frac{c}{a} > \frac{c}{b}$
  - E.  $\frac{a}{b} > 1$

50. The simplest form of  $\frac{2^{y+4} - 2(2^y)}{2(2^{y+3})}$  is:

- A.  $\frac{1}{4}$
- B.  $\frac{1}{2}$
- C.  $\frac{1}{8}$
- D.  $\frac{5}{8}$
- E.  $\frac{7}{8}$



## EST II - Individual Subject Test - Level 1

### Math - Answer key

Question Number	Correct Answer	Question Number	Correct Answer
1	A	32	A
2	B	33	D
3	B	34	A
4	B	35	A
5	B	36	A
6	C	37	E
7	D	38	C
8	D	39	B
9	B	40	C
10	A	41	B
11	B	42	E
12	D	43	C
13	A	44	D
14	E	45	D
15	E	46	A
16	C	47	E
17	C	48	C
18	C	49	D
19	D	50	E
20	E		
21	D		
22	A		
23	C		
24	A		
25	C		
26	B		
27	B		
28	A		
29	A		
30	B		
31	A		

**EST II – Individual Subject Test**

**Level 1**

**Student's Name** \_\_\_\_\_

**National ID** \_\_\_\_\_

**Test Center:** \_\_\_\_\_

**Subject:** Math

**Duration:** 60 minutes

50 Multiple Choice Questions

**Instructions:**

- Place your answer on the answer sheet. Mark only one answer for each of the multiple choice questions.
- Avoid guessing. Your answers should reflect your overall understanding of the subject matter.
- Calculator is allowed. When a calculator is used, be aware of switching between radian mode and median mode.
- Formula sheet is available at the end of the booklet for your reference.

THE FORMULAS BELOW MAY BE USEFUL IN ANSWERING QUESTIONS ON THIS TEST.

$S = 4\pi r^2$  is the formula for the surface area of a sphere with a radius of  $r$ .

$V = \frac{1}{3}\pi r^2 h$  is the formula for a right circular cone with a radius of  $r$  and a height of  $h$ .

$V = \frac{4}{3}\pi r^3$  is the formula for a sphere with a radius of  $r$ .

$V = \frac{1}{3}Bh$  is the formula for a pyramid with a base area of  $B$  and a height of  $h$ .

Dr. Ahmed Hassan

1. The remainder when  $t^{16} + 5$  is divided by  $t + 1$  is

- A. 6
- B. 0
- C. -1
- D. 4
- E. 1

$$f(1) = (1)^{16} + 5 = 6$$

2. If two roots of the equation  $ay^3 + by^2 + cy + d = 0$  (with a, b, and c integers) are 1 and  $2 - 8i$ , then the value of a is

- A.  $2 + 8i$
- B. -5
- C. 5
- D.  $4 + 8i$
- E.  $4 - 8i$

roots  $\rightarrow 1, 2+8i, 2-8i$

$$\text{Sum} = \frac{-b}{a}$$

$$-a = 2-8i + 2+8i + 1$$

$$a = -5$$

$$a = -1 + (-2+8i) + (-2-8i) = -5$$

$$c = (-1)(-2+8i)(-2-8i) = -68$$

$$b = (-1)(-2+8i)(-1) + (-2+8i)(-2-8i) = 72$$

3. The root(s) of the equation  $\sqrt{a+6} = -a$  is / are:

- A. 3
- B. -2
- C. 3 and -2
- D. 3 or -2
- E. No Roots

plug in

4. What are all p such that  $\frac{p+100}{p} \leq 1$ ?

- A.  $p > 0$
- B.  $p < 0$
- C.  $p \leq 0$
- D.  $-1 < p < 0$
- E.  $-1 \leq p < 0$

$$\frac{p}{p} + \frac{100}{p} \leq 1$$

$$1 + \frac{100}{p} \leq 1$$

$$\frac{100}{p} \leq 0$$

$p < 0$

5. The graph of  $t^2 - \sqrt{5}t - 2$  has its minimum value at which the approximate value of t

- A. 83
- B. 1.12
- C. 1.21
- D. 1.35
- E. 2.47

Vertex,  $x$

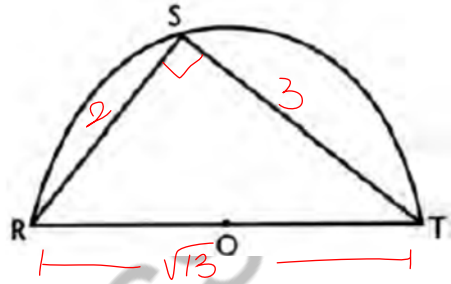
$$x = \frac{-b}{2a} = \frac{\sqrt{5}}{2} = 1.12$$

6. The diagonals of a parallelogram divide the figure into four triangles that are
- A. congruent
  - B. similar
  - C. equal in area
  - D. isosceles
  - E. equal in perimeter

7. In the adjacent figure,  $\overline{RT}$  is a diameter of the semicircle. If  $RS = 2$  and  $ST = 3$ , then the area of the semicircle is

- A.  $\frac{13\pi}{2}$
- B.  $\frac{13\pi}{4}$
- C.  $\frac{\pi}{6}$
- D.  $\frac{13\pi}{8}$
- E.  $\frac{\pi}{4}$

$$\frac{1}{2} \times \left(\frac{\sqrt{13}}{2}\right)^2 \times \pi = \frac{13\pi}{8}$$



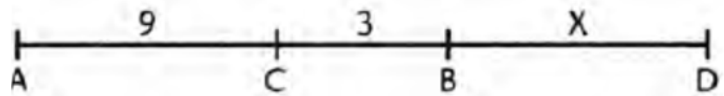
8. In the adjacent figure,  $AC = 9$ ,  $D$  is three times as far from  $A$  as from  $B$  and  $BC = 3$ . What is  $BD$ ?

- A. 9
- B. 12
- C. 18
- D. 6
- E. 15

$$DA = 3DB$$

$$9 + 3 + x = 3x$$

$$12 + x = 3x, x = 6$$



9. The graph of the equation  $y = 50 \cos 3x$  has a period, in radians, of

- A.  $3\pi$
- B.  $\frac{2\pi}{3}$
- C.  $\frac{2\pi}{5}$
- D.  $50\pi$
- E.  $\frac{50\pi}{3}$

$$\text{period} = \frac{2\pi}{3}$$

10. A right circular cone is cut into two equal parts. The lateral surface area of one of the parts is equal to thrice the lateral surface area of a hemisphere with a radius equal to 8 cm. What is the area of the right circular cone?

- A.  $768\pi \text{ cm}^2$   
 B.  $512\pi \text{ cm}^2$   
 C.  $384\pi \text{ cm}^2$   
 D.  $192\pi \text{ cm}^2$   
 E.  $128\pi \text{ cm}^2$

$$\frac{\pi r^2 + \pi rL}{2} = 3 \left( \frac{4\pi (8)^2}{2} \right)$$

$$= 768\pi$$

11. If  $a \neq b$ , then  $x = (a \sin t)$  and  $y = (b \cos t)$  represent the parametric equations of a(n):

- A. Circle  
 B. Ellipse  
 C. Straight line  
 D. Parabola  
 E. Hyperbola

$$x^2 = a^2 \sin^2 t, \quad y^2 = b^2 \cos^2 t$$

$$\frac{x^2}{a^2} = \sin^2 t, \quad \frac{y^2}{b^2} = \cos^2 t$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

add  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$   
 $\neq$   
 ellipse

12.  $2 \cos^3 B \sin B + 2 \sin^3 B \cos B$  is equal to

- A.  $2 \sin B$   
 B.  $\cos^2 B$   
 C.  $\cos 2B$   
 D.  $\sin 2B$   
 E.  $2 \cos B$

$$2 \sin B \cos B [\cos^2 B + \sin^2 B]$$

$$\downarrow \quad \quad \quad \downarrow$$

$$\sin 2B \quad \times \quad 1 = \sin 2B$$

13. The irrational number is

- A.  $\frac{3\sqrt{18}}{2\sqrt{6}} = 2.598076211$   
 B.  $\sqrt[3]{-27} = 3$   
 C.  $\sqrt{2}(3\sqrt{2} + 2\sqrt{8}) = 14$   
 D.  $\frac{2\sqrt{5}}{\sqrt{45}} = 2/3 \rightarrow 0.6$   
 E.  $\sqrt{\frac{1}{2}} \cdot \sqrt{\frac{25}{2}} = 5/2$

14. A car moved a distance of  $x$  meters to a distance of  $y$  meters. The percent of increase was

- A.  $\frac{100(y-x)}{y}$   
 B.  $\frac{100(x-y)}{x}$   
 C.  $\frac{y-x^x}{x}$   
 D.  $\frac{x-y}{x}$   
 E.  $\frac{100(y-x)}{x}$

$$\frac{y-x}{x} \times \frac{100}{1} = \frac{100(y-x)}{x}$$

$$\frac{G-S}{S} \times 100$$

15. The solution of  $3x^2 - xy = 3$  and  $6x - y = 10$  is

A.  $\frac{1}{3}$  and 3

B. 8 and -8

C.  $\frac{1}{3}$  and -8

D. 3 and 8

E.  $\frac{1}{3}$  and -8; 3 and 8

$$\begin{aligned} 6x - 10 &= y \\ 3x^2 - x(6x - 10) &= 3 \\ 3x^2 - 6x^2 + 10x - 3 &= 3 \\ -3x^2 + 10x - 3 &= 0 \end{aligned}$$

$x = 3 \text{ or } \frac{1}{3}$   
 $y = 8 \text{ or } -8$

16. If graphed on the same set of axes and  $0 \leq x \leq 2\pi$ , then  $y = \sin x$  and  $y = \cos x$  will intersect in quadrant(s)

A. I only

B. III only

C. I and III

D. II only

E. I and II

$$\sin x = \cos x$$

$90 + 2n\pi$

$n=0$   $2x = 90$   
 $x = 45 \leftarrow 1^{st}$

$n=1$   $2x = 90 + 2(1)(180) = 450$   
 $x = 225 \leftarrow 3^{rd}$

17. If A represents the set of rectangles and B the set of rhombi, then  $A \cap B$  represents the set of

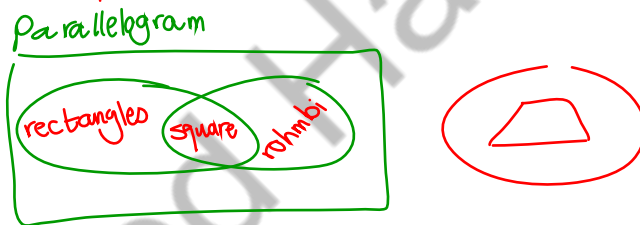
A. Trapezoids

B. Parallelograms

C. Squares

D. Rectangles

E. Quadrilaterals



18. The approximate area of the parallelogram DAWN, in the adjacent figure, is

A. 20

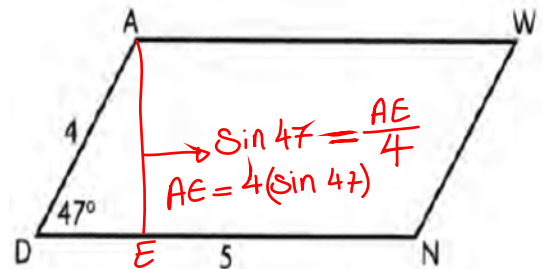
B. 17.25

C. 14.63

D. 11.5

E. 13.64

$$\begin{aligned} A &= B \times h \\ A &= 4(\sin 47^\circ) \times 5 \\ &= 14.63 \end{aligned}$$



19. If the line of equation  $5x + 12y - 60 = 0$  is tangent to the circle of equation  $(x - 1)^2 + (y - 3)^2 = r^2$ , then the value of r is

A.  $2\sqrt{3}$

B.  $\sqrt{3}$

C.  $\sqrt{10}$

D.  $\frac{19}{13}$

E.  $\frac{13}{12}$

Center (1,3)

Distance between line & point =

$$\frac{|5(1) + 12(3) - 60|}{\sqrt{5^2 + 12^2}} = \frac{19}{13}$$

20. If  $[\sqrt{2} (\cos 30^\circ + i \sin 30^\circ)]^2 = a + bi$ , then  $a = \dots$  and  $b = \dots$

A.  $a = 2, b = \sqrt{3}$

B.  $a = \frac{3}{2}, b = \frac{1}{2}$

C.  $a = 1, b = -\sqrt{3}$

D.  $a = \frac{3}{2}, b = -\frac{1}{2}$

E.  $a = 1, b = \sqrt{3}$

$30 \times 2$

$a = 2 \times \cos 60 = 1$

$b = 2 \times i \sin 60 = \sqrt{3}$

21. The probability of getting 80 % or more of the questions correct on a 10- question true-false exam merely by guessing is

A.  $\frac{5}{32}$

B.  $\frac{1}{16}$

C.  $\frac{16}{3}$

D.  $\frac{16}{7}$

E.  $\frac{7}{32}$

$P(\text{correct}) = (\frac{1}{2})$

2 option  $\rightarrow$  (V) or (X)

no. of q/s  $\rightarrow$  10

${}^{10}C_{10} (\frac{1}{2})^{10} + {}^{10}C_9 (\frac{1}{2})^{10} + {}^{10}C_8 (\frac{1}{2})^{10} = \frac{7}{128}$

22. The ratio of the diagonal of a cube to the diagonal of a face of this cube is

A.  $3: \sqrt{6}$

B.  $2: \sqrt{6}$

C.  $3: \sqrt{2}$

D.  $6: \sqrt{2}$

E.  $2: \sqrt{3}$

$\sqrt{3} : \sqrt{2}$

23. The intersection of the three lines of equations  $x=3y$ ,  $y=0$  and  $3x+y=7$  form a(n) ----- triangle

A. Acute

B. Obtuse

C. Right

D. Isosceles

E. Equilateral



24. The distance AB between the points A(-3, 7) and B(6, -5) in a coordinate system in which the y-axis is inclined  $60^\circ$  to the positive x-axis is

A.  $\sqrt{117}$

B.  $\sqrt{120}$

C.  $\sqrt{189}$

D.  $\sqrt{333}$

E.  $\sqrt{108}$

$\sqrt{(-3-6)^2 + (7-(-5))^2} - 2|-3-6| \times |7-(-5)| \times \cos 60$



25. The municipality in a certain hospital administered a survey on diabetes. Blood tests for 150 patients gave the following results in millimoles per liter:

Level of glucose in mmol/L	[6 – 8[	[8 – 10[	[10 – 12[	[12 – 14[	[14 – 16[
Frequency	50	10	20	30	40

The Median is

$$\frac{\sum f}{2} = \frac{150}{2} = 75 \leftarrow \text{Median}$$

- A. 11  
 B. 10  
 C. 11.5  
 D. 10.5  
 E. 12

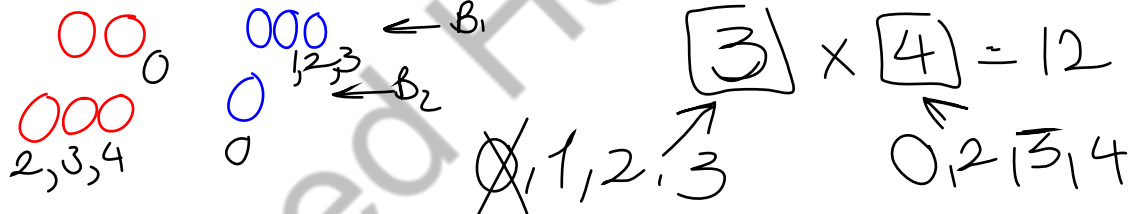
$$10 + \frac{2}{20} \times 15 = 11.5$$

26. A bag  $B_1$  contains two red balls numbered 0 and three blue balls numbered 1, 2, 3

A bag  $B_2$  contains three red balls numbered 2, 3, 4 and one blue ball numbered 0.

One ball is selected from  $B_1$ , then another ball is randomly selected from  $B_2$ . Thus a 2-digit number is formed. The number of possible outcomes is

- A. 25  
 B. 12  
 C. 20  
 D. 16  
 E. 144



27. The following table gives information about a population of smokers and of those having lung diseases due to smoking.

	Men		Men and Women	
	Smokers	Sick	Smokers	Sick
20 - 40 years	2500	30	4500	50
40 - 60 years	3000	50	5000	75
60 - 80 years	2000	80	4000	140
Total	7500	160	13500	265

An individual is selected at random from this population. The probability that this individual is sick knowing that he/she is between 60 and 80 years old is

- A.  $\frac{265}{13500}$   
 B.  $\frac{140}{4000}$   
 C.  $\frac{80}{2000}$   
 D.  $\frac{140}{13500}$   
 E.  $\frac{265}{4000}$

$$\frac{140}{4000}$$

28. The solution of the system  $\begin{cases} 3-x^2 \leq 0 \\ \frac{x-11}{x-4} \geq 2 \end{cases}$  is

- A.  $x \in [-3, -\sqrt{3}] \cup [\sqrt{3}, 4[$
- B.  $x \in ]-3, -\sqrt{3}] \cup [\sqrt{3}, 4[$
- C.  $x \in ]-3, -\sqrt{3}[ \cup [\sqrt{3}, 4[$
- D.  $x \in [-3, -\sqrt{3}] \cup [\sqrt{3}, 4[$
- E.  $x \in [-3, -\sqrt{3}] \cup ]\sqrt{3}, 4[$

$3-x^2 \leq 0 \rightarrow x = \pm 1.7$

$]-\infty, -1.7] \cup [1.7, \infty[$

$\frac{x-11}{x-4} \geq 2 \rightarrow \frac{(x-11)-2x+8}{x-4} \geq 0$

$\frac{-x-3}{x-4} \geq 0$

$]-3, 4[$

$]-3, -\sqrt{3}] \cup [\sqrt{3}, 4[$

29. The real numbers  $m, n$  and  $p$  such that  $P(x) = (x^2 - 2)(mx^2 + nx + p)$  will be identical to  $x^4 - x^3 - 3x^2 + 2x + 2$  are:

- A.  $m = 1, n = -1, p = -1$
- B.  $m = -1, n = -1, p = -1$
- C.  $m = 1, n = 1, p = 1$
- D.  $m = 1, n = 1, p = -1$
- E.  $m = -1, n = -1, p = 1$

$mx^4 + nx^3 + px^2 - 2mx^2 - 2nx - 2p$

$m=1$

$n=-1$

$p=-1$

30. A fair die is tossed and a card is randomly chosen from a deck of 52 cards. The probability that "a multiple of three appears and a king or an ace is selected" is:

- A.  $\frac{8}{312}$
- B.  $\frac{16}{312}$
- C.  $\frac{8}{52}$
- D.  $\frac{16}{52}$
- E.  $\frac{16}{36}$

$\frac{2}{6} \times \frac{8}{52}$

31. In a given school, the class of grade 12 is divided into three sections: Sociology and economics (SE), Life Sciences (LS) and General Sciences (GS). The students of this class are distributed as follows:

Sections/Students	Boys	Girls	Total
SE	17	21	38
LS	13	14	27
GS	14	5	19
<b>Total</b>	<b>44</b>	<b>40</b>	<b>84</b>

The number of ways that three students can be selected from this class if exactly two of them are girls and if the order is respected is

- A. 205,920
- B. 68,640
- C. 411,840
- D. 70,400
- E. 211,200

$3 \times [40 \times 39 \times 44]$

$G \downarrow G \downarrow B \downarrow$

GGB  
GBG  
BGG

32. A mining company extracts oil from an oilfield since the year 1963. The following table shows the quantity  $y_i$ , in tons, extracted during each year indicated by its rank  $x_i$ .

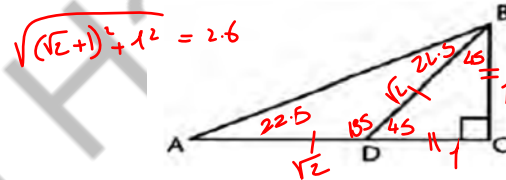
Year	1963	1968	1973	1978	1983	1988	1993	1998	2003	2008
Rank $x_i$	0	1	2	3	4	5	6	7	8	9
Quantity $y_i$ (in tons)	18.1	15.7	13.3	11	9.3	7.8	7.1	6.1	5.2	4.3

The means  $\bar{x}$  and  $\bar{y}$  of the variables  $x$  and  $y$  respectively are

- A.  $\bar{x} = 4.5$   $\bar{y} = 9.79$   
 B.  $\bar{x} = 4$   $\bar{y} = 9.79$   
 C.  $\bar{x} = 4.5$   $\bar{y} = 9$   
 D.  $\bar{x} = 5$   $\bar{y} = 9.79$   
 E.  $\bar{x} = 4.5$   $\bar{y} = 7.9$

33. The length of  $\overline{AB}$  in the adjacent right triangle at C such that  $AD = DB$ ,  $DC = BC = 1$  is

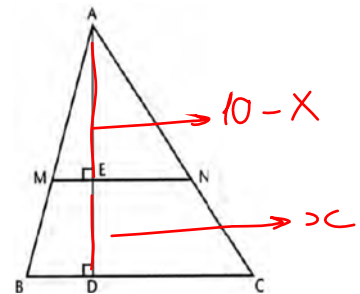
- A.  $\sqrt{10}$   
 B.  $2\sqrt{2}$   
 C.  $2\sqrt{10}$   
 D.  $\sqrt{4 + 2\sqrt{2}}$   
 E.  $4 + 2\sqrt{2}$



34. In the adjacent  $\triangle ABC$ ,  $AD = 10$ ,  $\overline{MN}$  is parallel to  $\overline{BC}$  and  $\overline{MN}$  bisects the area of  $\triangle ABC$ . The value of  $ED$

- A.  $10 - 5\sqrt{2}$   
 B.  $10 + 5\sqrt{2}$   
 C.  $5\sqrt{10} + 2\sqrt{2}$   
 D.  $5\sqrt{2}$   
 E.  $2\sqrt{10}$

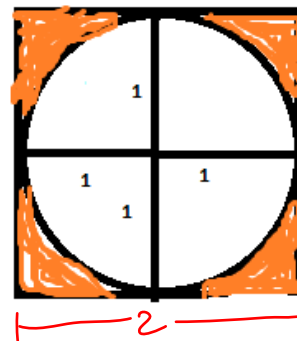
$$\frac{1}{2} = \left(\frac{10-x}{x}\right)^2$$



35. A circle is inscribed in the square as shown in the adjacent figure. The area of the shaded region is:

- A.  $4 - \pi$   
 B.  $4 + \pi$   
 C.  $1 - \pi$   
 D.  $1 + \pi$   
 E.  $2 - \pi$

area of square = 4  
 area of circle =  $\pi$



36. The total price of one kilogram of sugar and one kilogram of salt is 20 EGP. During an economic crisis, the price of sugar increased by 12 % and that of salt decreased by 15% such that the total price of 1 Kg of sugar and 1 Kg of salt became 22 EGP.

Reem bought 5 Kg of sugar and 2 Kg of salt during the crisis. How much did she pay, rounded to the nearest integer?

- A. 106 EGP  
 B. 100 EGP  
 C. 120 EGP  
 D. 96 EGP  
 E. 48 EGP

$$\begin{aligned}
 x + y &= 20 \\
 x(1+12\%) + y(1-15\%) &= 22 \\
 1.12x + 0.85y &= 22 \\
 \text{plug in } x &= 18.5, y = 1.4 \\
 103 \cdot 6 + 2 \cdot 38 &= 105.9
 \end{aligned}$$

37. Consider the polynomial

$$E(x) = (2x - 1)^2 + (x - 2)(1 - 2x)$$

$E(1 + \sqrt{2}) = c + d\sqrt{2}$ , where  $c$  and  $d$  are two integers such that

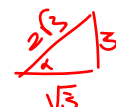
- A.  $c = d = 5$   
 B.  $c = 6, d = -5$   
 C.  $c = -6, d = -5$   
 D.  $c = -6, d = 5$   
 E.  $c = 6, d = 5$

$$= 6 + 5\sqrt{2}$$

38. In the  $xy$ -plane, consider a straight line ( $D$ ) passing through  $A(-\sqrt{3}; -3 - 2\sqrt{3})$  and making an acute angle  $\alpha$  with the  $x$  - axis such that  $\cos \alpha = \frac{\sqrt{3}}{3} \times \sin \alpha$ .

The value of  $\sin \alpha$  and  $\cos \alpha$  are respectively

- A.  $-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$   
 B.  $\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$   
 C.  $\frac{\sqrt{3}}{2}, \frac{1}{2}$   
 D.  $-\frac{\sqrt{3}}{2}, \frac{1}{2}$   
 E.  $\frac{\sqrt{3}}{2}, -\frac{1}{2}$

$$\begin{aligned}
 \frac{\cos \alpha}{\sin \alpha} &= \frac{\sqrt{3}}{3} \rightarrow \cot \alpha = \frac{\sqrt{3}}{3} \\
 \tan \alpha &= \frac{3}{\sqrt{3}} \\
 \cos \alpha &= \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2} \\
 \sin \alpha &= \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}
 \end{aligned}$$


39. Consider the three expressions:

$$A = (x - 1)(x - 2), B = (x - 2)(x - 3) \text{ and } C = (x - 3)(x - 4)$$

$A - 2B + C$  is equal to

- A. 1  
 B. 2  
 C. 3  
 D. 4  
 E. 5

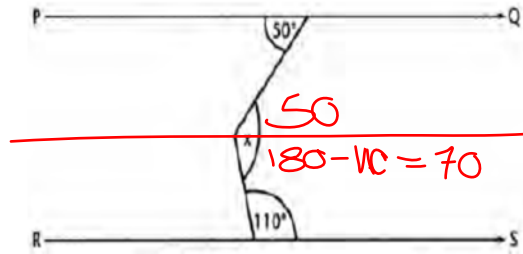
no (x)

$$\begin{aligned}
 A &= 2 \\
 B &= 6 \\
 C &= 12
 \end{aligned}
 \left. \vphantom{\begin{aligned} A \\ B \\ C \end{aligned}} \right\} 2 - 2 \times 6 + 12$$

40. In the adjacent figure, if the two lines (PQ) and (RS) are parallel, then x is equal to

- A.  $50^\circ$
- B.  $110^\circ$
- C.  $120^\circ$
- D.  $60^\circ$
- E.  $100^\circ$

$x = 70 + 50$



41. The largest real range of the function  $y = 1 - \frac{1}{x}$  is

- A.  $y > 0$
- B. All Real numbers for y except  $y = 1$
- C.  $y \neq 0$
- D.  $y \geq 0$
- E.  $y \leq 0$

point of symmetry  
(0,1)

Range =  $\mathbb{R} - \{1\}$

42. A company runs three production lines that together have an output of 54 parts/ hour. Twice the production of the second line is equal to the combined output of the other two lines, and the output of the third line is three parts per hour more than the second line.

The production rate of the first line x, the production rate of the second line y and that of the third one z are:

- A.  $x = 21, y = 18, z = 15$
- B.  $x = 18, y = 21, z = 15$
- C.  $x = 15, y = 21, z = 18$
- D.  $x = 21, y = 15, z = 18$
- E.  $x = 15, y = 18, z = 21$

$$\left. \begin{aligned} x + y + z &= 54 \\ 2y &= x + z \\ z &= 3 + y \end{aligned} \right\} \begin{aligned} x + y + z &= 54 \\ x - 2y + z &= 0 \\ 0 - y + z &= 3 \end{aligned} \quad \left. \begin{array}{l} \text{Mode} \\ 5, 2 \end{array} \right\}$$

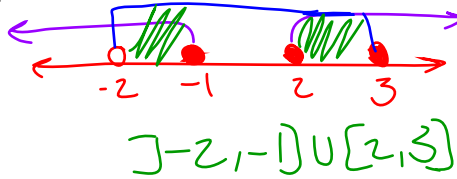
43. What is the equation of the tangent to the curve of function f defined by  $f(x) = x^2 + 6x + 8$  at a point of abscissa 1? (1,5)

- A.  $y = -3x + 18$
- B.  $y = 8x + 18$
- C.  $y = 8x + 7$
- D.  $y = -3x + 7$
- E.  $y = 8x - 7$

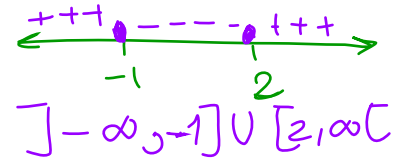
$y' = 2x + 6 \leftarrow (1) \text{ plug in}$   
 $y' = 8 \leftarrow \text{slope}$   
 $y = 8x + b \leftarrow (1,5) \text{ plug in}$   
 $y = 8x + b$

44. The solution of the system defined by  $\begin{cases} x^2 - x - 2 \geq 0 \\ \frac{x-3}{x+2} \leq 0 \end{cases}$  is

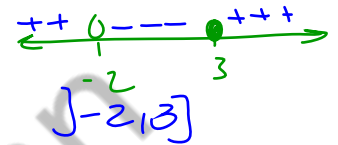
- A.  $] -\infty, -1] \cup [2, +\infty [$
- B.  $] -2, 3]$
- C.  $] -2, -1] \cup [2, 3 [$
- D.  $] -2, -1] \cup [2, 3]$**
- E.  $] -2, -1] \cup [2, 3 [$



$$x^2 - x - 2 \geq 0$$



$$\frac{x-3}{x+2} \leq 0$$



45.  $\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + 4x + 1} - 1}{x}$  is

- A. 2
- B.  $+\infty$
- C. 0
- D. 1**
- E.  $-\infty$

$$\sqrt{x^2} = x$$

$$y = 1 \rightarrow \frac{\sqrt{1}}{1} = 1$$

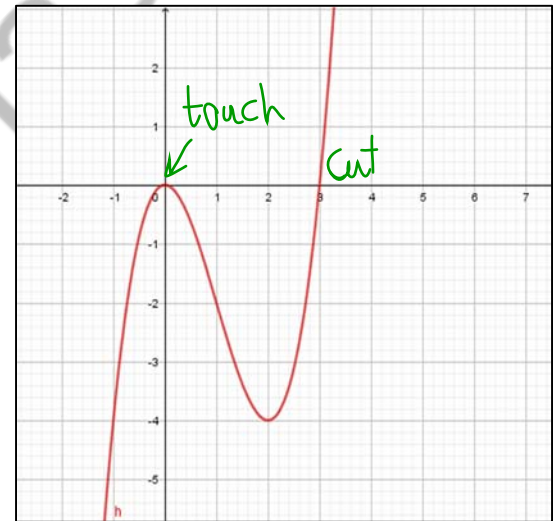
46. Suppose that the adjacent curve (h) is the curve representing the function f defined by  $f(x) = ax^3 + bx^2$ .

Graphically,

- A.  $a = 1$  and  $b = -3$**
- B.  $a = -1$  and  $b = -3$
- C.  $a = -3$  and  $b = 1$
- D.  $a = 1$  and  $b = 3$
- E.  $a = 3$  and  $b = -1$

$$x^2(ax + b)$$

$\downarrow$                        $\downarrow$   
 0                       $(x-3)$   
                           $a=1, b=-3$



47. The function f defined by  $f(x) = \frac{x^2 - 3x - 3}{x - 4}$  admits the center of symmetry

- A. (-4, 5)
- B. (4, -5)
- C. (-4, -5)
- D. (5, 4)
- E. (4, 5)**

Vertical  $\rightarrow x - 4 = 0$   
 $x = 4$

Slant  $\rightarrow$

$$\begin{array}{r|rrr} 4 & x^2 & x & C \\ & 1 & -3 & -3 \\ & & 4 & 4 \\ \hline & & 1 & 1 \end{array}$$

$$y = x + 1 \leftarrow x = 4$$

Center (4, 5) 13

48. On a given day, a department store sold 40 shirts. The shirts come in three styles: Casual, sports and dress. A casual shirt costs \$20, a sports shirt costs \$40, and a dress shirt costs \$60. The store sold \$1080 worth of these shirts on that day and the number of casual shirts exceeded by 20 the combined numbers of the sports and dress shirts.

The number of shirts of each type is:

- A. 6 casual shirts, 30 sport and 4 dress shirts  
 B. 30 casual shirts, 7 sport and 3 dress shirts  
 C. 30 casual shirts, 6 sport and 4 dress shirts  
 D. 4 casual shirts, 6 sport and 30 dress shirts  
 E. 6 casual shirts, 4 sport and 30 dress shirts

$$x + y + z = 40$$

$$20x + 40y + 60z = 1080$$

$$x = y + z + 20$$

$$x - y - z = 20$$

49. If  $a$ ,  $b$ , and  $c$  are three positive numbers and if  $a > b$ , which of the following is not necessarily true?

A.  $a + c > b + c$  ✓

B.  $a - c > b - c$  ✓

C.  $\frac{a}{c} > \frac{b}{c}$  ✓

D.  $\frac{c}{a} > \frac{c}{b}$  ✗

E.  $\frac{a}{b} > 1$  ✓

50. The simplest form of  $\frac{2^{y+4} - 2(2^y)}{2(2^{y+3})}$  is:

A.  $\frac{1}{4}$

B.  $\frac{1}{2}$

C.  $\frac{1}{8}$

D.  $\frac{5}{8}$

E.  $\frac{7}{8}$

$$\frac{2^{y+4} - 2^{y+1}}{2^{y+4}}$$

$$\frac{2^{y+4}}{2^{y+4}} - \frac{2^{y+1}}{2^{y+4}} \rightarrow 2^{y+4-y-1} = 2^{-3}$$

$$1 - 2^{-3} = \frac{7}{8}$$

# بسم الله الرحمن الرحيم وبه نستعين دائما

يسعدني تلقي طلباتك و استفساراتكم على هذه اللينكات



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**DR. AHMED HASSAN**



**EST II – Individual Subject Test**

**Level 1**

**Student's Name** \_\_\_\_\_

**National ID** \_\_\_\_\_

**Test Center:** \_\_\_\_\_

**Subject:** Math

**Duration:** 60 minutes

50 Multiple Choice Questions

**Instructions:**

- Place your answer on the answer sheet. Mark only one answer for each of the multiple choice questions.
- Avoid guessing. Your answers should reflect your overall understanding of the subject matter.
- Calculator is allowed. When a calculator is used, be aware of switching between radian mode and median mode.
- Formula sheet is available at the end of the booklet for your reference.

1. If  $f(x) = 2ax^2 + (3 - b)x + 7$  and  $g(x) = (a - 1)x^2 + 2bx + 7$  are identical, what are the zeros of function  $f$ ?

A.  $-1.54$  and  $4.54$

B.  $-2.19$  and  $3.19$

C.  $-1.27$  and  $2.77$

**D.  $-1.44$  and  $2.44$**

E.  $-2.44$  and  $1.44$

$2a = a - 1, a = -1$

$3 - b = 2b, b = 1$

$\rightarrow -2x^2 + 2x + 7$   
mode 5,3

2.  $3m, 2m + 5$ , and  $4m - 5$  are three consecutive terms of a geometric sequence. If  $m$  has a negative value, what is the value of the common ratio of this sequence?

A.  $-10$

**B.  $-2$**

C.  $-\frac{1}{2}$

D.  $\frac{1}{2}$

E.  $1$

$\frac{2m+5}{3m} = \frac{4m-5}{2m+5}$

$m = -\frac{5}{8}$

$\frac{2(-\frac{5}{8})+5}{3(-\frac{5}{8})} = -2$

3. What is the slope of the line perpendicular to the one passing through  $(-4, 5)$  and  $(3, 7)$ ?

**A.  $-\frac{7}{2}$**

B.  $-1$

C.  $-\frac{2}{7}$

D.  $\frac{2}{7}$

E.  $\frac{7}{2}$

$-\left(\frac{-4-3}{5-7}\right) = -\frac{-7}{2} = \frac{7}{2}$

4. If  $f(x) = \sqrt{x - 1}$  and  $g(x) = x^2$ , what is the domain of  $(f \circ g)(x)$ ?

A. All  $x$  such that  $x \geq 1$

B. All  $x$  such that  $x \leq -1$

**C. All  $x$  such that  $x \leq -1$  or  $x \geq 1$**

D. All  $x$  such that  $x \leq -1$  and  $x \geq 1$

E. All real numbers

$\sqrt{x^2-1}$

$x^2-1 \geq 0$

$x \leq -1, x \geq 1$

5. Using the adjacent diagram at the right, if  $3 \tan \beta = 2$ , what is the value of  $t$ ?

A.  $\frac{2}{3}$

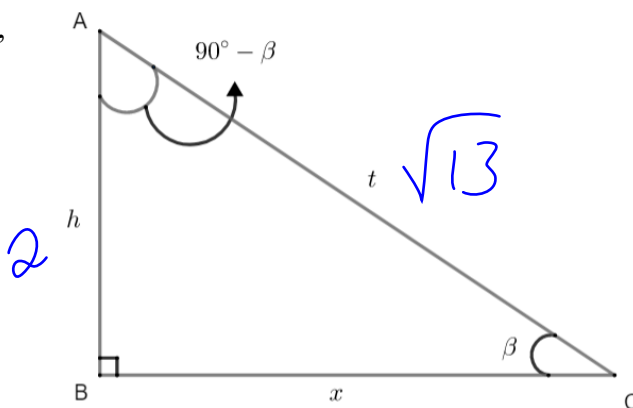
B.  $\frac{3}{2}$

C.  $2\sqrt{13}$

D.  $\sqrt{\frac{2}{3}}$

E.  $\sqrt{13}$

$\tan \beta = \frac{2}{3}$



6. If  $x + 4y = -10$  and  $2x = 35 + 3y$ , what is the value of twice the quotient of  $x$  and  $y$ ?

A. -4

B. -2

C.  $\frac{1}{2}$

D. 2

E. 4

Method 5, 1

$x = 10$

$y = -5$

$$2\left(\frac{10}{-5}\right) = -4$$

7. How many 2-digit numbers are even but do not have a zero digit?

A. 27

B. 30

C. 36

D. 40

E. 45

1 2 3 4 5 6 7 8 9

9 x 4

8. The length of the adjacent rectangle is equal to 14 cm, and the width is equal to 6 cm. What is the area of the shaded region? (figure is not drawn to scale)

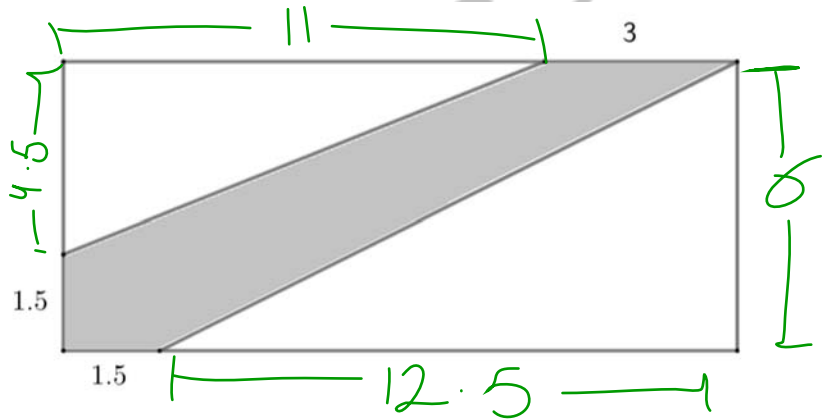
A.  $4.5 \text{ cm}^2$

B.  $21.75 \text{ cm}^2$

C.  $24.75 \text{ cm}^2$

D.  $37.5 \text{ cm}^2$

E.  $84 \text{ cm}^2$



$$14 \times 6 - \left[ \frac{1}{2}(11)(4.5) + \frac{1}{2}(3)(1.5) \right]$$

9. The adjacent figure at the right shows a shape made out of a cone and a cylinder. The total volume of this shape is  $39\pi$  cubic units. If the height of the cylinder is 4 units, and the radius is 3 units, what is the height of the cone?

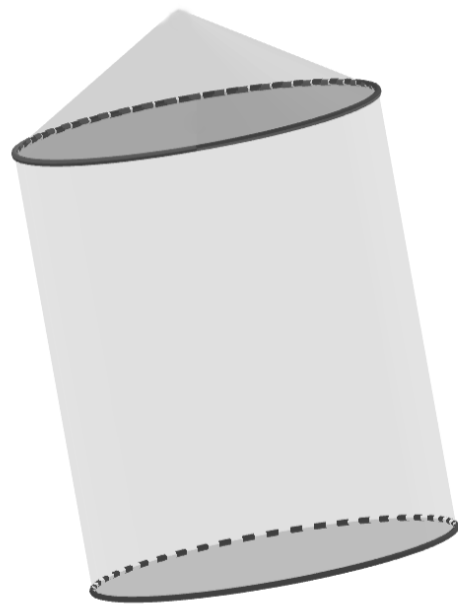
A. 1 unit

B. 2 units

C. 3 units

D. 4 units

E. 5 units



$$\frac{1}{3} \pi r^2 h + \pi r^2 h = 39\pi$$

$$\frac{1}{3} (3)^2 h + (3)^2 (4) = 39$$

$$h = 1$$

10. If  $|x - 3| = 2x$ , which of the following is the correct value of  $x$ ?

- A. -3
- B. -1
- C. 0
- D. 1**
- E. 3

) plug in

11. During the first 4 months of this academic year, Ross passed 70% of his math assignments and 55% of his chemistry assignments, independently one of another. What is the chance that on the next attempt of each subject, he will pass the math assignment but not the chemistry?

- A. 1
- B. 0.385
- C. 0.315**
- D. 0.165
- E. 0.135

Math 70% Chemistry 55%

12. What is the value of  $2x^3$  if  $3125 = 5^{3x-4}$ ?

- A. 3
- B. 6
- C. 27
- D. 54**
- E. 250

$\frac{5^{3x}}{5^4} = 5^5$   
 $5^{3x-4} = 5^5$   
 $3x-4=5$   
 $3x=9$   
 $x=3$   
 $2(3)^3 = 54$

13. What is the equation of the oblique asymptote of the function  $f(x) = \frac{3x^2 - 4x + 1}{2x + 1}$ ?

- A.  $y = 1.5x - 2$
- B.  $y = 1.5x - 2.75$**
- C.  $y = 1.5x$
- D.  $y = 1.5x + 2.75$
- E.  $y = 1.5x + 2$

$\frac{3x^2 - 4x + 1}{2x + 1}$   

$\frac{-1}{2}$	$x^2$	$2x$	$C$
	$3$	$-4$	$1$
		$-\frac{3}{2}$	$2.75$
	$3$	$-5.5$	$3.75$

( $\div 2$ )

14.  $x^2 - 6x + y^2 - 4y + 2 = 0$  represents the equation of a circle (C). What is the radius of this circle?

- A.  $\sqrt{2}$
- B. 2
- C.  $\sqrt{11}$**
- D. 5.5
- E. 11

C (3, 2)  
 $R = \sqrt{(9) + (4) - 2} = \sqrt{11}$

15.  $A(-5.84, 2.27, 0)$  and  $G(-3, 2, 0)$  are two vertices of a cube  $AGCFEDBH$ . What is the surface area of this cube?

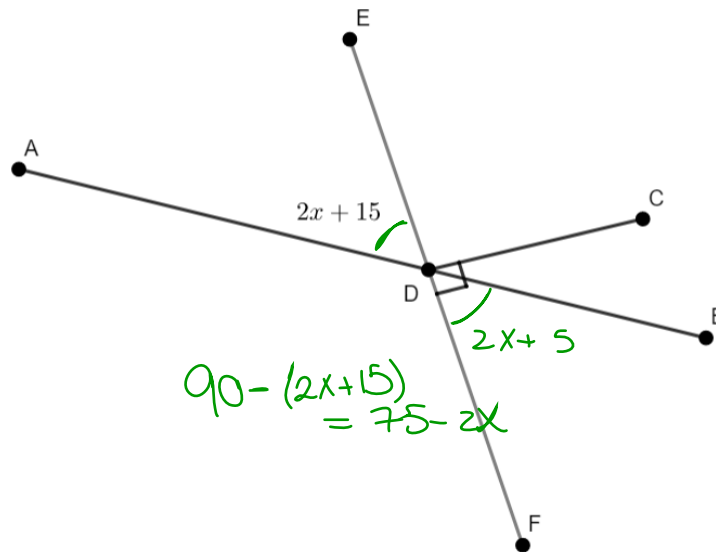
- A. 2.85 square units
- B. 15.41 square units
- C. 48.83 square units
- D. 237.47 square units
- E. 2384.46 square units

$$S = \sqrt{(-3 - 5.84)^2 + (2 - 2.27)^2}$$

$$= 2.85$$

$$S.A = 6(2.85)^2$$

16. Using the diagram below, what is the measure of  $\angle BDC$ ?



- A.  $2x + 105$
- B.  $2x + 75$
- C.  $2x - 75$
- D.  $105 - 2x$
- E.  $75 - 2x$

17. Which of the following is equivalent to the expression:  $2a^2 - 3a + 4ba - 6b$  ?

- A.  $(2a - 3)(a - 2b)$
- B.  $(2a - b)(a + 3)$
- C.  $(2 - b)(ab - 3)$
- D.  $(2a - 3)(a + 2b)$
- E.  $(2a + 3)(a - 2b)$

$$(2a - 3) + 2b(2a - 3)$$

$$(2a - 3)(a + 2b)$$

18. Jimmy decided to spend an average of \$40 a week on buying clothes. He spent \$25 the first week, and \$66 the second week. How much can he equally spend the third and the fourth week without surpassing his budget?

- A. \$69
- B. \$54.5
- C. \$34.5**
- D. \$45
- E. \$38

$$\frac{4(10) - 25 - 66}{2}$$

19. The height of a trapezoid is equal to  $2x + 1$ . Its small base is half the height, and the large base is triple the small one. What is the expression that represents the area of this trapezoid?

- A.  $2x + 1$
- B.  $(2x + 1)^2$**
- C.  $2(2x + 1)^2$
- D.  $2x - 1$
- E.  $4x^2 - 1$

$$\frac{3}{2}(2x+1) \quad A = \left(\frac{a_1+a_2}{2}\right)h \quad \left[ \frac{(2x+1)}{2} + \frac{3(2x+1)}{2} \right]$$

$$2x+1 = \frac{2(2x+1)^2}{2} = (2x+1)^2$$

20. What is the product of the solutions of  $x^4 - 3x^2 = -2$  ?

- A.  $-2\sqrt{2}$
- B.  $-2$
- C. 1
- D. 2**
- E.  $2\sqrt{2}$

$$- \left(\frac{-2}{1}\right) = 2$$

21. What are the transformations of  $f(x) = x^2$  when the graph of  $g(x) = 2x^2 - 2$  is sketched?

- A. It is vertically stretched with a vertical shift down 2 units**
- B. It is vertically stretched with a horizontal shift right 2 units
- C. It is vertically compressed with a vertical shift down 2 units
- D. It is vertically compressed with a vertical shift up 2 units
- E. It is horizontally stretched with a vertical shift down 2 units

22. Lora went to the mall and purchased the items that are shown in the adjacent table. What is the sum of the mean and the median of the list shown?

- A. 25.17
- B. 29.56
- C. 33.95
- D. 59.12**
- E. 101.72

Mean + median

$$33.95 + 2.17 = 59.12$$

Items	Price
Jeans	\$44.77 (5)
T-shirt	\$25.67 (4)
Sweatshirt	\$20.67 (2)
Backpack	\$24.67 (3)
Jacket	\$67.77 (6)
Hat	\$20.17 (1)

23. Which constant should be added to  $3x^2 - 8x$  in order to be made into a perfect square?

A.  $\frac{4}{3}$   
 B.  $\frac{16}{3}$   
 C.  $\frac{8}{3}$   
 D. 4  
 E. 16

$(\frac{-8}{2})^2 = 16$   
 $3x^2 - 8x + \frac{16}{3}$

24. The area of a parallelogram is  $16 m^2$ . The ratio of its height to base is 1:4. What is the dimension of its height?

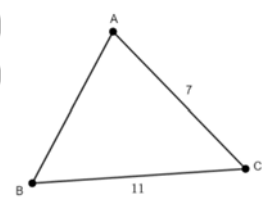
A.  $2 m$   
 B.  $4 m$   
 C.  $8 m$   
 D.  $16 m$   
 E.  $64 m$

$A = L \cdot W$   
 $16 = 4x^2$   
 $x = 2$

25. Which value for the dimension of segment  $AB$  could be true so that  $ABC$  can be a triangle?

A. 0.5  
 B. 1.5  
 C. 2.5  
 D. 3.5  
 E. 4.5

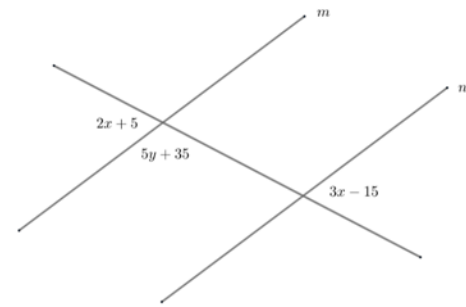
$11 - 7$   
 $11 + 7$   
 $4 < 4.5 < 18$



26. Using the adjacent diagram, if lines  $m$  and  $n$  are parallel, what is the value of  $y$ ?

A. 100  
 B. 80  
 C. 60  
 D. 40  
 E. 20

$2x + 5 = 3x - 15$   
 $x = 20$   
 $\Rightarrow 5y + 35 + 45 = 180$   
 $y = 20$



27. Which of the following is the inverse function of  $f(x) = x^2 - 4x + 4$ ?

A.  $f^{-1}(x) = -4 \pm \sqrt{x}$   
 B.  $f^{-1}(x) = -2 \pm \sqrt{x}$   
 C.  $f^{-1}(x) = \pm \sqrt{x}$   
 D.  $f^{-1}(x) = 2 \pm \sqrt{x}$   
 E.  $f^{-1}(x) = 4 \pm \sqrt{x}$

$x = y^2 - 4y + 4$   
 $x = (y - 2)^2$   
 $\sqrt{x} = y - 2$   
 $y = 2 \pm \sqrt{x}$

28. Which of the following is equivalent to  $f(x) = 343(7^{-2x})$  ?

A.  $f(x) = 7^{2+3x}$

B.  $f(x) = 7^{3-2x}$

C.  $f(x) = 7^{6x}$

D.  $f(x) = \frac{1}{7^{6x}}$

E.  $f(x) = \frac{1}{7^{3-2x}}$

$$7^3 \cdot 7^{-2x} = 7^{3-2x}$$

29. The perimeter of an equilateral triangle is equal to  $3\sqrt{7} - 6$ . What is the dimension of the height of the triangle?

A. 0.16

B. 0.31

C. 0.56

D. 0.65

E. 1.25

$$\text{Side length} = \frac{3\sqrt{7}-6}{3} = \sqrt{7}-2$$

$$H = \left(\frac{\sqrt{7}-2}{2}\right)\sqrt{3} = 0.56$$

30. Which of the following equations has roots equal to  $2\sqrt{2}$  and  $-\frac{7}{2}$  ?

A.  $0 = x^2 + x\left(\frac{-7+4\sqrt{2}}{2}\right) - 7\sqrt{2}$

B.  $0 = x^2 + x\left(\frac{7-4\sqrt{2}}{2}\right) - 7\sqrt{2}$

C.  $0 = x^2 + x\left(\frac{-7+4\sqrt{2}}{2}\right) + 7\sqrt{2}$

D.  $0 = x^2 + x\left(\frac{7-4\sqrt{2}}{2}\right) + 7\sqrt{2}$

E.  $0 = x^2 + x\left(\frac{7-2\sqrt{2}}{2}\right) - 7\sqrt{2}$

31. Jack bought a bag of 30-pieces of chocolates in which he found three different types of chocolate bars. If there are 11 bars of type A, 13 bars of type B, and the remaining are bars of type C, what is the probability that the first bar of chocolate that Jack will choose will be of type C?

A.  $\frac{4}{5}$

B.  $\frac{1}{2}$

C.  $\frac{1}{4}$

D.  $\frac{1}{5}$

E.  $\frac{1}{6}$

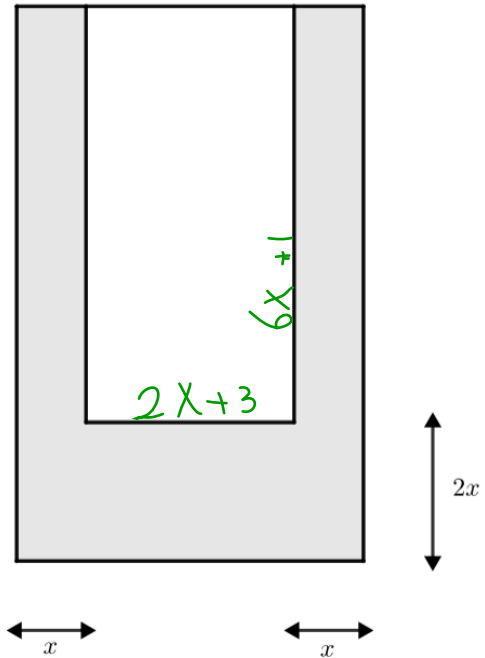
$$C = 30 - 11 - 13$$

$$\frac{6}{30} = \frac{1}{5}$$



32. Using the adjacent figure, the length of the larger rectangle is  $8x + 1$  while the width is equal to  $4x + 3$ . If the area of the shaded region is equal to 204 square units, what is the value of  $x$ ?

- A.  $\frac{1}{2}$   
 B. 2  
 C. 3  
 D. 6  
 E. 9



33. The adjacent table shows the heights of 10 randomly chosen buildings in Egypt. What is the sum of the interquartile range and the median of the data shown?

- A. 32  
 B. 110  
 C. 136.9  
 D. 142  
 E. 168.9

Building	Height (m)
Cairo Tower	187
Ministry of Foreign Affairs Building, Cairo	143
Nile City South Tower	142
El Maadi Residential Tower 16	140
Great Pyramid of Giza	138.8
National Bank of Egypt Tower 1	135
Residences Roda Island	118
Ramses Hilton & World Trade Center	110
Hilton World Trade Center Residences	110
Four Seasons Cairo at First Residence	103

$$[142 - 110] + \frac{138.8 + 135}{2}$$

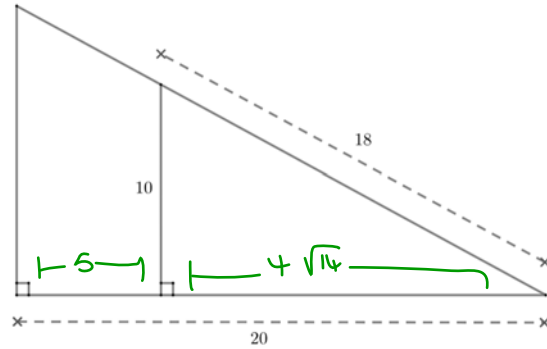
$\downarrow$  IQR                       $\downarrow$  Median  
 = 168.9

$Q_1$   
 $Q_2$   
 $Q_3$

$$\left(\frac{4\sqrt{14}}{20}\right)^2 = \frac{\frac{1}{2}(10)(4\sqrt{14})}{A}$$

34. The adjacent figure shows two similar triangles. What is the area of the larger triangle?

- A. 100 square units
- B. 120.27 square units
- C. 133.63 square units
- D. 240.53 square units
- E. 267.2 square units



35. Which of the following is not a solution of  $|-8 + 3x| - 2 < 3$ ?

- A.  $\frac{108}{25}$
- B.  $\frac{19}{5}$
- C.  $\frac{16}{5}$
- D. 2
- E.  $\frac{19}{20}$

$$\begin{aligned} |-8 + 3x| < 5 \\ -5 < -8 + 3x < 5 \\ 3 < 3x < 13 \\ 1 < x < \frac{13}{3} \end{aligned}$$

36. In a triangle  $TRD$ ,  $m\angle T = 66^\circ$  while  $\sec R = 1.887$ . What is the measure of angle  $D$ ?

- A.  $56^\circ$
- B.  $58^\circ$
- C.  $60^\circ$
- D.  $62^\circ$
- E.  $64^\circ$

$$\begin{aligned} \cos R &= \frac{1}{1.887} \\ 180 - 66 - 58 & \end{aligned}$$

37. The polynomial  $2x^3 - ax^2 - 7x + 6$  has no remainder when it is divided by  $x - 1$ . What is the value of  $a$ ?

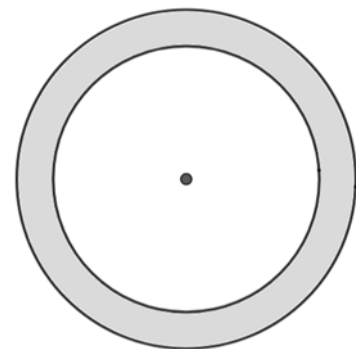
- A. -2
- B. -1
- C. 0
- D. 1
- E. 2

$$2 - a - 7 + 6 = 0$$

38. The area of the shaded region in the adjacent circle is equal to  $42.75\pi$  square units. The circumference of the smaller circle is equal to  $5\pi$  units. What is the radius of the larger circle?

- A. 14 units
- B. 7 units
- C. 6.25 units
- D. 5 units
- E. 2 units

$$\begin{aligned} \pi b^2 - \pi a^2 &= 42.75\pi \\ b^2 - a^2 &= 42.75 \\ b^2 - (2.5)^2 &= 42.75 \\ b &= 7 \end{aligned}$$



39. What is the equation of the line parallel to the line with equation  $\sqrt{2}x - 3y = 1$  and passing through the center of the circle with equation  $(x - 3\sqrt{2})^2 + (y^2 - 8y + 16) = 25$  ?

A.  $y = \frac{\sqrt{2}}{3}x + 2$   $(y-4)^2$

B.  $y = \frac{\sqrt{2}}{3}x - 2$

C.  $y = \frac{\sqrt{2}}{3}x$

D.  $y = -\frac{\sqrt{2}}{3}x + 2$

E.  $y = -\frac{\sqrt{2}}{3}x - 2$

$(3\sqrt{2}, 4)$   
 $\frac{\sqrt{2}}{3}x + C = y$

Slope  $\frac{\sqrt{2}}{3}$

40. In the adjacent figure,  $m\angle ABD = \frac{2}{3}m\angle CBE$ .

What is the value of  $x$  ?

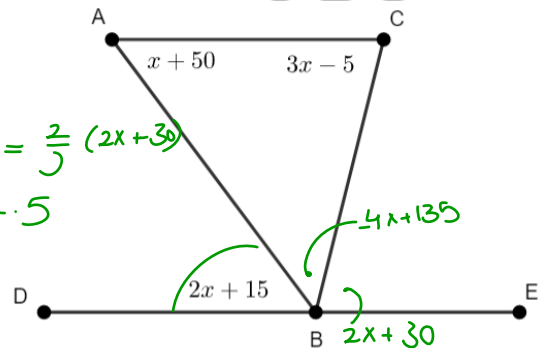
A. 3.75

B. 7.5

C. 15

D. 30.8

E. 42.5



41. Which of the following expressions is equivalent to  $\sqrt[3]{135x^5y^7} \cdot \sqrt{180x^3y}$  ?

A.  $18x^3y^2 \cdot \sqrt[6]{5^5y^4x}$

B.  $18x^2y^3 \cdot \sqrt[6]{5^5y^5x}$

C.  $18x^2y^3 \cdot \sqrt[6]{5^5y^4x}$

D.  $18x^3y^2 \cdot \sqrt[6]{5^5y^5x^2}$

E.  $18x^3y^2 \cdot \sqrt[6]{5^5y^5x}$

42. Given  $A = \begin{bmatrix} 3 & 1 \\ 0 & 5 \\ 2 & 4 \\ 3 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} -2 & 4 \\ 5 & 3 \\ 9 & 6 \\ 3 & -2 \end{bmatrix}$ . What is the matrix representing  $A + 2B$  ?

A.  $\begin{bmatrix} -1 & 9 \\ 10 & 11 \\ 20 & 16 \\ 9 & 0 \end{bmatrix}$

$3 + 2(-2) = -1$

B.  $\begin{bmatrix} 1 & -3 \\ 5 & 8 \\ 11 & 10 \\ 6 & 2 \end{bmatrix}$

C.  $\begin{bmatrix} 7 & -1 \\ 11 & 10 \\ 16 & 20 \\ 0 & 9 \end{bmatrix}$

D.  $\begin{bmatrix} 7 & -7 \\ -10 & -1 \\ -16 & -8 \\ -3 & 8 \end{bmatrix}$

E.  $\begin{bmatrix} 9 & 0 \\ 20 & 16 \\ 10 & 11 \\ -1 & 7 \end{bmatrix}$

43. Given  $A(2, 3, -7)$  and  $B(3m, -2m + 1, 6)$ . What is the value of  $m$  if  $AB^2 = 221$  ?

A. -2

B. 0

C. 1

D. 2

E. 4

$(3m-2)^2 + (-2m+1-3)^2 + (6+7)^2 = 221$   
 $m=2$

44. If  $f(x) = 2 \cdot 3^x + 2$ , which of the following statements is correct?

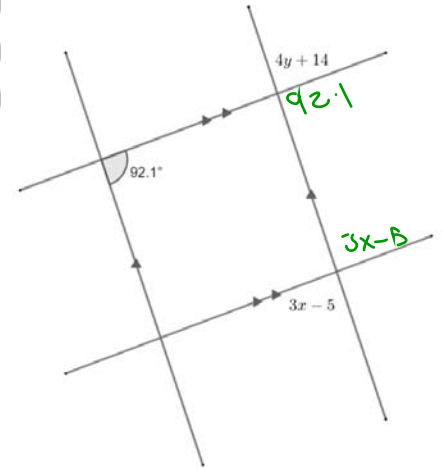
- A. The graph of  $f(x)$  is increasing and the equation of the vertical asymptote is  $x = 2$ .
- B. The graph of  $f(x)$  is increasing and the equation of the horizontal asymptote is  $y = 2$ .**
- C. The graph of  $f(x)$  is decreasing and the equation of the horizontal asymptote is  $y = 2$ .
- D. The graph of  $f(x)$  is decreasing and the equation of the vertical asymptote is  $x = 2$ .
- E. The graph of  $f(x)$  is increasing and there are no asymptotes.

45. Given  $f(x) = 3x^2 - 6x + 1$ . What is the product of the coordinates of the vertex of this function?

- A. -10
  - B. -1
  - C. -2**
  - D. 1
  - E. 2
- $x = \frac{-b}{2a} = 1$   
 $y = 3 - 6 + 1 = -2$   
 $(1)(-2) = -2$

46. Using the adjacent diagram, what is the value of  $\frac{2x}{y}$ ?

- A. 0.6
  - B. 0.83
  - C. 1.19
  - D. 1.67**
  - E. 3.35**
- $x = \frac{92.9}{10}$   
 $y = \frac{73.9}{40}$   
 $\frac{2x}{y} = 3.35$



47. If  $21x - i + 8y = 7xi + 2y - 1$ , what real values of  $x$  and  $y$  satisfy the equation?

- A.  $x = -\frac{1}{7}$  and  $y = \frac{1}{3}$**
  - B.  $x = \frac{1}{7}$  and  $y = -\frac{1}{3}$
  - C.  $x = -\frac{1}{7}$  and  $y = -\frac{1}{3}$
  - D.  $x = -7$  and  $y = 3$
  - E.  $x = \frac{1}{7}$  and  $y = \frac{1}{3}$
- $7x = -1, x = -\frac{1}{7}$   
 $21x + 8y = 2y - 1$   
 $y = \frac{1}{3}$

48. Which of the following expressions can never have a negative answer for any  $x$ ?

- A.  $|2x - 3(x - 5)| - 1$
- B.  $\frac{x^2 - x - 6}{x^4}$
- C.  $(-2|x - 3|)^2 - 3$
- D.  $\frac{x^2 + 3}{(x - 7)^2}$**
- E.  $(x - 1)^3$

49. Ahmad bought 7 pencils and 11 pens from a shop for \$11.2. The price of 5 pens is equal to the price of 7 pencils. What is the price of 3 pens?

- A. \$1.0  
 B. \$1.4  
 C. \$1.5  
 D. \$2.1  
 E. \$3.5

$$7\left(\frac{5y}{7}\right) + 11(v) = 11.2$$

$$y = 0.7 \quad \left| \begin{array}{l} 5y = 7x \\ x = \frac{5y}{7} \end{array} \right.$$

$$3(0.7) = 2.1$$

50. A group of 7 persons is to be chosen from 5 men and 4 women. If at least 3 women should be included, in how many ways can this be done?

- A. 4320 ways  
 B. 30 ways  
 C. 24 ways  
 D. 20 ways  
 E. 10 ways

$$5C_4 \times 4C_3$$

$$+ \frac{5C_3 \times 4C_4}{= 30}$$

Dr. Ahmed Hassan

THE FORMULAS BELOW MAY BE USEFUL IN ANSWERING QUESTIONS ON THIS TEST.

$S = 4\pi r^2$  is the formula for the surface area of a sphere with a radius of  $r$ .

$V = \frac{1}{3}\pi r^2 h$  is the formula for a right circular cone with a radius of  $r$  and a height of  $h$ .

$V = \frac{4}{3}\pi r^3$  is the formula for a sphere with a radius of  $r$ .

$V = \frac{1}{3}Bh$  is the formula for a pyramid with a base area of  $B$  and a height of  $h$ .

Dr. Ahmed Hassan

**EST II - Individual Subject Test**  
**Mathematics - Level 1 - Answer Key**

Question	Answer	Question	Answer
1.	D	26.	E
2.	B	27.	D
3.	A	28.	B
4.	C	29.	C
5.	E	30.	B
6.	A	31.	D
7.	C	32.	C
8.	B	33.	E
9.	A	34.	C
10.	D	35.	E
11.	C	36.	A
12.	D	37.	D
13.	B	38.	B
14.	C	39.	A
15.	C	40.	B
16.	E	41.	E
17.	D	42.	A
18.	C	43.	D
19.	B	44.	B
20.	D	45.	C
21.	A	46.	E
22.	D	47.	A
23.	B	48.	D
24.	A	49.	D
25.	E	50.	B





**EGYPTIAN  
SCHOLASTIC  
TEST**

## **EST II – Individual Subject Test**

### **Level 1**

**Student's Name** \_\_\_\_\_

**National ID** \_\_\_\_\_

**Test Center:** \_\_\_\_\_

**Subject: Math**

**Duration: 60 minutes**

**50 Multiple Choice Questions**

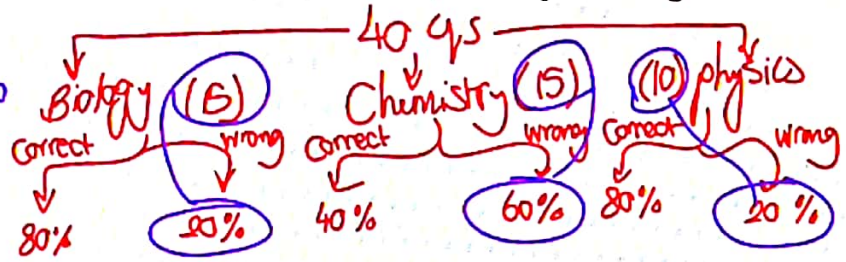
**Instructions:**

- Place your answer on the answer sheet. Mark only one answer for each of the multiple choice questions.
- Avoid guessing. Your answers should reflect your overall understanding of the subject matter.
- Calculator is allowed. When a calculator is used, be aware of switching between radian mode and median mode.
- Formula sheet is available at the end of the booklet for your reference.



6. On a science exam of 40 multiple choice questions, Elvis got 80% of the 15 biology questions correct, 60% of the 15 chemistry questions wrong and 20% of the 10 physics questions wrong. Knowing that Elvis answered all the questions, what percentage of all the questions did he get wrong?

- A. 35%  
 B. 65%  
 C. 50%  
 D. 72.5%  
 E. 54%



7. What is the value of  $x$  in the equality  $2ix(i-1) = (i-1)^2 - 2$  knowing that  $i = \sqrt{-1}$ ?

- A. -2  
 B. -1  
 C. 0  
 D. 2  
 E. 1

$$2i^2x - 2ix = -2 - 2i$$

$$2x = 2, x = 1$$

8. If  $|x - 2| < 4$ , then  $|x - 3|$  is less than

- A. 4  
 B. 2  
 C. 3  
 D. 5  
 E. 0

$$-4 < x - 2 < 4$$

$$-2 < x < 6 \quad | -5 > |3|$$

$$-5 < x - 3 < 3$$

$$0 < |x - 3| < 5$$

9. A bicycle store finds that  $N$ , the number of bikes sold, is related to  $d$ , the number of dollars spent on advertising by the relation  $N = 51 + 10 \ln\left(\frac{d}{10} + 2\right)$ .

If the average profit is \$35 per bike, is it worthwhile to spend \$1000 on advertising?

- A. No, because the profit will be less than the amount spent on advertising.  
 B. No, because the profit will be \$97.  
 C. Yes, because advertisement is important.  
 D. Yes, because the profit is bigger than \$1000.  
 E. No, because the advertisement amount is big with respect to the product being advertised.

$$N = 51 + 10 \ln\left(\frac{1000}{10} + 2\right)$$

$$= 97 \cdot 2$$

$$97 \times 35 = 3395$$

$$3395 > \frac{1000}{\text{profit}}$$

10. Using the two tables below, what is  $(g \circ f)(-2)$ ?

$x$	$f(x)$
-2	-1
4	-2
5	4

$x$	$g(x)$
5	4
-1	2
3	6

$$f(-2) = -1$$
$$g(-1) = 2$$

- A. 3
- B. 5
- C. -2
- D. -1
- E. 2**

11. The value, in US dollars, of a car is given by  $V = \beta e^{-\alpha t}$  where  $t$  represents the age of the car in years and  $\beta$  and  $\alpha$  are real numbers. The initial price of the car was \$8000, and its price after one year becomes \$6000. Then the value of  $\alpha$  is:

- A.  $\ln\left(\frac{1}{4}\right)$
- B.  $\ln\left(\frac{1}{3}\right)$
- C.  $\ln\left(\frac{2}{3}\right)$
- D.  $\ln\left(\frac{3}{4}\right)$
- E.  $\ln\left(\frac{4}{3}\right)$**

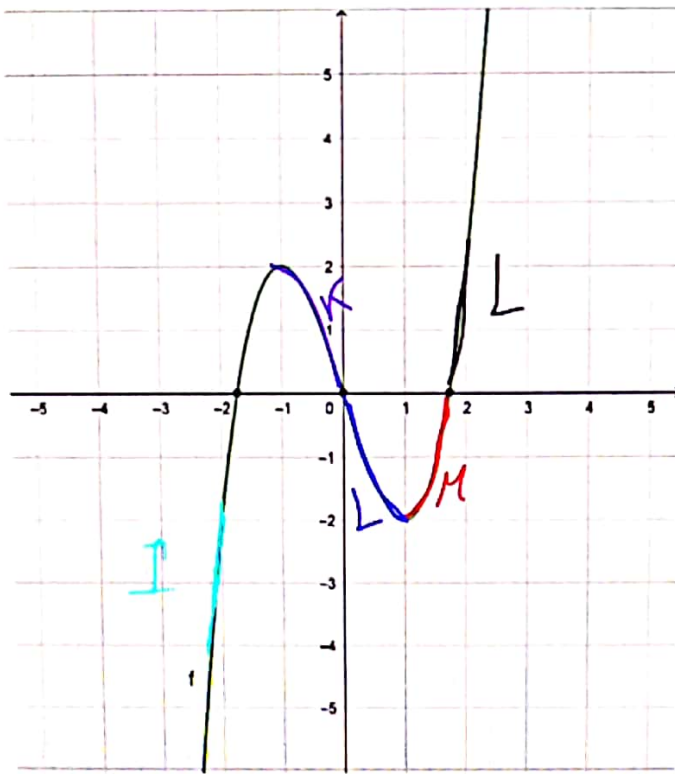
$$6000 = 8000 e^{-\alpha t}$$

$$\frac{3}{4} = e^{-\alpha t}$$

$$e^{\alpha} = \frac{4}{3}$$

$$\ln \alpha = \frac{4}{3}$$

$$\alpha = \ln\left(\frac{4}{3}\right)$$



12. The figure above represents a function  $f$  that cuts the  $x$ -axis at three points of abscissas  $-\sqrt{3}$ ,  $0$  and  $\sqrt{3}$ .

Consider the following intervals:  $I = ]-2, -\sqrt{3}[$ ,  $K = ]-1, 0[$ ,  $L = ]0, 1[$ ,  $M = ]1, \sqrt{3}[$  and  $N = ]\sqrt{3}, 2[$ .

In what interval is the function  $f$  considered to be **positive and decreasing function**?

- A. I
- B. K**
- C. L
- D. M
- E. N

13. Consider  $P(x) = ax^2 + bx + c$  ( $a \neq 0$ ), a polynomial of second degree in which

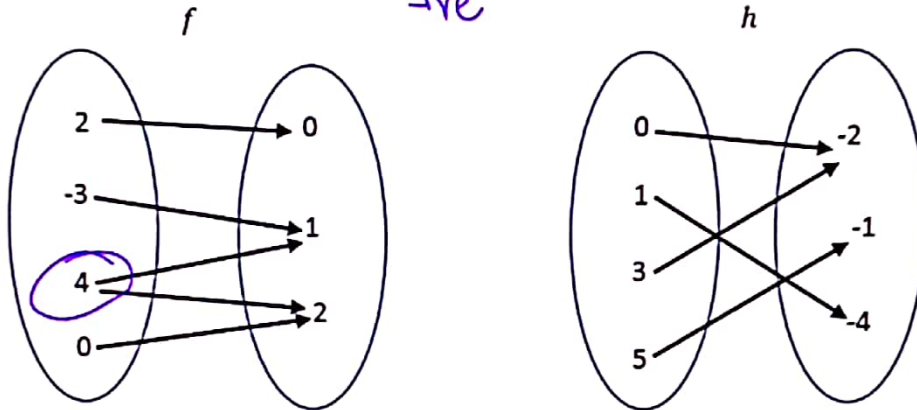
$P(0) = 1$ .  $C = 1$

If  $P(x+1) - P(x) = 4x$ , what are the values of  $a, b$  and  $c$ ?

- ~~A.~~  $a = 2, b = 2, c = 0$   $p(1) - p(0) = 4 \times 0$
- ~~B.~~  $a = 2, b = 2, c = 1$
- ~~C.~~  $a = 1, b = 2, c = 1$   $p(1) - 1 = 0$
- ~~D.~~  $a = -2, b = -2, c = 1$   $p(1) = 1$
- E.  $a = 2, b = -2, c = 1$   $a + b = 0$

14. If  $m$  is a real parameter, what is the number of the real roots of the equation  $x(x^2 + 1)(mx^2 - x - 2m) = 0$ ?

- A. 1
  - B. 2
  - C. 3
  - D. 0
  - E. 5
- $x = 0 \rightarrow 1$  solution  
 $(x^2 + 1) = 0, x^2 = -1 \rightarrow$  zero solution  
 $mx^2 - x - 2m$   
 $b^2 - 4ac \Rightarrow (x)^2 - 8m^2 \rightarrow 2$  solution  
 +ve



15. Which of the two relations above is a function and why?

- A.  $f$  because input 4 has two images
- B.  $f$  because each input has exactly one output
- C.  $h$  because input  $-2$  has two pre-images
- D.  $f$  or  $h$  because each relation is a function
- E.  $h$  because each input has only one image

16. Consider the functions below:

$$f(x) = 2x^3 - 4x + 1$$

$$g(x) = 3x^{0.5} + 3x^{-2} + 3$$

$$h(x) = -\sqrt{3}x^2 + 3x$$

$$k(x) = \frac{1}{3}x^2 + \frac{2}{x}$$

Which of these functions is a polynomial function?

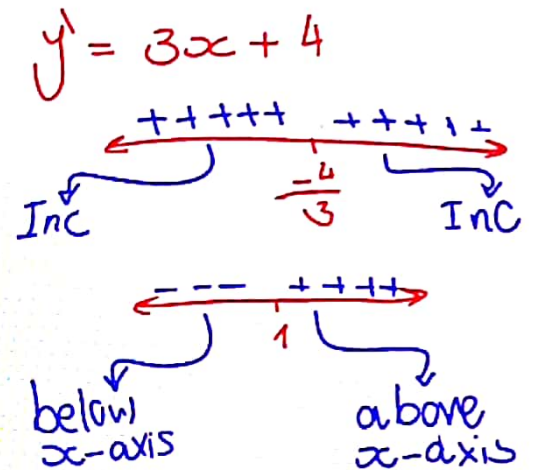
- A.  $f$  and  $g$
- B.  $h$  and  $g$
- C.  $f$  and  $h$**
- D.  $f, h$  and  $g$
- E.  $f, g, h$  and  $k$

17. In the  $xy$ -plane the function  $f$  defined by  $f(x) = x^3 + 4x - 5$  admits a graph (C). A computer program generated 5 graphs of which one is correct. The graphs are characterized by:

Graph 1	It admits 3 vertices.
Graph 2	It admits 2 maxima and 1 minimum.
Graph 3	It is strictly decreasing.
Graph 4	It is strictly increasing.
Graph 5	It is always positive.

Which graph is correct?

- A. Graph 1
- B. Graph 2
- C. Graph 3
- D. Graph 4**
- E. Graph 5



18. If  $f(x) = \sqrt[3]{x^3 + 3} + 3$  and  $f^{-1}(x) = \sqrt[3]{x-3}$  what is the value of  $x$ ?

- A. -3
- B. -2
- C. 0
- D. 3**
- E. 2

Handwritten work for Q18:  $x = \sqrt[3]{x-3}$  is written above the inverse function. An arrow points from this equation to the function definition. Below, the calculation is shown:  $\sqrt[3]{(-3)+3} + 3 = 3$ .

19. If  $(x + \frac{1}{x})^2 = 2$  what is the value of  $x^2 + \frac{1}{x^2}$ ?

- A. 4
- B. 8
- C. 2**
- D. 0.5
- E. 16

Handwritten work for Q19:  $(x + \frac{1}{x})^2 = 2$  is written above the question. Below, the expansion is shown:  $x^2 + 2 + \frac{1}{x^2} = 2$ , which simplifies to  $x^2 + \frac{1}{x^2} = 2$ .

20. If  $\log_2 16 = x + y$  and  $\log_x 4 = 2$ , then what is  $\log_2 y$ ?

- A. 0
- B. 1**
- C. 2
- D. 3
- E. 4

$(x+y)^2 = 16$   
 $x^2 = 4$   
 $x+y = 4$   
 $x = 2$   
 $y = 2$   
 $\log_2 2 = 1$

21. What is the domain of definition of  $\arcsin(3 - 4x)$ ?

- A.  $-\frac{1}{2} \leq x \leq \frac{1}{2}$
- B.  $-\frac{1}{2} < x \leq 1$
- C.  $\frac{1}{2} \leq x \leq 1$**
- D.  $-1 \leq x \leq -\frac{1}{2}$
- E.  $-1 \leq x \leq 1$

$-1 \leq 3 - 4x \leq 1$   
 $\frac{1}{2} \leq x \leq 1$

22. What is the parity and periodicity of function  $f$  defined by  $f(x) = 2 \sin\left(\frac{x}{2}\right)$ ?

- A. even and period  $4\pi$
- B. odd and period  $4\pi$**
- C. even and period  $\pi$
- D. odd and period  $\pi$
- E. odd and period  $2\pi$

$\sin$  is odd  
 period =  $\frac{2\pi}{1/2} = 4\pi$

23.  $\ln(x^2 - 2x + 2) > 0$  is verified in:

- A.  $]-\infty, +\infty[$
- B.  $]0, +\infty[$
- C.  $]0, 1]$
- D.  $]-\infty, \frac{-1}{2}[ \cup ]0, +\infty[$
- E.  $]-\infty, 1[ \cup ]1, +\infty[$**

$e^{x^2 - 2x + 2} > e^0$   
 $x^2 - 2x + 2 > 0$   
 $x^2 + 2x + 2 > 0$

$f(x) = \begin{cases} -8x - m & \text{for } x \leq 2 \\ 10x - 3 & \text{for } x > 2 \end{cases}$

24. For what value of  $m$ , if any, is the function  $f$  above continuous at  $x = 2$ ?

- A. 33
- B. 17
- C. 23
- D. -33**
- E. Doesn't exist

$f(2)^- = f(2)^+ = f(2)$   
 $10(2) - 3 = -8(2) - m$   
 $m = -33$



25. If  $x^2 = \sqrt{-2y - 5}$  and  $z = -8y^3$ , what is  $x$  in terms of  $z$ ?

A.  $x = \sqrt[3]{z} - 5$

B.  $x = \pm \sqrt{\sqrt[3]{z} - 5}$

C.  $x = \pm (\sqrt[3]{z} - 5)$

D.  $x = \pm \sqrt[6]{z} - \sqrt{5}$

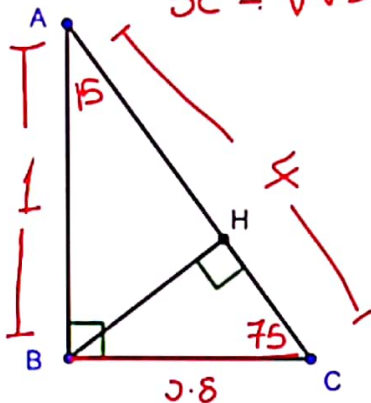
E.  $x = (\sqrt[3]{z} - 5)^2$

$x = \sqrt{-2y - 5}$

$y = \sqrt[3]{\frac{z}{-8}}$

$x = \sqrt{-2\left(\frac{\sqrt[3]{z}}{-8}\right) - 5}$

$x = \sqrt{\sqrt[3]{z} - 5}$



26. Triangle  $ABC$  is right angled at  $B$  such that  $\hat{BAC} = \theta$  and  $H$  is the foot of the altitude drawn from  $B$  to  $[AC]$ . If  $AC = 4$  and  $\theta = 15^\circ$  then  $BH =$

(The figure is not drawn to scale)

A.  $\frac{1}{3}$

B.  $\frac{1}{2}$

C. 1

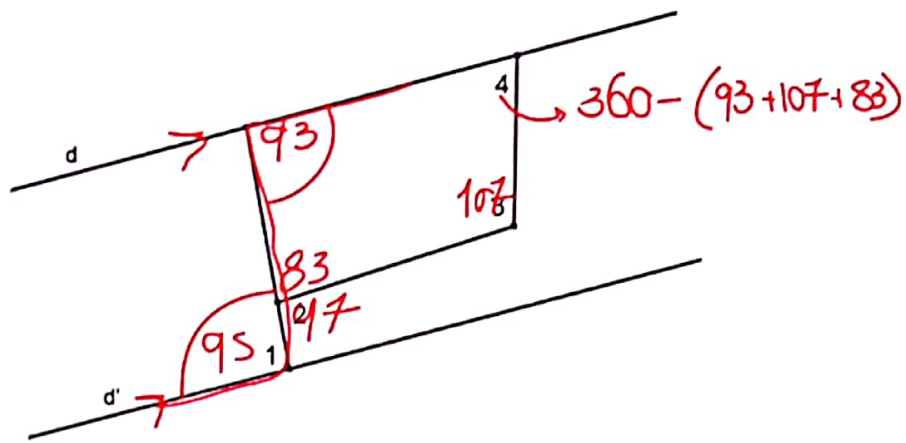
D.  $\frac{2}{3}$

E. 2

$\sin 15 = \frac{BC}{4}$        $\sin 75 = \frac{AB}{4}$   
 $\frac{\sin 15}{BC} = \frac{1}{4}$        $\frac{\sin 75}{AB} = \frac{1}{4}$

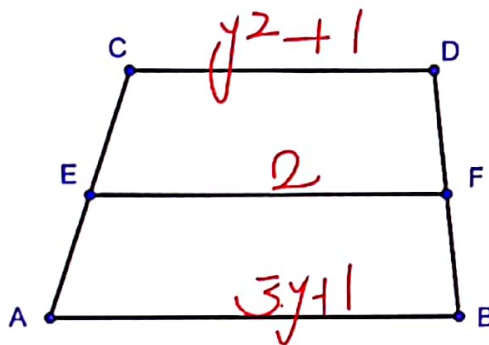
$\frac{\sin 15}{BC} = \frac{\sin 75}{AB} = \frac{1}{4}$

$BC = 1$  ,  $AB = 3.84$



27. In the figure above,  $d$  and  $d'$  are parallel lines,  $\angle 1 \cong 93^\circ$ ,  $\angle 3 = 107^\circ$  and  $\angle 2 = 97^\circ$ . What is the measure of angle 4? (The figure is not drawn to scale)

- A.  $77^\circ$
- B.  $63^\circ$
- C.  $73^\circ$
- D.  $83^\circ$
- E.  $67^\circ$



28. In the trapezoid above,  $CD = y^2 + 1$ ,  $AB = 3y - 1$  and  $EF = 2$ . If  $E$  and  $F$  are the midpoints of  $\overline{AC}$  and  $\overline{BD}$ , what is the value of  $y$ ? (The figure is not drawn to scale)

- A.  $-4$
- B.  $1$
- C.  $4$
- D.  $1$  or  $-4$
- E.  $1$  or  $4$

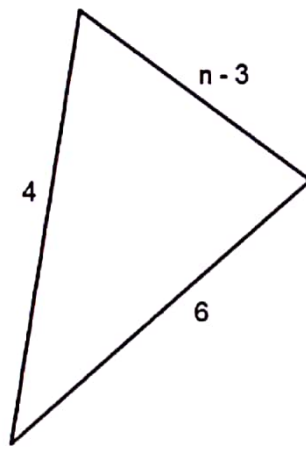
$$\frac{y^2 + 1 + 3y - 1}{2} = 2$$

$$y^2 + 3y - 4 = 0$$

$$y = 1 \text{ or } -4$$

$$2 < n-3 < 10$$

$$5 < n < 13$$

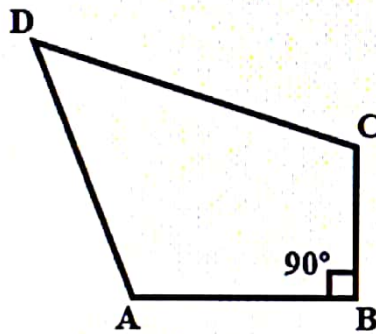
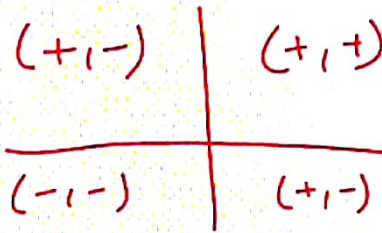


29. Which of the following numbers 2, 4, 8, 10 and 14 cannot be a value of  $n$  in the triangle above? (The figure is not drawn to scale)

- A. 14
- B. 2 and 4
- C. 2 and 14
- D. 2, 4 and 14**
- E. 8, 10 and 14

30. In the  $xy$ -plane, what quadrant(s) could point A lie in, if its  $x$ -coordinate and  $y$ -coordinate have opposite signs?

- A. Quadrant I
- B. Quadrant II
- C. Quadrants II and III
- D. Quadrants II and IV**
- E. Quadrants III and IV



31. In the quadrilateral ABCD above,  $\angle C - \angle D = 60^\circ$  and  $\angle A - \angle C - \angle D = 10^\circ$ . Find  $\angle A$ ,  $\angle C$  and  $\angle D$ . (The figure is not drawn to scale)

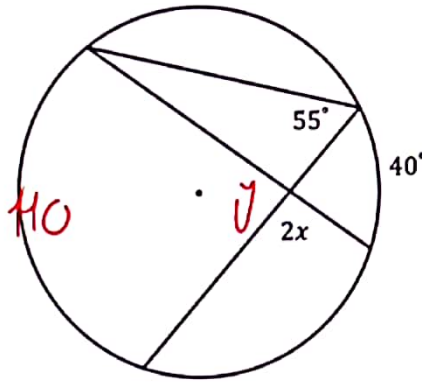
- A.  $\angle A \cong 140^\circ$  ;  $\angle C \cong 95^\circ$  ;  $\angle D \cong 35^\circ$**
- B.  $\angle A = 120^\circ$  ;  $\angle C = 85^\circ$  ;  $\angle D = 25^\circ$
- C.  $\angle A = 95^\circ$  ;  $\angle C = 140^\circ$  ;  $\angle D = 80^\circ$
- D.  $\angle A = 120^\circ$  ;  $\angle C = 115^\circ$  ;  $\angle D = 35^\circ$
- E.  $\angle A = 140^\circ$  ;  $\angle C = 35^\circ$  ;  $\angle D = 95^\circ$

$$\left. \begin{array}{l} A + C + D = 360 \\ 0 + C - D = 60 \\ A - C - D = 10 \end{array} \right\} \text{solve}$$

$$\frac{110 + 40}{2} = y$$

$$x = 52.5$$

$$x/2 = 26.25$$



32. In the figure above, what is the value of  $\frac{x}{2}$ ?

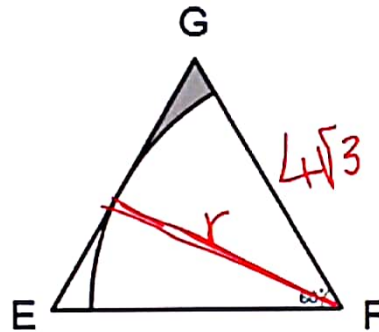
A.  $50^\circ$

B.  $66.25^\circ$

C.  $105^\circ$

D.  $52.5^\circ$

E.  $26.25^\circ$



$$r = h$$

33. A circular region centered at F is inscribed in equilateral triangle EFG as shown above. If the area of the triangle EFG is  $12\sqrt{3}$ , what is the approximate area of the shaded region?

A. 0.97

B. 18.85

C. 6.93

D. 1.94

E. 2.17

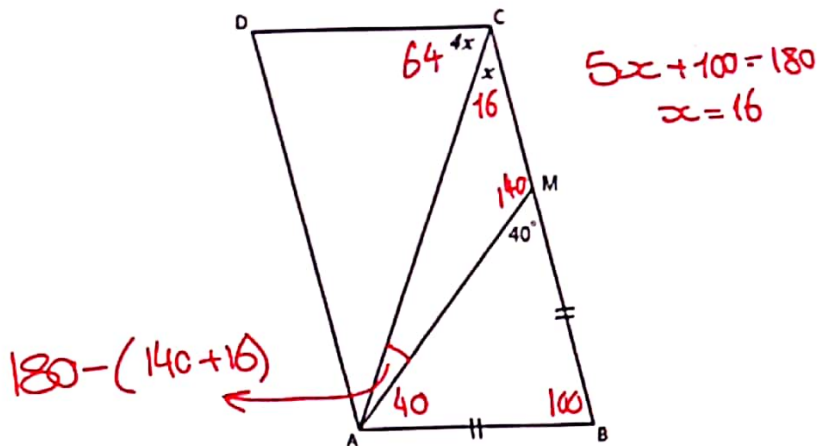
$$\left(\frac{s^2}{4}\right)\sqrt{3} = 12\sqrt{3}$$

$$s = 4\sqrt{3}$$

$$h = 6$$

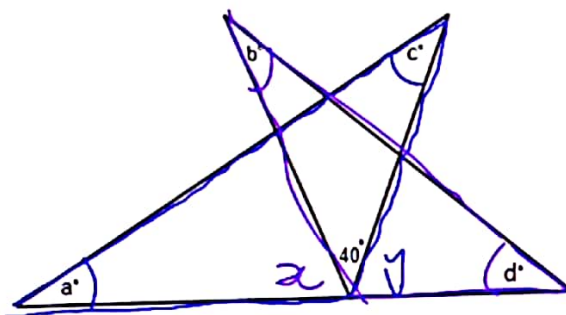
$$\frac{1}{2} (4\sqrt{3}) \times 6 = 12\sqrt{3} - \frac{30}{360} \times (6)^2 \pi$$

$$= \frac{12\sqrt{3} - 6}{2}$$



34. In the figure above, M is a point on side BC of parallelogram ABCD. What is the measure of  $\angle CAM$ ? (The figure is not drawn to scale)

- A.  $16^\circ$
- B.  $24^\circ$**
- C.  $64^\circ$
- D.  $104^\circ$
- E.  $20^\circ$



35. What is the measure of  $a + b + c + d$  in the figure shown above? (The figure is not drawn to scale)

- A. 140**
- B. 120
- C. 160
- D. 320
- E. Can't be calculated

$$a + c = y, \quad b + d = x$$

$$x + y + 40 = 180, \quad x + y = 140$$

36. Consider the two lines D and L of respective equations  $2x - y - 1 = 0$  and  $mx + (m - 1)y = 0$  where  $m$  is real number. For what value of  $m$ , line D is parallel to line L?

- A. -1
- B. 2
- C.  $\frac{3}{2}$
- D.  $\frac{2}{3}$**
- E. -2

$$2x - y - 1 = 0$$

$$m(x) + (m - 1)y = 0$$

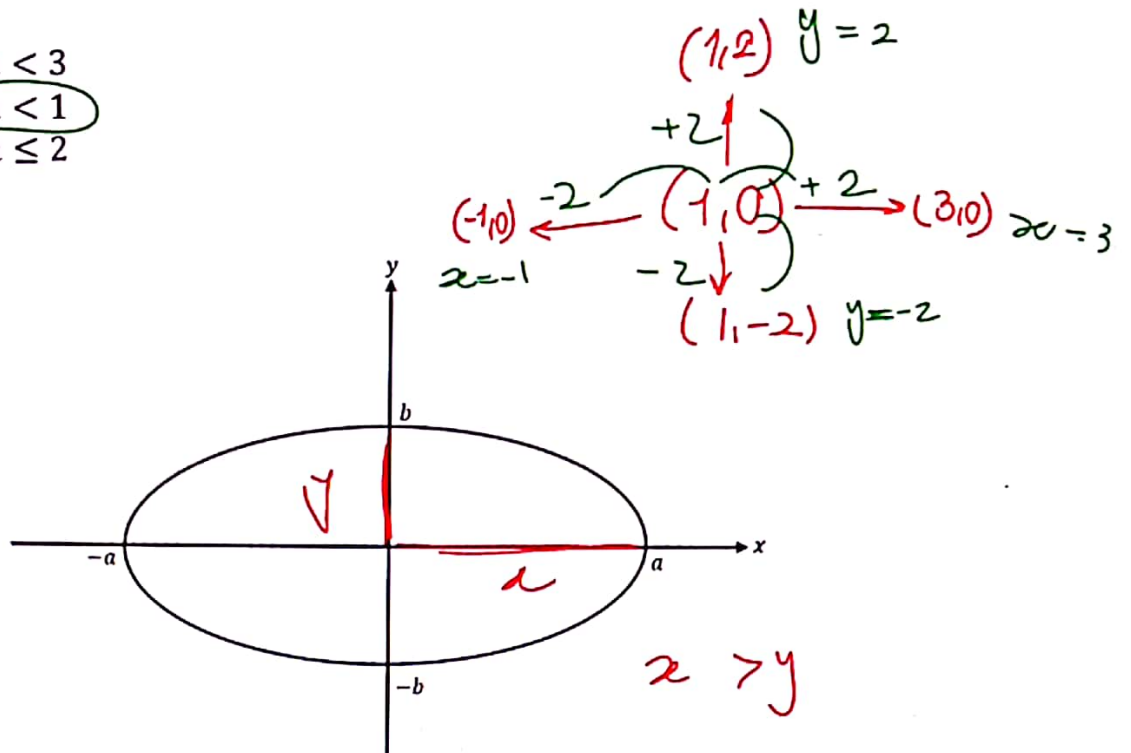
$$\frac{-2}{-1} = \frac{2}{1} \neq \frac{-m}{m-1}$$

$$-m = 2m - 1, \quad m = \frac{2}{3}$$

Same slope ←

37. In the  $xy$  - plane, (C) is a circle of center A(1, 0) and radius 2. (d) is the line of equation  $x + m = 0$ . For what values of  $m$  does the line (d) cut circle (C) in two distinct points?

- A.  $m < 2$
- B.  $-1 < m < 3$
- C.  $-3 < m < 1$**
- D.  $-2 \leq m \leq 2$
- E.  $m > 3$



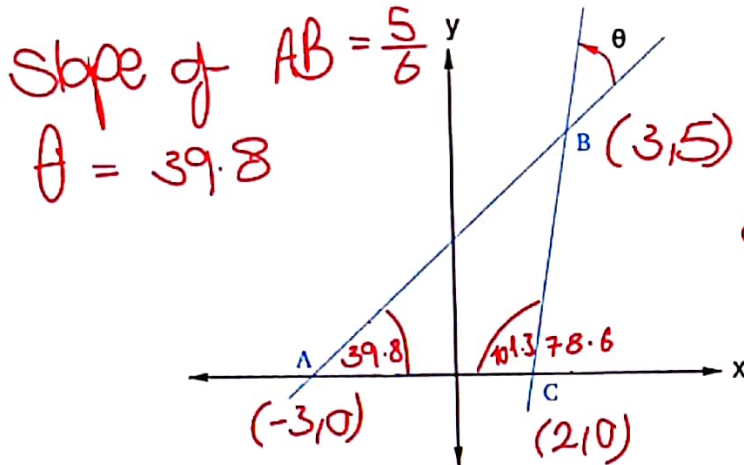
38. Knowing that the area of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $A = \pi ab$ , what is the equation of the ellipse if  $a + b = 12$  and  $A = 35\pi$ ?

- A.  $\frac{x^2}{25} + \frac{y^2}{49} = 1$
- B.  $\frac{x^2}{8^2} + \frac{y^2}{4^2} = 1$
- C.  $\frac{x^2}{7^2} + \frac{y^2}{5^2} = 1$**
- D.  $\frac{x^2}{35^2} + \frac{y^2}{12^2} = 1$
- E.  $\frac{x^2}{7^2} - \frac{y^2}{5^2} = 1$

Handwritten notes for problem 38:

- $ab = 35$
- $a + b = 12$

39. In the  $xy$ -plane, consider the points  $A(-3, 0)$ ,  $B(3, 5)$  and  $C(2, 0)$  as shown in the figure below. What is the approximate measure, in degrees, of the angle  $\theta$ ?



- A.  $39^\circ$   
 B.  $40^\circ$   
 C.  $62^\circ$   
 D.  $79^\circ$   
 E.  $28^\circ$

40. The point  $K(1, -3)$  is rotated about the origin through an angle of  $90^\circ$  in an anti-clockwise direction. What are the coordinates of the image of  $K$ ?

- A.  $(-1, -3)$   
 B.  $(3, 1)$   
 C.  $(-3, -1)$   
 D.  $(-3, 1)$   
 E.  $(1, 3)$

Clockwise  $270$   
 $\rightarrow (1, -3) \Rightarrow (3, 1)$

41. What is the nature of triangle  $ABC$  if  $A(2, 3, 1)$ ,  $B(-1, 3, 0)$  and  $C(2, 3, -1)$ ?

- A. Isosceles  
 B. Right  
 C. Right isosceles  
 D. Equilateral  
 E. Scalene

$AB = \sqrt{10}$   
 $BC = \sqrt{10}$   
 $AC = 2$

42. A hollow cube of internal edge 20 cm is filled with identical spherical marbles of diameter 0.6 cm each, and it is assumed that  $\frac{1}{8}$ th space of the cube remains unfilled. What is the number of marbles, rounded to the nearest integer, that the cube can accommodate?

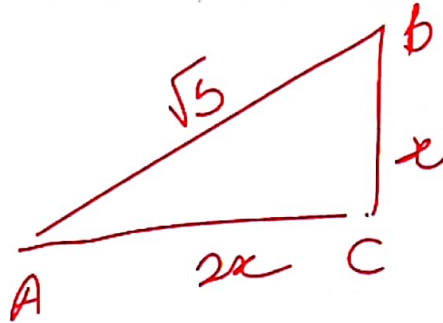
- A. 70,000  
 B. 61,894  
 C. 70,736  
 D. 7,737  
 E. 65,235

$\frac{7}{8} \times \frac{(20)^3}{\frac{4}{3} (\pi) (0.3)^3}$

43. Triangle  $ABC$  is right at  $C$ . If  $BC = x$ ,  $AC = 2x$  where  $x$  is a positive real number and angle  $B\hat{A}C = \theta$ , then  $\sin 2\theta =$

- A.  $\frac{1}{5}$
- B.  $\frac{2}{5}$
- C.  $\frac{3}{5}$
- D.  $\frac{4}{5}$
- E. 1

Handwritten notes for Q43:  
 $2 \cos \theta \sin \theta$   
 $2 \left( \frac{2}{\sqrt{5}} \right) \left( \frac{1}{\sqrt{5}} \right)$



44. Which could be a value of  $\sin \theta$  if  $\frac{7\pi}{4} < \theta < 2\pi$ ?

- A.  $-\frac{4}{5}$
- B. -1
- C.  $-\frac{1}{2}$
- D.  $-\frac{1}{4}$
- E.  $\frac{1}{2}$

45. If  $\cos\left(\frac{\pi}{2} - \theta\right) = \frac{2}{\sqrt{3}}$ , then what is the value of  $\frac{\sin \theta}{\csc \theta}$ ?  $= \sin^2 \theta$

- A.  $\frac{2}{\sqrt{3}}$
- B. 1
- C.  $\frac{4}{3}$
- D.  $\frac{\sqrt{3}}{2}$
- E.  $\frac{3}{4}$

Handwritten notes for Q45:  
 $\sin \theta = \frac{2}{\sqrt{3}}$   
 $\left(\frac{2}{\sqrt{3}}\right)^2$

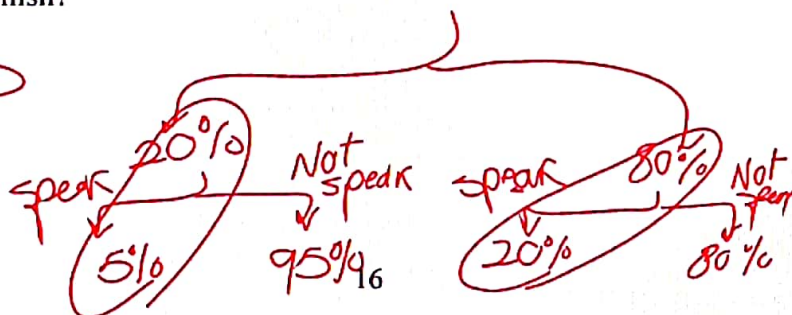
46. If the mean of a normal distribution is 55 and the standard deviation is 5, then almost all of the scores are likely to fall between:

- A. 40 and 70
- B. 50 and 60
- C. 30 and 55
- D. 55 and 80
- E. 60 and 90

47. A survey is done in an enterprise having 20% administrators and 80% employees. We know that 5% of the administrators and 20% of the employees speak Spanish.

A person is chosen randomly from the enterprise, what is the probability that he/she speaks Spanish?

- A. 0.26
- B. 0.17
- C. 0.25
- D. 0.08
- E. 0.48





48. A box contains 24 marbles of red, green and yellow colors. If two marbles are drawn randomly and simultaneously from the box, the probability that both are red is  $\frac{15}{92}$ . What is the number of the **non-red** marbles?

- A. 10
- B. 16
- C. 14**
- D. 20
- E. 18

↓  
 $24 - 10 = 14$

$$\frac{15}{92} = \frac{R}{24} \times \frac{R-1}{23}$$

$$R = 10$$

49. Which of the numbers below is not the mean, median, mode or range of the data set 5, 2, 4, 5, 10, 16, 12, 18, 15, 13?

- A. 10
- B. 11
- C. 5
- D. 8**
- E. 16

$2, 4, 5, 5, 10, 12, 13, 15, 16, 18$

Mean = 10    Median = 11  
 Mode = 5  
 Range = 16

50. A regression analysis between sales (Y) and advertising (X) across all the branches in Egypt of a major company resulted in the following equation  $Y = 40,000 + 6.25 X$ .

If the advertising budgets of the two branches in Giza and Alexandria **differ** by 200,000 EGP, then what will be the predicted difference in their sales?

- A. 12.5 EGP
- B. 80,000 EGP
- C. 160,000 EGP
- D. 250,000 EGP
- E. 1,250,000 EGP**

$$6.25 \times 200,000 = 1,250,000$$

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