3 3

Math Test—No Calculator

25 MINUTES, 20 QUESTIONS

Turn to Section 3 of your answer sheet to answer the questions in this section.

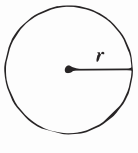
DIRECTIONS

For questions 1–15, solve each problem, choose the best answer from the choices provided, and fill in the corresponding circle on your answer sheet. **For questions 16–20**, solve the problem and enter your answer in the grid on the answer sheet. Please refer to the directions before question 16 on how to enter your answers in the grid. You may use any available space in your test booklet for scratch work.

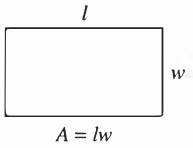
NOTES

1. The use of a calculator **is not permitted**.
2. All variables and expressions used represent real numbers unless otherwise indicated.
3. Figures provided in this test are drawn to scale unless otherwise indicated.
4. All figures lie in a plane unless otherwise indicated.
5. Unless otherwise indicated, the domain of a given function f is the set of all real numbers x for which $f(x)$ is a real number.

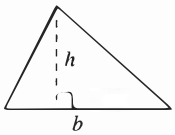
REFERENCE



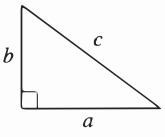
$A = \pi r^2$
$C = 2\pi r$



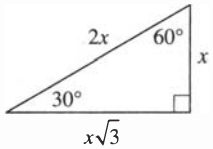
$A = lw$

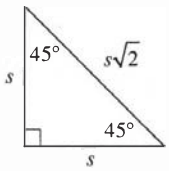


$A = \frac{1}{2}bh$

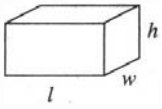


$c^2 = a^2 + b^2$

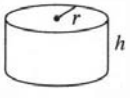




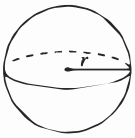
Special Right Triangles



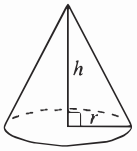
$V = lwh$



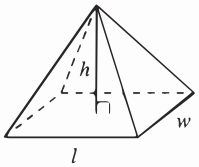
$V = \pi r^2 h$



$V = \frac{4}{3}\pi r^3$



$V = \frac{1}{3}\pi r^2 h$



$V = \frac{1}{3}lwh$

The number of degrees of arc in a circle is 360.
The number of radians of arc in a circle is 2π.
The sum of the measures in degrees of the angles of a triangle is 180.

CONTINUE

3 3

1

If $\frac{3}{4}a+\frac{2}{3}b=7$, what is the value of $9a+8b$?

A) 64

B) 72

C) 84

D) 96

2

Raj can exchange 15 Euros for 11 British Pounds. At this exchange rate, approximately how many British Pounds should he receive in exchange for 100 Euros?

A) 21

B) 73

C) 137

D) 340

3

If $ax+b=cx+d$, which of the following gives the value of x in terms of a and b?

A) $x=\frac{c}{a}+d-b$

B) $x=c+d-\frac{b}{a}$

C) $x=\frac{(d-b)}{ac}$

D) $x=\frac{d-b}{a-c}$

4

Jeff received a gift card worth \$50 and plans to use it to download music and movies. Each song download costs \$1.25 and each movie costs \$3. If Jeff downloads 4 songs and 1 movie each month, which of the following indicates the number of dollars, d, left on Jeff's gift card after m months?

A) $d=53-5m$

B) $d=50-2m$

C) $d=50-8m$

D) $d=45-3m$

5

$$2x+y=3x+4$$

$$x+5y=2$$

Based on the system of equations above, what is the value of $x+y$?

A) -3

B) -2

C) -1

D) 1

6

$$4(2x+3)^2+2x$$

Which of the following is equivalent to the expression above?

A) $4x^2+14x+9$

B) $8x^2+12x+9$

C) $16x^2+48x+36$

D) $16x^2+50x+36$

7

$$y=ax^3+2x^2+5x+d$$

The graph of the function above in the xy-plane has an x-intercept at $x=3$ and a y-intercept at $y=2$. What is the value of a?

A) $-\frac{35}{27}$

B) $-\frac{31}{27}$

C) $\frac{31}{27}$

D) $\frac{35}{27}$

CONTINUE

3 3

8

$$\frac{1}{6}+\frac{1}{x}=\frac{1}{4}$$

Mrs. Perry can paint a fence in 6 hours, but if her son Jason helps her, they can finish it in 4 hours. If the equation above models this situation, what does the term $\frac{1}{x}$ represent?

A) The number of hours it takes Mrs. Perry and Jason to paint the fence working together

B) The number of hours it would take Jason to paint the fence alone

C) The part of the job Jason completes in one hour

D) The part of the job Mrs. Perry completes in one hour

9

Luis budgeted no more than \$500 to purchase a suit and shirts for his new job. He found the suit for \$264 and shirts for \$24 each. Which of the following could be used to find the number of shirts, x, Luis could purchase while staying within his budget?

A) $500 \leq 264 - 24x$

B) $500 \geq 264x + 24$

C) $500 \geq 264 + 24x$

D) $500 \leq 264x - 24$

10

$$f(x) = ax^2 + b$$

In the function above, a and b are constants, $f(0) = 2$, and $f(1) = 5$. What is the value of $f(-3)$?

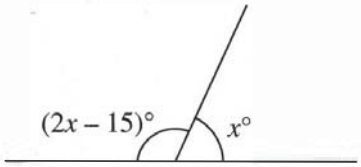
A) -43

B) -25

C) 29

D) 47

11



In the figure above, what is the measure of the larger angle?

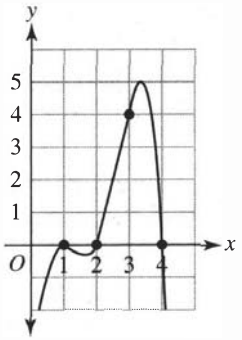
A) 65°

B) 80°

C) 115°

D) 125°

12



The function $y = g(x)$ is graphed in the xy-plane above. Which of the following equations could describe $g(x)$?

A) $g(x)=(x+1)(x+2)(x-4)^2$

B) $g(x)=-(x-1)^2(x-2)(x-4)$

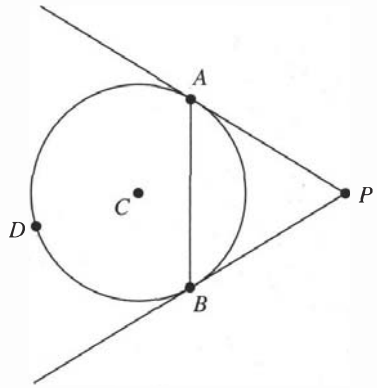
C) $g(x)=(x-1)^2(x-2)(x-4)$

D) $g(x)=-(x+1)^2(x+2)(x+4)$

CONTINUE

3 3

13



In the figure above, two tangent segments, AP and BP, are drawn to the circle with center C, and the measure of arc ADB is 240°. Which of the following must be true?

A) ΔABP is a right triangle

B) $AB > AP$

C) ΔABP is an equilateral triangle

D) $BP < AP$

14

$$y = 3x - 4$$
$$2x + y = 1$$

When graphed in the xy-plane, the lines described by the equations above each include a diameter of a circle. If the circle includes the point (–2, –5), which of the following is the equation of the circle?

A) $(3x-4)^2 + \left(\frac{1}{2} - \frac{1}{2}y\right)^2 = 29$

B) $(x+2)^2 + (y+5)^2 = 29$

C) $(x-1)^2 + (y+1)^2 = 29$

D) $(x-1)^2 + (y+1)^2 = 25$

15

$$\frac{3-2i}{4+5i}$$

Which of the following expressions is equivalent to the expression above? ($i = \sqrt{-1}$)

A) $-\frac{2}{9} - \frac{23}{9}i$

B) $\frac{2}{41} - \frac{23}{41}i$

C) $\frac{22}{41} + \frac{7}{9}i$

D) $\frac{22}{41} - \frac{23}{9}i$

CONTINUE

3 3

16

A bag of apples and oranges contains twice as many apples as oranges. If there are 15 total pieces of fruit in the bag, how many apples are in the bag?

17

A shade of green paint called *Groovy Green* is made by combining yellow paint and blue paint so that the ratio, by volume, of yellow to blue paint is 12 to 5. How many tablespoons of blue paint are needed to make 34 cups of Groovy Green? (1 cup = 16 tablespoons)

18

Points A, B, and C do not lie on the same line. If the distance from A to B is 4 units, and the distance from B to C is 5 units, then what is the largest possible integer distance between points A and C?

19

Pei-Sze has an average score of 89 on the six tests she has taken in her Physics class. She has two more tests left to take, and wants to raise her average to at least 90. If all tests are weighted equally, what is the minimum possible average score Pei-Sze must get on the two remaining tests to raise her average for the eight tests to at least 90?

20

If $x=2\sqrt{3}$ and $5x=\sqrt{3y}$, what is the value of y?

STOP

If you finish before time is called, you may check your work on this section only. Do not turn to any other section of the test.

4 4

Math Test—Calculator

55 MINUTES, 38 QUESTIONS

Turn to Section 4 of your answer sheet to answer the questions in this section.

DIRECTIONS

For questions 1–30, solve each problem, choose the best answer from the choices provided, and fill in the corresponding circle on your answer sheet. **For questions 31–38**, solve the problem and enter your answer in the grid on the answer sheet. Please refer to the directions before question 31 on how to enter your answers in the grid. You may use any available space in your test booklet for scratch work.

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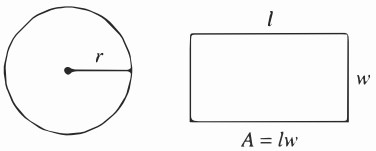

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$C = 2\pi r$

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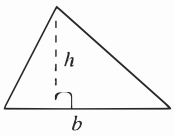
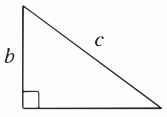


$A = \frac{1}{2}bh$



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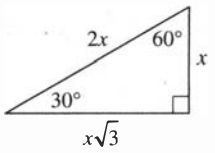
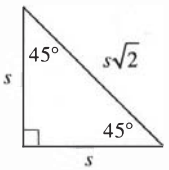
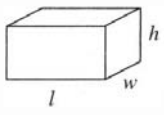
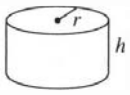




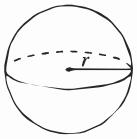
Special Right Triangles



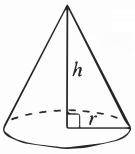
$V = lwh$



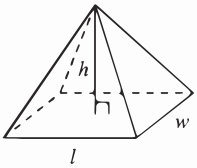
$V = \pi r^2 h$



$V = \frac{4}{3}\pi r^3$



$V = \frac{1}{3}\pi r^2 h$



$V = \frac{1}{3}lwh$

The number of degrees of arc in a circle is 360.
The number of radians of arc in a circle is 2π.
The sum of the measures in degrees of the angles of a triangle is 180.

CONTINUE

4 4

1

When Shondra makes an online purchase, a 15% discount is applied to the retail price and then a 6.25% tax is added to this discounted price. Which of the following represents the amount Shondra pays, in dollars, for an item with a retail price of x dollars?

A) $x - 0.15 + 0.0625x$

B) $1.0625x - 0.15x$

C) $1.0625(0.85x)$

D) $x - 0.15(1.0625x)$

2

In Connecticut, the highest recorded temperature over the past 100 years was 109°F in 1995 and the lowest recorded temperature was −37°F in 1943. How many degrees higher than the lowest recorded temperature was the highest recorded temperature?

A) 72°F

B) 95°F

C) 124°F

D) 146°F

3

At Petro's Restaurant, the owner will sometimes push smaller tables together to form one long rectangular table. If she pushes two tables together, she can seat 10 people. If she pushes three tables together, she can seat 14. Which equation best describes the relationship between the number of people, P, who can be seated at t tables that have been pushed together?

A) $P = 4t + 2$

B) $P = 5t$

C) $P = t + 4$

D) $P = 4t - 1$

4

Mark took a test with two kinds of questions: some worth 5 points and some worth 8 points. He answered a total of 14 questions correctly and earned a total of94 points. How many 5-point questions did Mark answer correctly?

A) 9

B) 8

C) 7

D) 6

5

A pool in the shape of a right rectangular prism holds 450 cubic feet of water. If the length and width of the pool are 10 feet and 15 feet, respectively, what is the depth of the water in the pool?

A) 3

B) 6

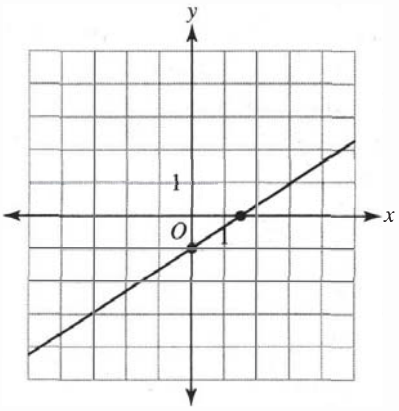
C) 7

D) 10

CONTINUE

4 4

6



Which of the following equations best describes the graph above?

A) $y=\frac{2}{3}x+\frac{3}{2}$

B) $y=2x-1$

C) $2x-3y=0$

D) $2x-3y=3$

7

$$f(x)=4x^2-7x+3$$
$$g(x)=2x^2-5x-4$$

If $h(x)=f(x)-g(x)$, which of the following is equivalent to $2h(x)$?

A) $4x^2-4x+14$

B) $4x^2-4x-2$

C) $12x^2-24x-2$

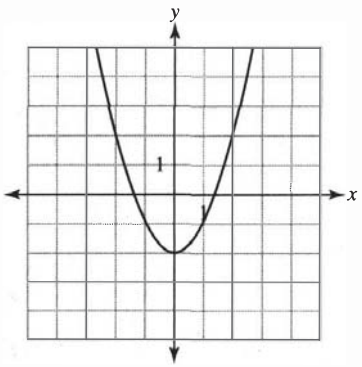
D) $4x^2-4x-14$

8

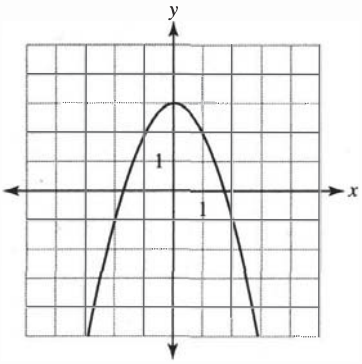
$$y+2=x^2-k$$

In the equation above, k is a constant less than zero. Which of the following graphs in the xy-plane could represent the solutions to this equation?

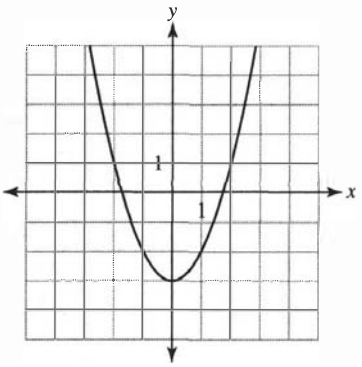
A)



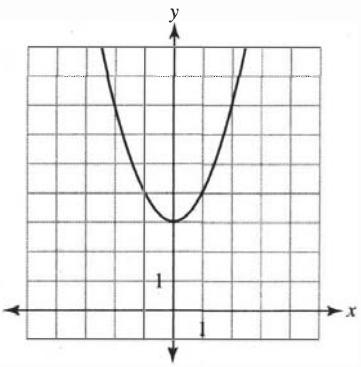
B)



C)



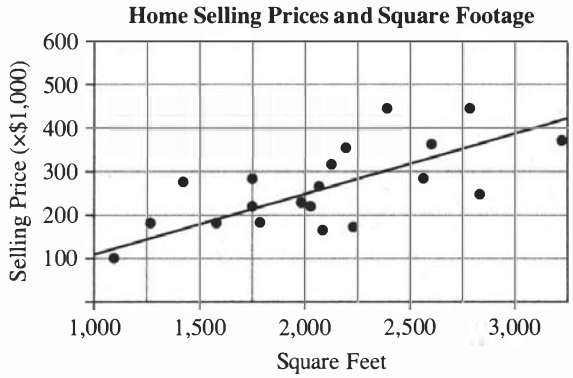
D)



CONTINUE

4 4

Questions 9 and 10 refer to the following information.



The scatterplot above shows the selling price and area, in square feet, for 20 homes in a suburban area, as well as the line of best fit for the data.

9

The Lees have budgeted between $200,000 and $300,000 for their new home. Approximately what percentage of the houses shown in the graph are priced within the range of this budget?

A) 35%

B) 40%

C) 45%

D) 55%

10

The line of best fit for these data has the equation $y = 0.12x + 3.37$, where x represents the total square footage of the house, and y represents the selling price, in thousands of dollars, for the house. Which choice best describes the meaning of the number 0.12 in this equation?

A) The selling price of the smallest house in the area is about $1,200.

B) The average selling price of a house, in thousands of dollars, is about 12% of the size of the house in square feet.

C) For every increase of 1 square foot in the size of the house, the average selling price increases by about $120.

D) For every increase of 1 square foot in the size of the house, its average selling price increases by about $0.12.

11

Approximately 3 million U.S. students graduated from high school in 2014, and of those, 70% of the women and 65% of the men enrolled in college. The U.S. school population is approximately 49% male and 51% female. Which of the following is the best estimate of the number of 2014 female high school graduates who did not enroll in college?

A) 459,000

B) 509,000

C) 907,000

D) 1,080,000

CONTINUE

4  4

12

$$x^{-2}\left(\frac{x+x+x}{x+x}\right)$$

Which of the following is equivalent to the expression above for all positive values of x?

A) $\frac{1}{x}$

B) $\frac{3x^3}{2}$

C) $\frac{3}{2x}$

D) $\frac{3}{2x^2}$

13

$$h(x, y)=\frac{Kx^2}{y}$$

If K is a constant in the definition of the function h above, and $h(m, n) = 2$, what is the value of $h(3m, 2n)$?

A) 4.5

B) 6

C) 9

D) 18

14

STATE	1990 Population	2000 Population	2010 Population
Alabama	4.041	4.447	4.780
Georgia	6.478	8.186	9.688
Louisiana	4.220	4.469	4.533
North Carolina	6.629	8.049	9.535
South Carolina	3.487	4.012	4.625

The table above shows the populations, in millions, of 5 Southern states, according to the U.S. Census for 1990, 2000, and 2010. How many of the states shown saw a population increase of 10% or more from 2000 to 2010?

A) One

B) Two

C) Three

D) Four

15

The legs of a right triangle have measures 15 and 36. What is the sine of the smallest angle in this triangle?

A) $\frac{15}{36}$

B) $\frac{15}{39}$

C) $\frac{36}{15}$

D) $\frac{36}{39}$

CONTINUE

4 4

16

$$-\frac{2}{x}<-\frac{1}{3}$$

Which of the following describes all solutions of the inequality above?

A) $x>-6$

B) $x<-6$

C) $x>6$ or $x<-6$

D) $0<x<6$

17

The current price of a share of stock A is one-fifth the price of a share of stock B. If the price of stock A were to increase at a constant rate of \$5 per month and the price of stock B were to decrease at a constant rate of \$3 per month, then in 6 months the two stock prices would be equal. What is the current price of a share of stock A?

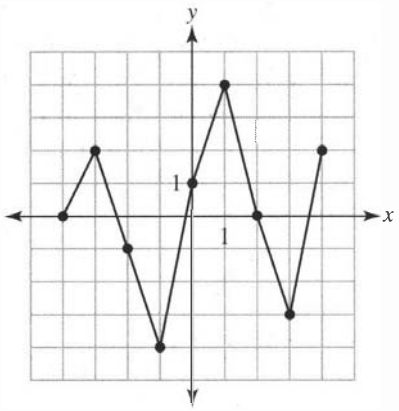
A) \$12

B) \$18

C) \$42

D) \$60

18



The function $y = f(x)$ is graphed on the xy-plane above. On the interval $-4 \le x \le 4$, for how many distinct values of x does $f(x)=\frac{1}{2}$?

A) Zero

B) Three

C) Four

D) Five

19

Mr. Johnson collected data on salaries of 20 randomly selected employees in his company. He found that their median salary was \$37,500, but their average salary was \$49,500. Which of the following would best explain the discrepancy between the median and the average values?

A) More than 20 employees were included when calculating the median.

B) One or more very high salaries pulled the average up.

C) One or more very low salaries pulled the median down.

D) Fewer salaries were included in calculating the median than the average.

CONTINUE

4 4

20

In a poll of n students at a local college, 45% of these students identify themselves as liberals. Of these, 60% support universal health care. If 405 students of these students are liberals who support universal health care, what is the value of n?

A) 109

B) 675

C) 900

D) 1,500

21

Light travels through a vacuum at a speed of approximately 186,000 miles per second. Approximately how many miles will a ray of light travel through a vacuum in one day?

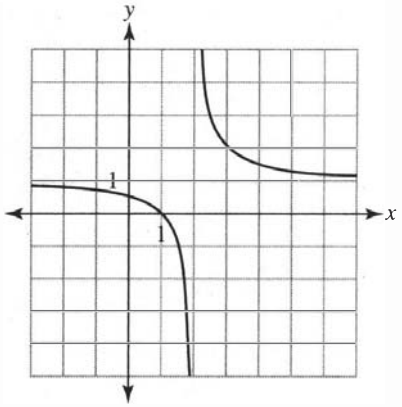
A) 6.7×10^8 miles

B) 1.6×10^{10} miles

C) 3.2×10^{10} miles

D) 4.6×10^{13} miles

22



$$f(x)=\frac{x-1}{x-2}$$

The function $y = f(x)$ is graphed on the xy-plane above. If the equation $y = x - 1$ is drawn on the same set of axes, which of the following is a point of intersection of the two graphs?

A) (0, 1)

B) (0, 0.5)

C) (1.5, –1)

D) (3, 2)

23

$$f(x) = x^2 - 6x - 475$$

$$g(x) = 3 - 4x$$

Given the definitions of the functions above, if $f(2a) = 3g(a)$, which of the following could be the value of a?

A) –9

B) 11

C) 12

D) 121

CONTINUE

4 4

24

$$y = x^2 + k$$
$$2x + y = 5$$

When the equations above are graphed in the xy-plane, they intersect in exactly one point. What is the value of k?

A) 6

B) 5

C) 1

D) 2.5

25

Every box of Weitz water crackers has a label that indicates it contains 6.5 ounces of crackers. However, industry standards allow these boxes to contain anywhere between 6.45 and 6.75 ounces of crackers. If x represents the number of ounces of crackers inside a box of Weitz water crackers that meets industry standards, which of the following expresses all possible values of x?

A) $|x - 6.60| \leq 0.10$

B) $|x - 6.60| \leq 0.15$

C) $|x - 6.50| \leq 0.05$

D) $|x - 6.50| \leq 0.25$

26

$$a + b < b < a - b$$

Let a and b be numbers that satisfy the inequality above. Which of the following must be true?

I. $a < 0$

II. $b < a$

III. $b < 0$

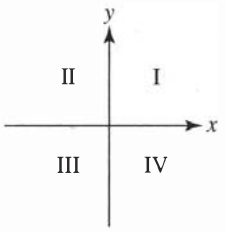
A) I only

B) I and III only

C) II and III only

D) I, II, and III

27



If the system of inequalities $y \leq 2x + 3$ and $y > x - 4$ are graphed in the xy-plane above, which quadrant contains no solutions to the system?

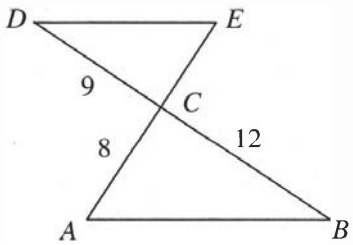
A) Quadrant II

B) Quadrant III

C) Quadrant IV

D) There are solutions in all four quadrants.

28



In the figure above, segment AB is parallel to segment DE, and segment AE is perpendicular to segment DB. What is the area of triangle DCE?

A) 27

B) 28

C) 32

D) 36

CONTINUE

4 4

29

If the average of a and b is x, the average of b and c is $2x$, and the average of a and c is $3x$, what is the average of a, b, and c, in terms of x?

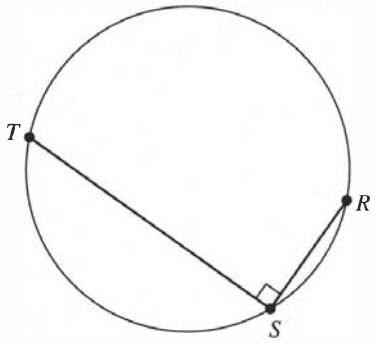
A) $\frac{2x}{3}$

B) $\frac{5x}{3}$

C) $2x$

D) $6x$

30



In the figure above, $SR = 10$ and $TS = 24$. If the area of the circle is $k\pi$, what is the value of k?

A) 13

B) 26

C) 169

D) 531

CONTINUE

4  4

31

The points (1, 3) and (4, k) lie on a line with a slope of $\frac{2}{3}$. What is the value of k?

32

The maximum load that a certain boat can carry is 4,000 pounds. The boat is to carry n identical crates, each weighing 60 pounds. If the crew and equipment weigh a total of 950 pounds, what is the maximum value for n that will still keep the weight of the load beneath the maximum?

Questions 33 and 34 refer to the following information.

Opinions on Immigration Reform				
Age Group	Support	Oppose	No Opinion	Total
21–40	68	22	10	100
41–60	55	39	6	100
61+	30	45	25	100
Total	153	106	41	300

A questionnaire about immigration reform was given to 300 people whose opinions were tabulated in the table above.

33

If one of the supporters of immigration reform is chosen at random from the table above, what is the probability that he or she is in the 21–40 age group?

34

The "indifference factor" for a group of people is defined as the number of people in the group who indicate no opinion on an issue divided by the number of people in the group who do indicate an opinion on the issue. According to the table, how much greater is the "indifference factor" on the issue of immigration reform for the 61+ age group than for the 21–40 age group?

CONTINUE

4 4

35

If $(x-a)(x-b)=x^2-9x+7$ for all values of x, what is the value of $a+b$?

36

Degrees Awarded by Hawthorne University in 2015				
	Bachelors	Masters	Doctorate	Total
Male	45	48	27	120
Female	55	52	23	130
Total	100	100	50	250

The table above shows the numbers of bachelor's, master's, and doctoral degrees awarded by Hawthorne University in 2015, broken down by gender. What fraction of the males who earned a degree from Hawthorne University in 2015 earned doctorates?

Questions 37 and 38 refer to the following information.

Ms. Hamid has established a trust fund for her daughter, Aisha. Aisha will be allowed to withdraw 4% annually from this fund once she reaches 21 years of age. Ms. Hamid uses the formula

$$A=5{,}000r^n$$

to approximate the amount of money, in dollars, remaining in the trust fund n years after Aisha turns 21.

37

What should Ms. Hamid use for the value of r?

38

To the nearest cent, how much more money, in dollars, can Aisha withdraw from the trust fund in the second year after she turns 21 than she can in the third year? (Note: Disregard the $ sign when gridding your answer.)

STOP

If you finish before time is called, you may check your work on this section only. Do not turn to any other section of the test.

SAT PRACTICE TEST 1 ANSWER KEY

Section 1: Reading

1. C
2. C
3. B
4. D
5. B
6. A
7. C
8. B
9. C
10. D
11. D
12. A
13. C
14. C
15. A
16. D
17. A
18. D
19. C
20. A
21. B
22. C
23. A
24. D
25. B
26. D
27. A
28. D
29. D
30. C
31. B
32. A
33. C
34. C
35. A
36. A
37. A
38. B
39. D
40. D
41. B
42. C
43. B
44. C
45. A
46. D
47. B
48. D
49. B
50. C
51. D
52. A

Total Reading Points (Section 1)

Section 2: Writing and Language

1. C
2. C
3. B
4. C
5. A
6. D
7. B
8. A
9. C
10. D
11. A
12. A
13. C
14. D
15. B
16. D
17. A
18. C
19. B
20. C
21. D
22. C
23. D
24. C
25. A
26. D
27. C
28. B
29. C
30. B
31. A
32. D
33. C
34. A
35. C
36. C
37. B
38. A
39. B
40. D
41. B
42. B
43. D
44. B

Total Writing and Language Points (Section 2)

Section 3: Math (No Calculator)

1. C
2. B
3. D
4. C
5. B
6. D
7. A
8. C
9. C
10. C
11. C
12. B
13. C
14. D
15. B
16. 10
17. 160
18. 8
19. 93
20. 100

Section 4: Math (Calculator)

1. C
2. D
3. A
4. D
5. A
6. D
7. A
8. D
9. B
10. C
11. A
12. D
13. C
14. C
15. B
16. D
17. A
18. D
19. B
20. D
21. B
22. D
23. B
24. A
25. B
26. B
27. D
28. A
29. C
30. C
31. 5
32. 50
33. 4/9 or .444
34. 2/9 or .222
35. 9
36. 9/40 or .225
37. .96
38. 7.68

Total Math Points (Section 3 + Section 4)

36. **C** **Verb Tense, Mood, and Voice**

This phrase must indicate a clear idea as well as coordinate logically with the prepositional phrase that follows. The original phrase is in the passive voice, which is stylistically weak, and it separates the prepositional phrase from the noun phrase it modifies, *the "rules of the game."* Choice D commits the same error and is in the wrong mood to boot. Choice B is in the wrong tense. Only choice C is clear and effective.

37. **B** **Clarity of Expression**

The underlined phrase is an adverbial phrase that modifies the verb *grows*. The context of the sentence requires that it indicate some way that an *economy* grows. The only choice that does this logically and idiomatically is choice B.

38. **A** **Clarity of Expression/Cohesiveness**

The original phrasing most logically supports the main claim of the paragraph because it is the only one that expresses the idea that *uncertainty hinders economic activity.*

39. **B** **Diction**

The correct choice must express what *a* country might do with *laws* in order to *ensure robust property rights.* The only reasonable choice is C, *adopting.*

40. **D** **Coordination**

Choice D establishes the correct cause-and-effect relationship between the two parts of the sentence.

41. **B** **Coordination/Cohesiveness**

The new sentence indicates an additional benefit provided by countries with strong economic institutions, so this sentence should follow the sentence that indicates the *first* economic benefit of such countries, which is sentence 2.

42. **B** **Clarity of Expression/Diction**

This sentence conveys the relative weakness of economic institutions in the second country. The only word among the choices that indicates this relative weakness is B, *tenuous.*

43. **D** **Redundancy**

Since the sentence begins with the word *Additionally,* choices A, B, and C are all redundant.

44. **B** **Subject-Verb Agreement**

The subject of this clause, *the stability, enforcement, and predictability,* is plural and so requires a plural verb. Choice D provides a verb that agrees with the subject, but forms an illogical idea. Choices A and C include verbs that disagree with the subject.

Section 3: Math (No Calculator)

1. **C** **Algebra (linear equations) EASY**

$$\frac{3}{4}a+\frac{2}{3}b=7$$

Multiply by 12 (the common denominator):

$$12\left(\frac{3}{4}a+\frac{2}{3}b=7\right)$$

Distribute: $\frac{36}{4}a+\frac{24}{3}b=84$

Simplify: $9a+8b=84$

2. **B** **Algebra (ratios) EASY**

Set up a proportion: $\frac{15 \text{ Euros}}{11 \text{ Pounds}}=\frac{100 \text{ Euros}}{x \text{ Pounds}}$

Cross-multiply: $15x=1{,}100$

Divide by 15: $x=73.33$

3. **D** **Algebra (solving equations) EASY**

$$ax+b=cx+d$$

Subtract cx and b from each side: $ax-cx=d-b$

Factor out x: $x(a-c)=d-b$

Divide by $(a-c)$: $x=\frac{d-b}{a-c}$

4. **C** **Algebra (equation writing) EASY**

In one month, Jeff spends 4($1.25) = $5 on music and $3 on a movie, for a total of $8 per month. Therefore, in m months, he spends 8m$. The amount remaining on his card after m months is, therefore, $50 minus the amount he spends in m months: $d=50-8m$.

5. **B** **Algebra (systems) EASY**

First equation: $2x+y=3x+4$

Subtract $2x$: $y=x+4$

Substitute $y=x+4$ into the second equation:

$$x+5(x+4)=2$$

Distribute: $x+5x+20=2$

Simplify: $6x+20=2$

Subtract 20: $6x=-18$

Divide by 6: $x=-3$

Substitute $x=-3$ into the equation to solve for y:

$$y=-3+4=1$$

Therefore, $x+y=-3+1=-2$.

6. **D** **Advanced Mathematics (polynomials) MEDIUM**

$$4(2x+3)^2+2x$$

Factor: $4(2x+3)(2x+3)+2x$

FOIL: $4(4x^2+6x+6x+9)+2x$

Simplify: $4(4x^2+12x+9)+2x$

Distribute: $16x^2+48x+36+2x$

Combine like terms: $16x^2+50x+36$

7. **A** **Advanced Mathematics (analyzing quadratics) MEDIUM**

If the graph has a y-intercept of $y=2$, then $y=2$ when $x=0$:

$$2 = a(0)^3 + 2(0)^2 + 5(0) + d$$

Simplify: $2 = d$

If the graph has an x-intercept at $x=3$, then $x=3$ when $y=0$:

$$0 = a(3)^3 + 2(3)^2 + 5(3) + 2$$

Simplify: $0 = 27a + 18 + 15 + 2$

Simplify: $0 = 27a + 35$

Subtract 35: $-35 = 27a$

Divide by 27: $-\frac{35}{27} = a$

8. **C** **Algebra (expressing relationships) MEDIUM**

Equations about work are framed in terms of the part of the job each participant can complete in one unit of time. If Mrs. Perry can do the job in 6 hours, she does one sixth of it in an hour. Together, she and Jason do the job in 4 hours, so they do one fourth in an hour. The time it would take Jason alone is unknown, so if it is represented by x, $\frac{1}{x}$ will represent the part of the job Jason can do in one hour.

9. **C** **Algebra (inequalities) MEDIUM**

If each shirt costs \$24, then Luis's total expenditure for x shirts is \$24x. If he also purchases a suit for \$264, the total expenditure is $264 + 24x$ dollars. If this must be less than or equal to 500, then $500 \geq 264 + 24x$.

10. **C** **Advanced Mathematics (functions) MEDIUM-HARD**

If $f(0) = 2$: $2 = a(0)^2 + b$

Simplify: $2 = b$

If $f(1) = 5$: $5 = a(1)^2 + b$

Simplify: $5 = a + b$

Substitute $b = 2$: $5 = a + 2$

Subtract 2: $3 = a$

Therefore, the function is: $f(x) = ax^2 + b = 3x^2 + 2$

Therefore: $f(-3) = 3(-3)^2 + 2 = 27 + 2 = 29$

11. **C** **Additional Topics (angles) MEDIUM**

Since the two angles form a linear pair, their sum is 180°:

$$2x - 15 + x = 180$$

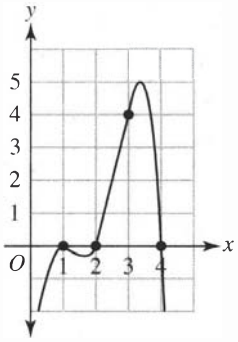
Combine like terms: $3x - 15 = 180$

Add 15: $3x = 195$

Divide by 3: $x = 65$

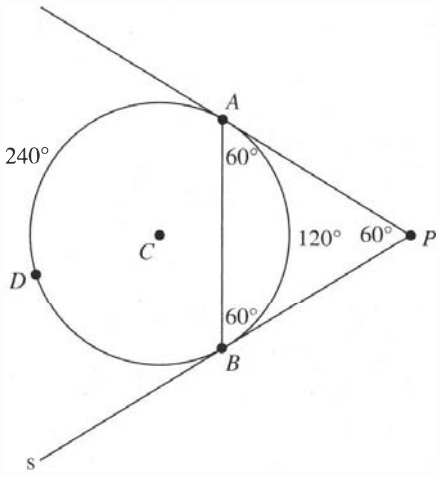
Therefore, the larger angle has a measure of $2(65) - 15 = 130 - 15 = 115°$.

12. **B** **Advanced Mathematics (polynomials) MEDIUM-HARD**



Notice that every choice is a polynomial function in factored form. Since the graph clearly has x-intercepts at $x = 1$, $x = 2$, and $x = 4$, then by the Factor Theorem the polynomial must have factors of $(x-1)$, $(x-2)$, and $(x-4)$. This rules out choices A and D. Notice, also, that the graph contains the point (3, 4). This point satisfies the function in B because $g(3) = -(3-1)^2(3-2)(3-4) = -(4)(1)(-1) = 4$. Notice that the function in choice C gives $g(3) = -4$.

13. **C** **Additional Topics (angles in circles) HARD**



We can analyze this triangle completely by just marking up the diagram with the measurements we are given. First, let's mark arc ADB with its measure: 240°. Since a circle has 360°, the minor arc AB must have measure $360° - 240° = 120°$. Now we must recall an important theorem called the Inscribed Angle Theorem: **an angle that has its vertex on a circle intercepts an arc that is twice its measure**. Since points A and B are both on the circle, we can use this theorem. Since angle ABP intercepts an arc of 120°, it must have a measure of $120/2 = 60°$. We can say the same of angle BAP. Then, since the angles in a triangle must have a sum of 180°, the remaining angle in the triangle must have a measure of $180 - 60 - 60 = 60°$. Since all of the angles in triangle ABP are equal, it is an equilateral triangle.

14. **D** **Additional Topics (circles) HARD**

First notice that each choice is an equation in the form $(x-h)^2+(y-k)^2=r^2$, which is the equation of a circle with radius (h, k) and radius r. We can find the radius (h, k) by finding the intersection point of the two lines (because all diameters of a circle must intersect at the center).

Substitute first linear equation ($y=3x-4$) into the second: $2x+(3x-4)=1$

Simplify: $5x-4=1$

Add 4: $5x=5$

Divide by 5: $x=1$

Substitute $x=1$ into first equation to find the y-coordinate of the center: $y=3(1)-4=-1$

Therefore, the center of the circle is $(1, -1)$, so the equation must have the form $(x-1)^2+(y+1)^2=r^2$.

This rules out choices A and B. We can finalize the equation by noticing that it must be satisfied by the point $(-2, -5)$:

$(-2-1)^2+(-5+1)^2=r^2$

Simplify: $(-3)^2+(-4)^2=9+16=25=r^2$

Therefore, the equation is: $(x-1)^2+(y+1)^2=25$

15. **B** **Special Topics (complex numbers) HARD**

$$\frac{3-2i}{4+5i}$$

Multiply the numerator and denominator by the conjugate of the denominator:

$$\frac{3-2i}{4+5i}\times\frac{4-5i}{4-5i}$$

FOIL the top and bottom:

$$\frac{3-2i}{4+5i}\times\frac{4-5i}{4-5i}=\frac{12-15i-8i+10i^2}{16-25i^2}$$

Combine terms:

$$\frac{12-23i+10i^2}{16-25i^2}$$

Substitute -1 for i^2:

$$\frac{12-23i+10(-1)}{16-25(-1)}=\frac{12-23i-10}{16+25}$$

Combine terms:

$$\frac{2-23i}{41}$$

Distribute division to get standard $a+bi$ form: $\frac{2}{41}-\frac{23}{41}i$

16. **10** **Algebra (ratios/word problems) EASY**

Let a represent the number of apples in the bag, and let r represent the number of oranges in the bag. If there are twice as many apples as oranges, then $a=2r$. If there are 15 total pieces of fruit in the bag, then $a+r=15$. Substitute $2r$ for a and solve:

$r+2r=15$

Combine terms: $3r=15$

Divide by 3: $r=5$ oranges

The bag contains twice as many apples as oranges, so it contains $2(5)=10$ apples.

17. **160** **Algebra (ratios) EASY**

Let's leave the conversion for the last step, and first find the number of cups of blue paint required. First set up a proportion in which x represents the number of cups of blue paint required. Since this will produce 34 total cups of paint, this mixture must also contain $34-x$ cups of yellow paint.

$$\frac{12}{5}=\frac{(34-x)\text{ cups of yellow paint}}{x\text{ cups of blue paint}}$$

Cross-multiply: $12x=5(34-x)$

Distribute: $12x=170-5x$

Add $5x$: $17x=170$

Divide by 17: $x=10$

Therefore, it requires 10 cups of blue paint, or $10(16)=160$ tablespoons of blue paint.

18. **8** **Additional Topics (triangles) MEDIUM**

If the three points are not all on the same line, they are the vertices of a triangle. If the distance from A to B is 4 units, and the distance from B to C is 5 units, then the distance between A and C must be less than the sum of these two differences, so less than 9 miles. The largest integer less than but not equal to 9 is 8.

19. **93** **Data Analysis (central tendency) MEDIUM-HARD**

If Pei-Sze has an average of 89 for 6 tests, she has earned $6(89)=534$ points thus far. To earn an average of 90 for all 8 tests, she needs a total of $8(90)=720$ points. That means she needs an additional $720-534=186$ points. If she earns those 186 points on 2 tests, she will average $186\div 2=93$ points per test.

20. **100** **Advanced Mathematics (radicals) HARD**

First equation: $x=2\sqrt{3}$

Multiply by 5: $5x=10\sqrt{3}$

Second equation: $5x=\sqrt{3y}$

Use the Transitive Property of Equality: $\sqrt{3y}=10\sqrt{3}$

The square root of a product is the product of the square roots. $\sqrt{3}\sqrt{y}=10\sqrt{3}$

Divide both sides by $\sqrt{3}$: $\sqrt{y}=10$

Square both sides: $y=100$

Section 4: Math (Calculator)

1. **C** **Algebra (representing quantities) EASY**

If x represents the retail price of the order, then $x-0.15x=0.85x$ is the price after the 15% discount. If a 6.25% tax is added, the final price $0.85x+0.0625(0.85x)=1.0625(0.85x)$.

2. **D** **Data Analysis (data spread) EASY**

This requires simply finding the difference between −37°F and 109°F: 109°F − (−37°F) = 146°F

3. **A** **Algebra (linear functions) EASY**

We are given that $P = 10$ when $t = 2$ and $P = 14$ when $t = 3$. The only equation among the choices that is satisfied by both of these ordered pairs is A: $P = 4t + 2$.

4. **D** **Algebra (word problems) EASY**

If Mark answered 14 questions correctly, and if x is the number of 5-point questions he answered correctly, then $14 - x$ must be the number of 8-point questions he answered correctly. He earned $5x$ points for the 5-point questions and $8(14 - x)$ points for the 8-point questions. If he earned a total of 94 points:

$$5x + 8(14 - x) = 94$$

Distribute: $5x + 112 - 8x = 94$
Combine like terms: $112 - 3x = 94$
Subtract 112: $-3x = -18$
Divide by −3: $x = 6$

5. **A** **Additional Topics (volume) EASY**

The pool holds 450 cubic feet of water with a length and a width of 10 feet and 15 feet. The volume of water can be expressed in the equation:

$$V = lwh$$

Substitute $V = 450$, $l = 10$, and $w = 15$: $450 = (10)(15)(h)$
Simplify: $450 = 150h$
Divide by 150: $3 = h$

6. **D** **Algebra (linear analysis) EASY**

The graph shows clearly that the line has an x-intercept at (1.5, 0) and a y-intercept at (0, −1), so we want to find the linear equation that has these features. One way to tell is simply to substitute these ordered pairs into the choices, and eliminate those that don't work. Let's start by testing (0, −1):

A) $-1 = \frac{2}{3}(0) + \frac{3}{2} = \frac{3}{2}$ (Nope. Eliminate.)

B) $-1 = 2(0) - 1 = -1$ (Yes. Keep.)

C) $2(0) - 3(-1) = 3 = 0$ (Nope. Eliminate.)

D) $2(0) - 3(-1) = 3 = 3$ (Yes. Keep.)

Next we can test (1.5, 0) in the remaining choices:

B) $0 = 2(1.5) - 1 = 2$ (Nope. Eliminate.)

D) $2(1.5) - 3(0) = 3 - 0 = 3$ (Yes.)

7. **A** **Advanced Mathematics (polynomials) MEDIUM**

$$2h(x) = 2f(x) - 2g(x)$$

Use definitions of $f(x)$ and $g(x)$:

$$= 2(4x^2 - 7x + 3) - 2(2x^2 - 5x - 4)$$

Distribute: $8x^2 - 14x + 6 - 4x^2 + 10x + 8$
Combine like terms: $4x^2 - 4x + 14$

8. **D** **Advanced Mathematics (quadratic functions) MEDIUM**

This equation represents a quadratic function in x, so its graph is a parabola. Since all of the choices are parabolic graphs, we must analyze the equation further to find the correct graph.

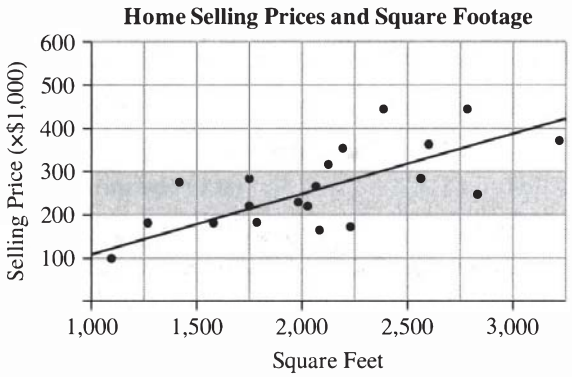
Given equation: $y + 2 = x^2 - k$
Subtract 2: $y = x^2 - k - 2$
Express in vertex form: $y = (x - 0)^2 + (-k - 2)$

This means that this is an "open-up" parabola with a vertex at $(0, -k - 2)$, which is on the y-axis.

If k is negative: $k < 0$
Multiply by −1 (and "flip" the inequality): $-k > 0$
Subtract 2: $-k - 2 > -2$

This means that $-k - 2$ (the y-coordinate of the vertex) is greater than −2, so the vertex must be on the y-axis above the point (0, −2). The graphs in choices B and D both satisfy this criterion, but since the graph in B is open down instead of up, the correct answer is D.

9. **B** **Data Analysis (scatterplots) MEDIUM**



The shaded region shows the range of prices that are within the Lees' budget. Eight of the 20 data points are in this region, and these represent 8/20 = 0.40 = 40% of the total.

10. **C** **Data Analysis (scatterplot) MEDIUM-HARD**

This is a linear equation in slope-intercept form, and 0.12 represents the slope. The slope is the rate of change of the y variable with respect to the x variable. This means that the average selling price of the house increases by (0.12) thousand dollars for every additional square foot, or $120 per square foot.

11. **A** **Data Analysis (percentages) MEDIUM**

Of 3 million graduates, approximately (0.51)(3 million) = 1.53 million are female. Of those, 70% enrolled in college. Therefore, 30% are <u>not</u> enrolled: 30% of 1.53 million young women is (0.3)(1.53 million) = 0.459 million or approximately 459,000.

12. **D** **Advanced Mathematics (exponentials) MEDIUM-HARD**

Original expression: $x^{-2}\left(\frac{x+x+x}{x+x}\right)$

Rewrite x^{-2} as $\frac{1}{x^2}$: $\frac{1}{x^2}\left(\frac{x+x+x}{x+x}\right)$

Simplify: $\frac{1}{x^2}\left(\frac{3x}{2x}\right)$

Simplify: $\frac{1}{x^2}\left(\frac{3}{2}\right)$

Multiply fractions: $\frac{3}{2x^2}$

13. **C** **Problem Solving and Data Analysis (variation) MEDIUM-HARD**

Definition of function: $h(x, y)=\frac{Kx^2}{y}$

If $h(m, n) = 2$: $\frac{Km^2}{n}=2$

We want to evaluate $h(3m, 2n)$:

$$h(3x, 2y)=\frac{K(3m)^2}{2n}=\frac{9Km^2}{2n}=\left(\frac{9}{2}\right)\left(\frac{Km^2}{n}\right)$$

Substitute $\frac{Km^2}{n}=2$: $h(3x, 2y)=\left(\frac{9}{2}\right)(2)=9$

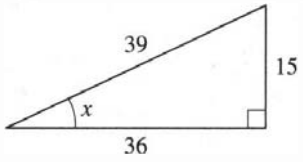
14. **C** **Problem Solving and Data Analysis (set relations) MEDIUM**

Since the question asks only about the change from 2000 to 2010, we simply need to calculate 10% of the 2000 populations and compare this to the difference in population from 2000 to 2010. (Remember that calculating 10% of a decimal is easy: just move the decimal one place to the left.

	10% of 2000 pop.	Increase from 2000 to 2010
Alabama	0.4447	0.333
Georgia	0.8186	1.502
Louisiana	0.4469	0.064
North Carolina	0.8049	1.486
South Carolina	0.4012	0.613

For three of these states—Georgia, North Carolina, and South Carolina—the increase is greater than 10%.

15. **B** **Additional Topics (trigonometry) MEDIUM**



If the two legs of a right triangle have lengths 15 and 36, we can find the hypotenuse, h, with the Pythagorean Theorem: $15^2 + 36^2 = h^2$

Simplify: $225 + 1{,}296 = h^2$

Simplify: $1{,}521 = h^2$

Take the square root: $39 = h$

The question asks for the sine of the smallest angle. The smallest angle in a triangle is always across from the smallest side, so the smallest angle in this triangle must be the one marked x. The sine ratio is defined as the opposite side divided by the hypotenuse, so $\sin x = 15/39$.

16. **D** **Algebra (inequalities) MEDIUM**

Original inequality: $-\frac{2}{x}<-\frac{1}{3}$

Multiply by −1 and "flip": $\frac{2}{x}>\frac{1}{3}$

Notice that this inequality tells us that $\frac{2}{x}$ is a positive number, so x must be a positive number, and $3x$ (the common denominator) must also be a positive number. This means that if we multiply both sides by $3x$, we don't need to "flip" the inequality.

Multiply by $3x$ (and don't flip): $6 > x > 0$

17. **A** **Problem Solving and Data Analysis (rates) MEDIUM-HARD**

Let x denote the current price of stock A, and $5x$ denote the current price of stock B. In 6 months, stock A will have gained \$5 per month, so it will have gained a total of (\$5)(6) = \$30, so its new price will be $x + 30$. After the same six months, stock B will have lost \$3 per month, so it will have lost a total of (\$3)(6) = \$18, giving it a new price of $5x - 18$. If the two stock prices are then the same:

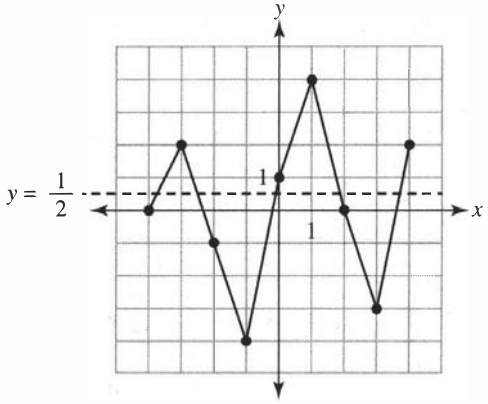
$x + 30 = 5x - 18$

Subtract x: $30 = 4x - 18$

Add 18: $48 = 4x$

Divide by 4: $12 = x$

18. **D** **Advanced Mathematics (functions) MEDIUM-HARD**



Drawing a horizontal line at $y = ½$ shows that the function intersects with this line 5 times.

19. **B** **Data Analysis (central tendency) MEDIUM-HARD**

The median of a set is not affected by extreme values, but the average is. Consider a simple set of numbers like 1, 2, and 3. It should be clear that the median and the average of this set are both 2. Compare this to the set 1, 2, and 300. The median of this set is still 2, but the average is now much larger: 101. If the average of a set is much larger than its median, the likely explanation is that some very large outliers are pulling the average up without changing the median.

20. **D** **Problem Solving and Data Analysis (percents) MEDIUM-HARD**

If n students were polled, then the 45% of these who identify as liberals can be represented as $0.45n$. The 60% of this group that support universal health care can be expressed as $0.60(0.45n) = 0.27n$. If there are 405 students in this group, then $0.27n = 405$. Dividing by 0.27 gives $n = 1{,}500$.

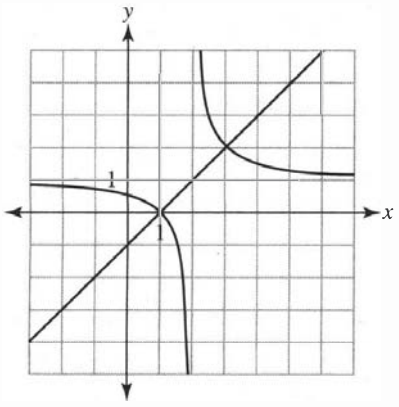
21. **B** **Problem Solving and Data Analysis (conversions) MEDIUM-HARD**

This is a fairly straightforward conversion problem. Just make sure you set up your conversion factors carefully and convert the answer to scientific notation.

$$1 \text{ day} \times \frac{24 \text{ hours}}{1 \text{ day}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{60 \text{ seconds}}{1 \text{ minute}}$$

$$\times \frac{186{,}000 \text{ miles}}{1 \text{ second}} = 1.6 \times 10^{10} \text{ miles}$$

22. **D** **Advanced Mathematics (functions) MEDIUM-HARD**



Since the graph is given, one way to solve this problem is to simply graph the line $y = x - 1$ and notice that the two graphs appear to intersect at (1, 0) and (3, 2). Then we can confirm that these are solutions by plugging these values back into the equations and confirming that they work.

If you prefer the algebraic method, you can solve the equation $x - 1 = \frac{x-1}{x-2}$ for x: $x - 1 = \frac{x-1}{x-2}$

Multiply by $(x-2)$: $(x-1)(x-2) = x-1$
FOIL: $x^2 - 3x + 2 = x - 1$
Subtract x and add 1: $x^2 - 4x + 3 = 0$
Factor: $(x-1)(x-3) = 0$
Solve with the Zero Product Property: $x = 1$ or $x = 3$
Plugging these values of x back into either equation gives the ordered pair solutions (1, 0) and (3, 2).

23. **B** **Advanced Mathematics (functions) HARD**

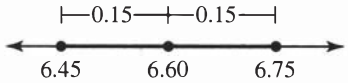
Given functions: $f(x) = x^2 - 6x - 475$
$g(x) = 3 - 4x$
If $f(2a) = 3g(a)$: $(2a)^2 - 6(2a) - 475 = 3(3 - 4(a))$
Simplify: $4a^2 - 12a - 475 = 9 - 12a$
Add $12a$: $4a^2 - 475 = 9$
Add 475: $4a^2 = 484$
Divide by 4: $a^2 = 121$
Take the square root: $a = \pm 11$

24. **A** **Advanced Mathematics (non-linear systems) MEDIUM-HARD**

Although the question gives us information about the graphs of these equations, it is impossible to graph the first equation precisely, because k is unknown. Therefore, it is probably easier to treat this as an algebraic system of equations with only one solution.

Given equations: $y = x^2 + k$
$2x + y = 5$
Substitute $y = x^2 + k$ into the second equation:
$2x + (x^2 + k) = 5$
Express in standard quadratic form: $x^2 + 2x + k = 5$
Subtract 5: $x^2 + 2x + (k - 5) = 0$
Recall that if a quadratic equation has only one solution, then its discriminant, $b^2 - 4ac$, must equal 0.
The discriminant is 0: $(2)^2 - 4(1)(k-5) = 0$
Simplify: $4 - 4k + 20 = 0$
Combine like terms: $24 - 4k = 0$
Add $4k$: $24 = 4k$
Divide by 4: $6 = k$

25. **B** **Algebra (absolute values) MEDIUM-HARD**



It helps a lot to draw the number line and indicate the range of "industry standard weights." Since the problem states that any weight between 6.45 and 6.75 ounces satisfies industry standards, we can represent this range with the dark line above. Notice that the midpoint of this segment is (6.45 + 6.75)/2 = 6.60, and that this midpoint is 0.15 away from each endpoint. This means that every value in this set is 0.15 units or less away from 6.60. Another way to put this is that "the distance from x to 6.60 is less than or equal to 0.15" or $|x - 6.60| \leq 0.15$.

26. **B** **Problem Solving and Data Analysis (numerical reasoning) HARD**

Original inequality: $a+b<b<a-b$

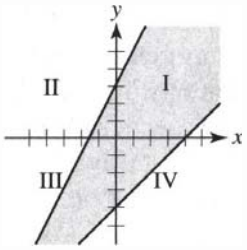
Subtract a: $b<b-a<-b$

Subtract b: $0<-a<-2b$

Multiply by -1 and "flip": $0>a>2b$

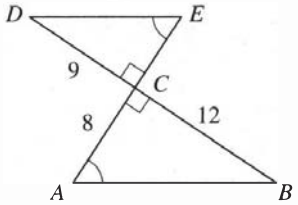
This indicates that both a and b must be negative, so we have proven that statements I and III must be true. This rules out choices A and C. To choose between B and D, we must determine whether b must be less than a. Here, we might choose simple values for a and b that satisfy the inequality. Notice that $a=-1$ and $b=-1$ work, because $0>-1>2(-1)$. Since this shows that a and b can be equal, statement II is not necessarily true. Therefore, the correct answer is B, I and III only.

27. **D** **Algebra (graphing systems) HARD**



Since both linear inequalities are written in slope-intercept form, you should be able to graph each line based on its slope and y-intercept, and then shade accordingly. The first line has a slope of 2 and a y-intercept of 3, and is shaded below, to catch all of the y values that are less than or equal to this line. The second line has a slope of 1 and a y-intercept of -4, and is shaded above, to catch all of the y values that are greater than this line. This gives the shaded region shown above, which clearly contains points in all four quadrants.

28. **A** **Additional Topics (Triangles) HARD**



Because AB is parallel to DE, alternate interior angles A and E are congruent, as shown above. Also, since AE is perpendicular to DB, the triangles contain right angles as shown. This proves that the triangles are similar, and so corresponding sides must be proportional. We can find the length of CE by setting up a proportion:

$$\frac{CE}{AC}=\frac{DC}{CB}$$

Substitute values: $\frac{CE}{8}=\frac{9}{12}$

Cross-multiply: $12(CE)=72$

Divide by 12: $CE=6$

Therefore, the area of triangle DCE is $(6)(9)/2=27$.

29. **C** **Problem Solving and Data Analysis (averages) MEDIUM-HARD**

Recall the average formula: $\text{average}=\frac{\text{sum}}{\#\text{ of numbers}}$

And therefore: $\text{sum}=(\text{average})(\#\text{ of numbers})$

Therefore, if the average of a and b is x:

$a+b=x(2)=2x$

If the average of b and c is $2x$: $b+c=2x(2)=4x$

If the average of a and c is $3x$: $a+c=3x(2)=6x$

Add these three equations together:

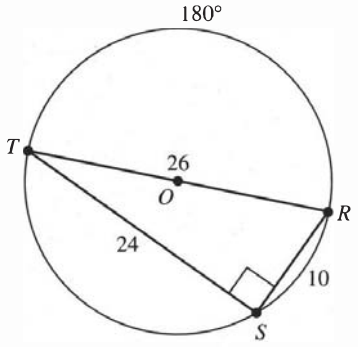
$2a+2b+2c=2x+4x+6x=12x$

Divide by 2: $a+b+c=6x$

The average of a, b and c is $\frac{a+b+c}{3}$:

$$\frac{a+b+c}{3}=\frac{6x}{3}=2x$$

30. **C** **Additional Topics (Circles) HARD**



First recall the Inscribed Angle Theorem: if an angle is inscribed in a circle, it intercepts an arc that is twice the measure of the angle. Since angle RST has a measure of 90°, it must intercept an arc of 180°, which is a semicircle. This means that TR is a diameter. We can find the measure of this diameter by using the Pythagorean Theorem: $(TR)^2=10^2+24^2$

Simplify: $(TR)^2=676$

Take the square root: $TR=26$

Since this is the diameter of the circle, the radius must be $26\div 2=13$, and the circle has an area of $\pi(13)^2=169\pi$. Therefore $k=169$.

31. **5** **Algebra (graphs of lines) EASY**

Slope formula: $\text{slope}=\frac{y_2-y_1}{x_2-x_1}$

Substitute known values: $\frac{2}{3}=\frac{k-3}{4-1}$

Simplify: $\frac{2}{3}=\frac{k-3}{3}$

Multiply by 3: $2=k-3$

Add 3: $5=k$

32. **50** **Problem Solving and Data Analysis (inequalities) MEDIUM**

If the total weight of the crew, equipment, and cargo cannot exceed 4,000 pounds: $950 + 60n < 4{,}000$
Subtract 950: $60n < 3050$
Divide by 60: $n < 50.83$
Since n must be a whole number, the greatest value it can take is 50.

33. **4/9 or .444** **Problem Solving and Data Analysis (probability) MEDIUM**

The table shows 153 respondents in support of immigration support, 68 of which are in the 21–40 age group. Therefore, the probability of a supporter being in the 21–40 age group is 68/153 = 4/9 =.444.

34. **2/9 or .222** **Data Analysis (analysis of tabular data) MEDIUM**

The "indifference factor" for the 61+ age group is 25 ÷ (30 + 45) = 1/3 or .333. The "indifference factor" for the 21–40 age group is 10 ÷ (68 + 22) = 1/9 or .111. The difference is .333 – .111 = .222 or 2/9.

35. **9** **Algebra (identities) MEDIUM**

$(x-a)(x-b) = x^2 - 9x + 7$
FOIL: $x^2 - ax - bx + ab = x^2 - 9x + 7$
Combine like terms: $x^2 - (a+b)x + ab = x^2 - 9x + 7$
If this equation is true for all values of x, then the two quadratics must be identical, and so their coefficients must match perfectly. This means that $a + b = 9$ and $ab = 7$.

36. **9/40 or .225** **Data Analysis (percentages) MEDIUM**

According to the table, 27 men were awarded doctorates. The question asks what percentage *of the males*, so the fraction is 27/120 = .225 or 9/40.

37. **.96** **Advanced Mathematics (exponential functions) MEDIUM**

The formula is designed to find the amount of money remaining in the account. If 4% is withdrawn, 96% will remain. Ms. Hamid should use $r = 0.96$.

38. **7.68** **Algebra (extended analysis) HARD**

According to the formula, the trust fund started with $5,000, so Aisha can withdraw (0.04)($5,000) = $200 the first year. This means that the fund will have $5,000 – $200 = $4,800 remaining, so Aisha can withdraw (0.04)($4,800) = $192 the second year. This means that the fund will have $4,800 – $192 = $4,608 remaining, so Aisha can withdraw (0.04)($4,608) = $184.32 the second year. This is a difference of $192 – $184.32 = $7.68.

3 3

Math Test—No Calculator

25 MINUTES, 20 QUESTIONS

Turn to Section 3 of your answer sheet to answer the questions in this section.

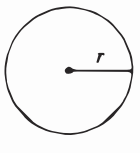
DIRECTIONS

For questions 1–15, solve each problem, choose the best answer from the choices provided, and fill in the corresponding circle on your answer sheet. **For questions 16–20**, solve the problem and enter your answer in the grid on the answer sheet. Please refer to the directions before question 16 on how to enter your answers in the grid. You may use any available space in your test booklet for scratch work.

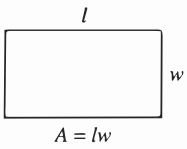
NOTES

1. The use of a calculator **is not permitted**.
2. All variables and expressions used represent real numbers unless otherwise indicated.
3. Figures provided in this test are drawn to scale unless otherwise indicated.
4. All figures lie in a plane unless otherwise indicated.
5. Unless otherwise indicated, the domain of a given function f is the set of all real numbers x for which $f(x)$ is a real number.

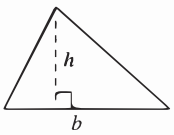
Reference



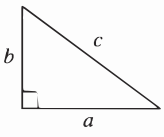
$A = \pi r^2$
$C = 2\pi r$



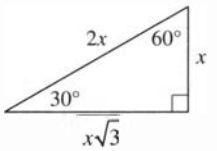
$A = lw$

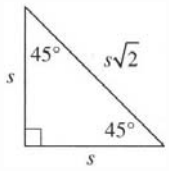


$A = \frac{1}{2}bh$

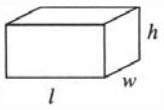


$c^2 = a^2 + b^2$

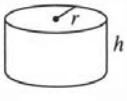




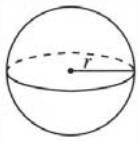
Special Right Triangles



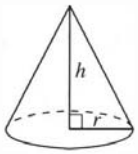
$V = lwh$



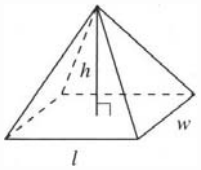
$V = \pi r^2 h$



$V = \frac{4}{3}\pi r^3$



$V = \frac{1}{3}\pi r^2 h$



$V = \frac{1}{3}lwh$

The number of degrees of arc in a circle is 360.
The number of radians of arc in a circle is 2π.
The sum of the measures in degrees of the angles of a triangle is 180.

CONTINUE

3

3

1

If $2(x+2)=3x-1$, what is the value of x?

A) 5

B) 3

C) $\frac{5}{6}$

D) $\frac{3}{8}$

2

If $\frac{1}{2}x+\frac{1}{3}y=7$, what is the value of $6x+4y$?

A) 21

B) 42

C) 84

D) 128

3

$$\frac{3(k-5)}{4}=\frac{5+2k}{3}$$

In the equation above, what is the value of k?

A) 14

B) 29

C) 65

D) 82

4

$$x=5y+5$$
$$2x-y=19$$

Which of the following ordered pairs (x, y) satisfies the system of equations above?

A) (1, 10)

B) (1, 5)

C) (10, 15)

D) (10, 1)

5

Dosages of children's medications are calculated by the child's weight. The dosage of amoxicillin for a one year-old child is 40 milligrams per kilogram of body weight per day. Which of the following functions models the dosage, d, in milligrams of amoxicillin per day for a one-year-old child weighing p pounds? (1 kilogram = 2.2 pounds)

A) $d=2.2p+40$

B) $d=2.2(40p)$

C) $d=\frac{2.2p}{40}$

D) $d=\frac{40p}{2.2}$

6

Calvin is hanging photographs that are 9 inches wide in a horizontal row, and he wants 2 inches of space between the photographs. He wants to create the display with no more than 12 feet between the outmost edges of the first and last photographs. Which inequality expresses the number, n, of these photographs Calvin can include in the display?

A) $9n+2(n-1)\leq 144$

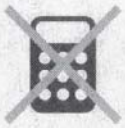
B) $9n+2(n-1)\leq 12$

C) $11n\leq 144$

D) $11n\leq 12$

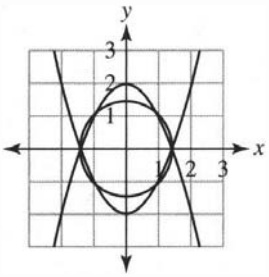
CONTINUE

3



3

7



$$y = x^2 - 2$$
$$y = 2 - x^2$$
$$x^2 + y^2 = 2$$

The system of three equations shown above is graphed in the xy-plane. How many solutions does this system have?

A) Zero

B) Two

C) Four

D) Six

8

$$\left(\frac{1}{10} + \frac{1}{5}\right)x = 1$$

Mr. Hong has two hoses available to fill a pool. The blue hose alone can fill his pool in 10 hours. The red hose can fill the pool in half that time. Mr. Hong wonders how long it would take to fill the pool using both hoses at once. The equation above describes this situation. Which of the following is the best interpretation of x in this equation?

A) The rate, in pools per hour, at which both hoses together fill the pool

B) The number of hours it takes to fill the pool using both hoses

C) The number of pools that both hoses, working together, can fill in one hour

D) The total number of gallons of water required to fill the pool

9

$$2i^2 + 3i^3 - 4i^4 + 5i^5$$

Which of the following is equivalent to the complex number shown above? ($i = \sqrt{-1}$)

A) $-6 - 8i$

B) $-6 + 2i$

C) $2 + 2i$

D) $2 + 8i$

10

In a certain sequence, each term after the first term is 4 less than three times the previous term. The third term of the sequence is 20. What is the sum of the second term and the fourth term?

A) 2

B) 48

C) 56

D) 64

11

$$f(x) = x^3 + 2x^2 - 16x - 32$$
$$g(x) = x^2 + 6x + 8$$

When $y = f(x)$ and $y = g(x)$ are graphed together on the xy-plane, the graph of $y = f(x)$ passes through both x-intercepts of the graph of $y = g(x)$. Which of the following ordered pairs (x, y) is the third x-intercept of $y = f(x)$?

A) (2, 0)

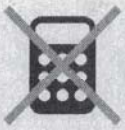
B) (4, 0)

C) (−8, 0)

D) (−10, 0)

CONTINUE

3



3

12

If $x = a^2 - b^2$ and $y = 2a - 2b$, where $a \neq b$, which of the following is equal to $\frac{10x}{y}$?

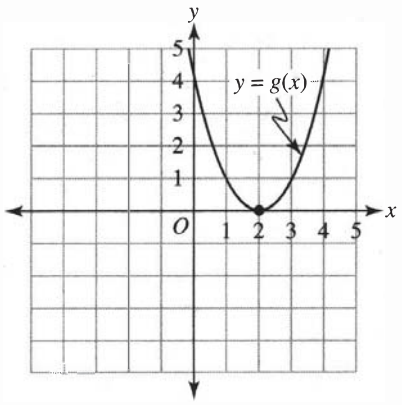
A) 5

B) $5a - b$

C) $5a - 5b$

D) $5a + 5b$

13



The figure above shows the graph of quadratic function $y = g(x)$ in the xy-plane. If this graph were shifted up one unit, it would coincide with the function $y = h(x)$ (not shown). Which of the following is the equation that defines $h(x)$?

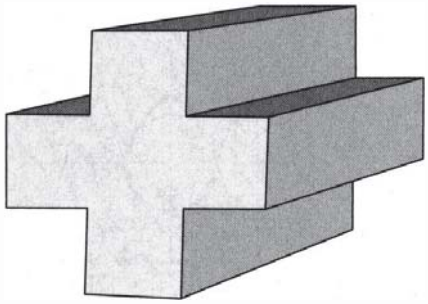
A) $h(x) = (x - 2)^2 + 1$

B) $h(x) = (x + 2)^2 + 1$

C) $h(x) = (x - 1)^2 + 1$

D) $h(x) = x^2 + 3$

14



The metal beam shown above is a right prism with parallel faces in the shape of a cross. Each cross-shaped face of is composed of five identical squares, each with a side length of 6 inches. If the beam has a depth of 72 inches, what is the volume, in cubic feet, of the beam? (1 foot = 12 inches)

A) 1.5 cubic feet

B) 7.5 cubic feet

C) 10.8 cubic feet

D) 12.9 cubic feet

15

$$4^{4x} = 64a^4$$

In the equation above, x and a are positive numbers. If $2^{2x} = na$, what is the value of n?

A) 2

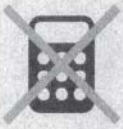
B) $\sqrt{6}$

C) $\sqrt{8}$

D) 4

CONTINUE

3



3

16

If $t^2 = 5t$ and $t = 2x - 1$, what is one possible value of x?

17

Alliyah takes a test that consists of some 3-point questions and some 5-point questions. She answered 20 questions correctly, and earned 86 points in total. How many 5-point questions did she answer correctly?

18

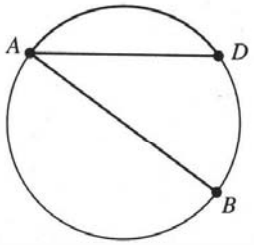
If $\frac{x}{x^2-9}-\frac{1}{x+3}=\frac{1}{4x-12}$, what is the value of x?

19

$$m^2+5=x$$
$$9y^2=x-5$$

Based on the system of equations above, what is the value of $\left|\frac{m}{y}\right|$?

20



Segment AB is a diameter of the circle above, and $AB=\frac{20}{\pi}$. If the measure of $<BAD$ is $\frac{\pi}{5}$ radians, what is the length of arc AD?

STOP

If you finish before time is called, you may check your work on this section only. Do not turn to any other section of the test.

4 4

Math Test—Calculator

55 MINUTES, 38 QUESTIONS

Turn to Section 4 of your answer sheet to answer the questions in this section.

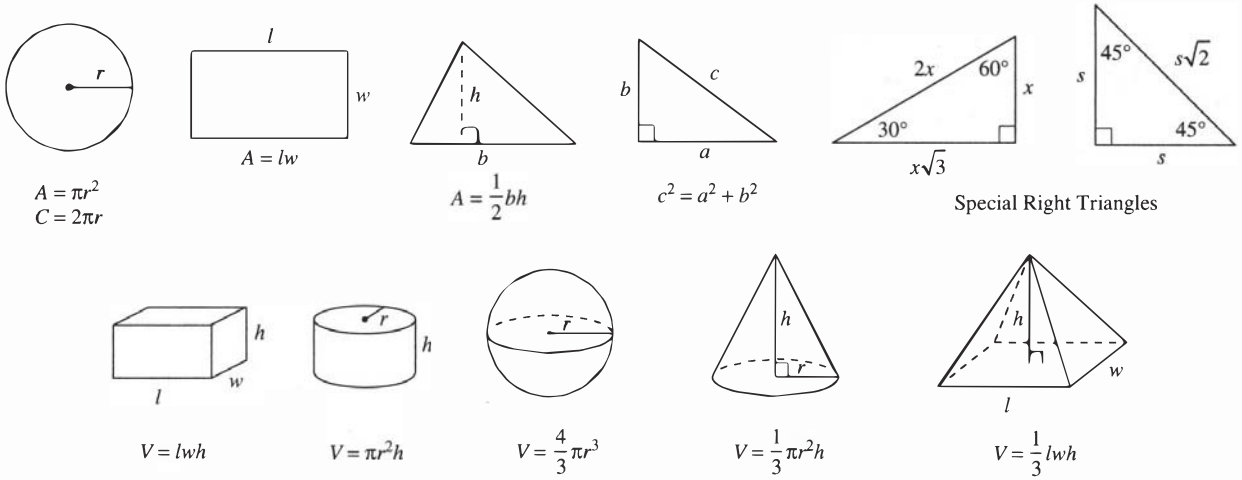
DIRECTIONS

For questions 1–30, solve each problem, choose the best answer from the choices provided, and fill in the corresponding circle on your answer sheet. **For questions 31–38**, solve the problem and enter your answer in the grid on the answer sheet. Please refer to the directions before question 31 on how to enter your answers in the grid. You may use any available space in your test booklet for scratch work.

NOTES

1. The use of a calculator **is permitted**.
2. All variables and expressions used represent real numbers unless otherwise indicated.
3. Figures provided in this test are drawn to scale unless otherwise indicated.
4. All figures lie in a plane unless otherwise indicated.
5. Unless otherwise indicated, the domain of a given function f is the set of all real numbers for which $f(x)$ is a real number.

REFERENCE



$A = \pi r^2$
$C = 2\pi r$

$A = \frac{1}{2}bh$

$c^2 = a^2 + b^2$

Special Right Triangles

$V = lwh$

$V = \pi r^2 h$

$V = \frac{4}{3}\pi r^3$

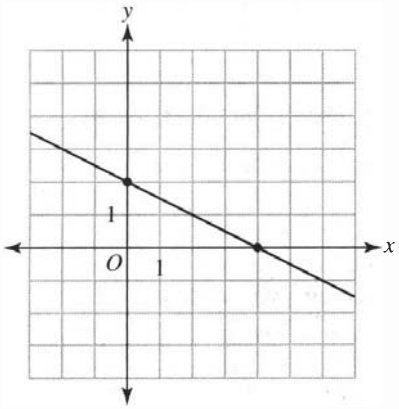
$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}lwh$

The number of degrees of arc in a circle is 360.
The number of radians of arc in a circle is 2π.
The sum of the measures in degrees of the angles of a triangle is 180.

4 4

1



Which of the following is an equation representing the line shown in the figure above?

A) $y=\frac{1}{2}x+2$
B) $3x+6y=12$
C) $y=-2x+2$
D) $3y=6x+12$

2

Gretchen has $4,000 to invest in a mutual fund. Shares of the fund cost $3.19 per share, and the brokerage charges a $75 fee for the entire transaction. What is the maximum number of shares of this mutual fund that Gretchen can purchase?

A) 53
B) 1,179
C) 1,230
D) 1,231

3

What is the radius of the largest sphere that will fit inside a cube with a volume of 64 cubic inches?

A) 2 inches
B) 3 inches
C) 4 inches
D) 6 inches

4

If the function $f(x)=x^3-x^2+px-3$ has a zero at $x=1$, what is the value of p?

A) 3
B) 1
C) 0
D) –3

5

1, 2, 3, 4, 5

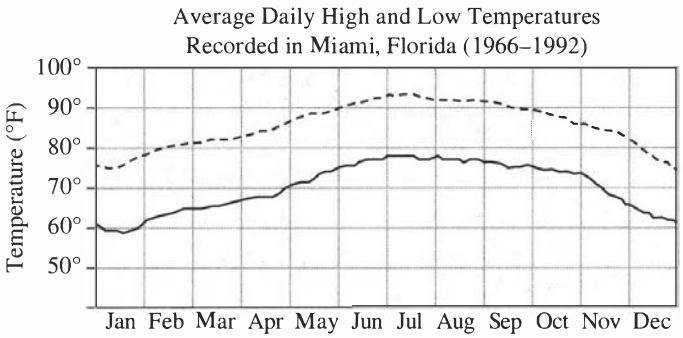
How many pairs of different numbers, chosen from the list above, have a sum that is odd?

A) Four
B) Five
C) Six
D) Seven

CONTINUE

4 4

Questions 6 and 7 refer to the following information.



The graph above shows average daily high and low temperatures recorded in Miami, Florida, during the years 1966–1992.

6

In the period between 1966 and 1992, during which of the following months was the average daily low temperature in Miami, Florida, closest to 75°F?

A) January
B) April
C) October
D) November

7

In the month for which the average daily high temperature in Miami was closest to 80°F, what was the average daily low temperature for the month?

A) 60°F
B) 63°F
C) 72°F
D) 74°F

8

Which of the following inequalities is equivalent to $-3x - 5y < 4y - 6$?

A) $x < -3y - 2$
B) $x > -3y + 2$
C) $x < 3y - 2$
D) $x > 3y + 2$

9

Anton's lemonade juice machine squeezes lemons at a constant rate of 75 lemons per hour. His brother, Luigi, has a lemonade juice machine that squeezes lemons at a constant rate of 80 lemons per hour. How many more minutes does it take Anton's machine to squeeze 40 lemons than it takes Luigi's machine?

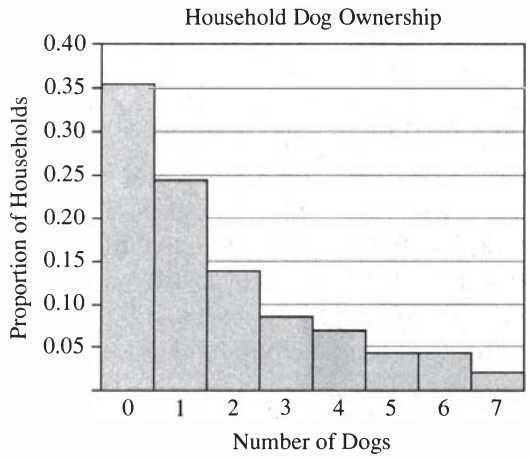
A) 2
B) 3
C) 4
D) 5

CONTINUE

4 4

Questions 10 and 11 refer to the following information.

The histogram below shows the proportion of households in a large suburban area that own a certain number of dogs.



10

According to a local ordinance, no household in this area may own more than 3 dogs. What is the approximate percentage of households in this area that are in violation of the ordinance?

A) 9%

B) 17%

C) 26%

D) 83%

11

Using the data in the histogram, a researcher calculates the median number of dogs per household and the average number of dogs per household for this suburban area. Based on the graph, which of the following statements must be true?

A) The median is equal to the average.

B) The median is greater than the average.

C) The median is less than the average.

D) The median could be greater than or less than the average, depending on the population of the suburban area.

12

Graham University admits one-third of those who apply, and of those admitted, 40% enroll. In 2013, a total of 46,815 students applied to Graham University. Which of the following is the best estimate of the number of students who will be admitted but will not enroll?

A) 6,242

B) 9,363

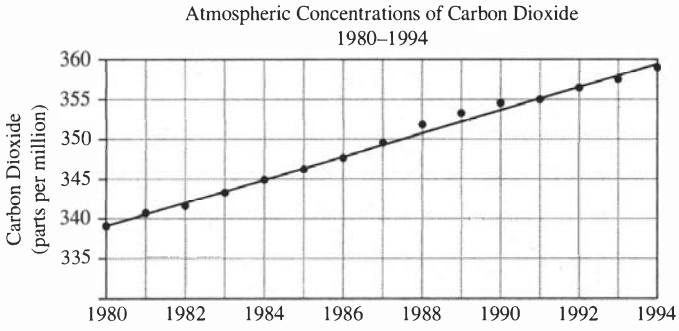
C) 15,605

D) 18,726

CONTINUE

4 4

13



According to the data in the scatter plot above, what was the approximate rate of increase in the atmospheric concentration of carbon dioxide, in parts per million per decade? (1 decade = 10 years)

A) 1.5

B) 3.0

C) 6.5

D) 14.5

14

Vie's security code consists of four integers, separated by hyphens, that satisfy the following rules:

- Exactly one of the four numbers is even.
- One of the four numbers is her age.
- Exactly one of the four numbers is triple the value of one of the other numbers.
- Exactly one of the four numbers is prime.

Which of the following could be Vie's security code?

A) 66-63-22-13

B) 51-17-11-18

C) 55-77-33-11

D) 50-63-39-13

Questions 15 and 16 refer to the following information.

Ms. Lee administered a 50-point quiz to her trigonometry class. After scoring the quiz, she decided that the scores were not as strong as she had hoped, so she gave the class a second quiz on the same material the next day. She told the class that she would count only the higher of the two scores for each student. The table below shows the scores on both quizzes for six students in the class.

Student	1	2	3	4	5	6
Quiz 1	40	34	34	34	36	20
Quiz 2	48	40	32	38	32	26

15

What is the average final score on the quiz for these six students?

A) 33

B) 36

C) 37

D) 38

16

If the final score of a seventh student were included in this set, the average of the seven scores would be 35. What is the final quiz score for the seventh student?

A) 21

B) 23

C) 24

D) 25

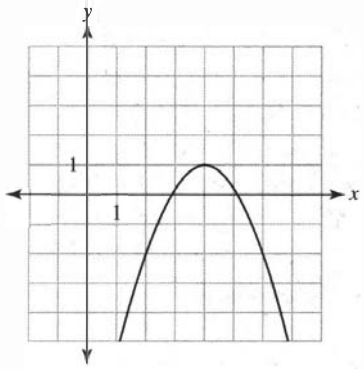
CONTINUE

4 4

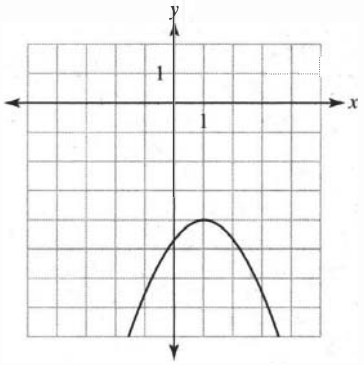
17

Which of the following could be the graph of $y - 4 = k(x - 1)^2$ in the xy-plane if k is a constant less than zero?

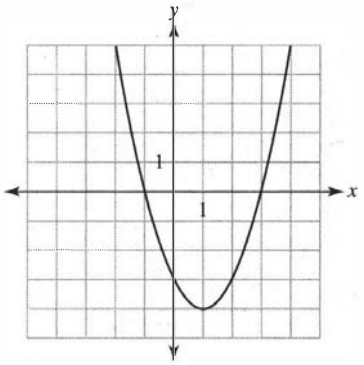
A)



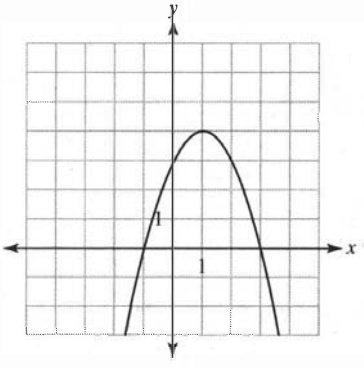
B)



C)



D)



18

When graphed in the xy-plane, the functions $f(x) = x^2 - x - 2$ and $g(x) = kx - 6$ intersect at the point $(-2, a)$. What is the value of k?

A) -5

B) -1

C) 3

D) 8

19

Nationwide Movie Theater Admissions		
Year	Total Cash Receipts (billions of dollars)	Total Attendance (billions)
2006	5.5	1.25
2007	5.9	1.34
2008	6.4	1.39
2009	7.0	1.48
2010	7.5	1.47
2011	7.7	1.40

The table above shows the total cash receipts for ticket sales, in billions of dollars, and the number of people attending, in billions, for movie theaters nationwide from 2006 to 2011. Based on the table, by how much did the receipts per attendee increase from 2006 to 2011?

A) \$0.11 per person

B) \$0.24 per person

C) \$1.10 per person

D) \$2.20 per person

CONTINUE

4 4

20

The current price of unleaded gasoline is \$2.10 per gallon and is expected to rise about \$0.16 per month for the foreseeable future. The current price of a gallon of diesel fuel is 30% higher than that of gasoline but is expected to increase only \$0.09 per month. If these trends hold, in how many months will the per-gallon price of the two fuels be equal?

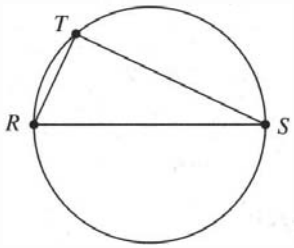
A) 9

B) 10

C) 11

D) 12

21



The segment RS is a diameter of the circle above. If the circumference of this circle is $\frac{13\pi}{2}$ and $RT = 2.5$, what is the area of triangle RTS?

A) 7.5

B) 8.125

C) 15.0

D) 16.25

22

$$a = 3k$$
$$b = 6k + 5$$
$$c = k$$
$$d = 4k - 5$$

Line l passes through the points (a, b) and (c, d), where a, b, c, and d satisfy the equations above, where $k > 0$. If line m is perpendicular to line l, what is the slope of line m, in terms of k?

A) $-\frac{k}{k+10}$

B) $-\frac{k}{k+5}$

C) $\frac{k+5}{k}$

D) $\frac{k+10}{k}$

23

If $\frac{3x}{y^2} = \frac{24y^{-2}}{x^{-2}}$, what is the value of x?

A) $\frac{1}{8}$

B) $\frac{1}{6}$

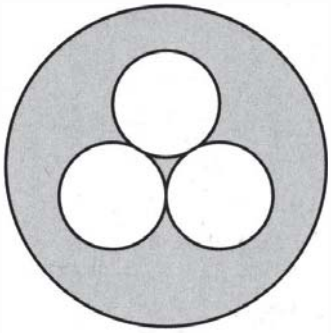
C) $\frac{1}{4}$

D) $\frac{1}{2}$

CONTINUE

4 4

24



In the figure above, each of the three smaller circles is tangent to the other two and has a diameter of 2 centimeters. The larger circle has a diameter of 6 centimeters. What fraction of the area of the larger circle is shaded?

A) $\frac{4}{9}$

B) $\frac{1}{2}$

C) $\frac{5}{9}$

D) $\frac{2}{3}$

25

If $18 - 2k^2 = 16k$, and $k < 0$, what is the value of k^2?

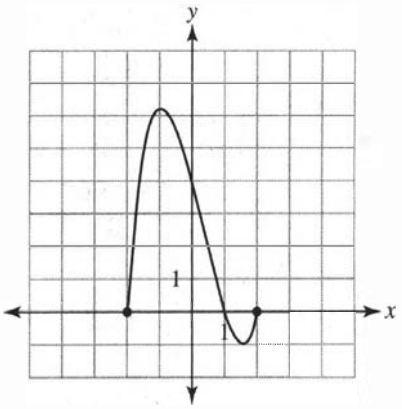
A) 1

B) 9

C) 36

D) 81

26



$f(x) = x^3 - x^2 - 4x + 4$

The function $y = f(x)$ is graphed in the xy-plane above on the interval $-2 \le x \le 2$. Which of the following indicates the range of x values on this interval for which $f(x) > 2x + 4$?

A) $-2 < x < 0$

B) $-2 < x < 1$

C) $-2 < x < 2$

D) $1 < x < 2$

CONTINUE

4 4

27

Stephanie, Daphne, and Brian each have one AP test to take. One test is in AP Biology, one is in AP U.S. History, and one is in AP Calculus. The tests are on three different days next week: Monday, Tuesday, and Wednesday. Stephanie's test is before Brian's test, but after the U.S. History test. Brian is not taking AP Biology. Which choice correctly indicates which test Stephanie is taking and on what day?

	Subject	Test Day
A)	AP Biology	Monday
B)	AP Biology	Tuesday
C)	AP Calculus	Tuesday
D)	AP U.S. History	Monday

28

If x is the radian measure of an angle, where $0 < x < \frac{\pi}{2}$ and $\tan x = k$, which of the following is the value of $\tan(\pi - x)$?

A) k

B) $-k$

C) $\pi - k$

D) $k - \pi$

Questions 29 and 30 refer to the following information.

The table below shows the women's world-record times for the 100 meters from 1911 to 1936.

Year	1911	1913	1920	1921	1925	1928	1932	1933	1934	1935	1936
Record Time (sec)	18.8	13.1	13.0	12.8	12.4	12.0	11.9	11.8	11.7	11.6	11.5

29

What was the average rate of change, in seconds per year, for the world-record women's 100-meter time over the period from 1911 to 1936?

A) −0.292

B) −0.281

C) −0.261

D) −0.130

30

By 1973, the world-record time for the women's 100 meters dropped to 10.8 seconds. The change from 1936 to 1973 is equal to the change from 1925 to what year?

A) 1926

B) 1928

C) 1934

D) 1935

CONTINUE

4  4

31

A line containing the points $(0, -5)$ and $(a, 27)$ has a slope of 4. What is the value of a?

32

As part of a presentation for a client, an architect constructs a model of a proposed building, using a scale of 1 inch to 64 feet. The architect wants to include a representation of an oak tree that would be preserved during the building process. If the oak tree is actually 96 feet tall, what is the height, in inches, of the model representation of this tree?

33

The equation $t^2 + 5t = 14$ has solutions $t = a$ and $t = b$. If $a < b$, what is the value of $b - a$?

34

Among the registered voters in Curtsville, 30% are registered as Democrats, 25% are registered as Republicans, and 45% are registered as Independents. In this year's election, 58% of the registered Democrats, 60% of the registered Republicans, and 64% of registered Independents cast their votes. If 372 registered Republicans cast their votes in Curtsville in this year's election, how many registered voters does Curtsville have?

35

What is the only positive integer value for n that satisfies the inequality $2 < \left|\frac{1}{3} - \frac{2}{3}n\right| < 3$?

36

What is the least positive integer m such that $\frac{1}{\sqrt{50}} - \frac{2}{m} > 0$?

CONTINUE

Questions 37 and 38 refer to the following information.

A "fair" coin is defined as a coin that, when tossed, has a 0.5 probability of landing heads and a 0.5 probability of landing tails. If a coin is not fair, one of those probabilities is greater than the other, but their sum is still 1.

Julia and Ivan use two methods to determine whether or not a coin is fair. Method 1 consists of performing 20 trials, each of which consists of tossing the coin 5 times and recording the number of times a head appears. The average of these 20 results is called x. The best estimate for the probability of this coin landing heads is then calculated with the formula $p=\frac{x}{5}$.

Method 2 consists of performing 20 trials, each of which involves tossing the coin until it lands heads and then recording number of the toss on which the first head was observed. The average of these 20 results is then called y. The best estimate for the probability of the coin landing heads is then calculated with the formula $p=\frac{1}{y}$.

37

Julia uses Method 2 to test a coin and discovers that, on average, the first head appears on toss 2.5. What is the best estimate for the probability that this coin will land tails?

38

Ivan uses Method 1 to test a coin and discovers that, on average, the coin lands heads 3.75 times out of 5 tosses. Based on Ivan's best estimate from this method, how many tosses, on average, should it take until the first head appears?

STOP

If you finish before time is called, you may check your work on this section only. Do not turn to any other section of the test.

SAT PRACTICE TEST 2 ANSWER KEY

Section 1: Reading

1. B
2. C
3. A
4. B
5. C
6. B
7. C
8. D
9. A
10. B
11. D
12. D
13. C
14. A
15. C
16. C
17. B
18. B
19. D
20. A
21. C
22. A
23. C
24. B
25. A
26. B
27. B
28. D
29. C
30. D
31. A
32. A
33. C
34. A
35. B
36. B
37. A
38. B
39. B
40. C
41. C
42. B
43. C
44. A
45. B
46. C
47. D
48. C
49. B
50. C
51. B
52. A

Total Reading Points (Section 1)

Section 2: Writing and Language

1. B
2. D
3. A
4. A
5. C
6. B
7. C
8. B
9. B
10. A
11. D
12. B
13. C
14. A
15. C
16. D
17. A
18. D
19. B
20. B
21. B
22. A
23. D
24. B
25. A
26. C
27. A
28. D
29. A
30. D
31. C
32. D
33. D
34. D
35. B
36. A
37. C
38. D
39. B
40. D
41. C
42. C
43. B
44. B

Total Writing and Language Points (Section 2)

Section 3: Math (No Calculator)

1. A
2. C
3. C
4. D
5. D
6. A
7. B
8. B
9. B
10. D
11. B
12. D
13. A
14. B
15. C
16. 3 or 1/2 or .5
17. 13
18. 9
19. 3
20. 6

Section 4: Math (Calculator)

1. B
2. C
3. A
4. A
5. C
6. C
7. B
8. B
9. A
10. B
11. C
12. B
13. D
14. D
15. C
16. B
17. D
18. A
19. C
20. A
21. A
22. B
23. A
24. D
25. D
26. A
27. B
28. B
29. A
30. C
31. 8
32. 1.5
33. 9
34. 2480
35. 4
36. 15
37. 3/5 or .6
38. 4/3 or 1.33

Total Math Points (Section 3 + Section 4)

than offering a *historically accurate depiction of the attack.*

40. **D** **Clear Expression of Ideas/Diction**

Picasso's vivid images *call forth* or *elicit* or *evoke* a sense of agony.

41. **C** **Standard English Conventions/Apostrophes**

The core (subject and verb) of this sentence is *critics agree*, so the apostrophe in the original phrasing and in choice D is incorrect. Choice B is incorrect because it creates a sentence with two uncoordinated verbs.

42. **C** **Clarity of Expression/Diction**

Though all choices are grammatically correct, only C maintains the appropriately serious tone of the rest of the paragraph. The words in the remaining choices are inappropriately informal or playful.

43. **B** **Clear Expression of Ideas/ Logical Comparisons**

This phrase is establishing a comparison between Picasso and Einstein, and therefore must be logical and follow the Law of Parallelism. Only choice B creates a clear, logical, and parallel comparison.

44. **B** **Clear Expression of Ideas**

Choice B provides the best conclusion because it alludes directly to the subject of the passage as a whole, "Guernica," while also indicating the legacy of Picasso.

Section 3: Math (No Calculator)

1. **A** **Heart of Algebra (solving equations) EASY**

$2(x+2)=3x-1$

Distribute: $2x+4=3x-1$

Subtract $2x$: $4=x-1$

Add 1: $5=x$

2. **C** **Additional Topics (systems) EASY**

Original equation: $\frac{1}{2}x+\frac{1}{3}y=7$

Multiply by 12: $12\left(\frac{1}{2}x+\frac{1}{3}y\right)=84$

Distribute: $6x+4y=84$

3. **C** **Algebra (solving equations) EASY**

Original equation: $\frac{3(k-5)}{4}=\frac{5+2k}{3}$

Cross multiply: $9(k-5)=20+8k$

Distribute: $9k-45=20+8k$

Subtract $8k$ and add 45: $k=65$

4. **D** **Algebra (systems) EASY**

There are several methods we could use to solve this system. One method is simply to "test" each ordered pair in the choices to see which one satisfies both equations. If you prefer to do it algebraically, notice that the answer choices tell us that we should solve the system for y instead of x. Why? Because all of the y-coordinates are different in the choices, but some of the x-coordinates repeat. Therefore, the best algebraic strategy is to substitute for x and solve for y.

Original equations: $x=5y+5$ and $2x-y=19$

Substitute $x=5y+5$ into the second equation:

$2(5y+5)-y=19$

Distribute: $10y+10-y=19$

Combine like terms: $9y+10=19$

Subtract 10: $9y=9$

Divide by 9: $y=1$

Notice that the only choice with a y-coordinate of 1 is choice D (10, 1). Substituting $x=10$ and $y=1$ into both equations confirms that this ordered pair satisfies both equations.

5. **D** **Advanced Mathematics (two-variable relationships) EASY**

This is a straightforward conversion problem:

$$p \text{ pounds} \times \frac{1 \text{ kilogram}}{2.2 \text{ pounds}} \times \frac{40 \text{ milligrams amoxicillin}}{1 \text{ kilogram}}$$

$$= \frac{40p}{2.2} \text{ milligrams of amoxicillin}$$

6. **A** **Algebra (inequalities) MEDIUM**

If Calvin hangs n photographs, there will be $n-1$ spaces between them, and each space is 2 inches wide. Since each photograph is 9 inches wide, the total width of the display is $9n+2(n-1)$ inches. If this display can be no wider than 12 feet (or 144 inches), then $9n+2(n-1)\le 144$.

7. **B** **Advanced Mathematics (quadratics) MEDIUM**

Solutions of the system coincide with those points at which all three graphs intersect. There are 2 such points of intersection: $(-\sqrt{2},0)$ and $(\sqrt{2},0)$.

8. **B** **Advanced Mathematics (functions) MEDIUM**

The key to this problem is seeing that the equation is a standard "rate × time = work" equation. The blue hose takes 10 hours to fill the pool, so it fills the pool at a rate of $\frac{1}{10}$ pool per hour. The red hose requires 5 hours to do the job, so it fills at a rate of $\frac{1}{5}$ pool per hour. Together,

then the two hoses working together fill the pool at a rate of $\frac{1}{10}+\frac{1}{5}$ pool per hour. The right side of the equation shows the total amount of work done, which is 1 pool. Therefore, x represents the total time, in hours, needed to fill one pool using both hoses.

9. **B** **Special Topics (imaginary numbers) MEDIUM**

Original expression: $2i^2+3i^3-4i^4+5i^5$
Based on the definition that $i=\sqrt{-1}$, we can show that $i^2=-1$, $i^3=-i$, $i^4=1$, and $i^5=i$.
Substitute these values into original expression: $2(-1)+3(-i)-4(1)+5(i)$
Simplify: $-2-3i-4+5i$
Combine like terms: $-6+2i$

10. **D** **Advanced Mathematics (sequences and inverse operations) MEDIUM-HARD**

The third term of the sequence is 20. To find the fourth term, we can simply apply the rule of the sequence: "multiply the previous term by 3 and subtract 4." This gives 3(20) − 4 = 56 as the fourth term. The second term can be found by applying the *inverse* of the sequence rule: "*add* 4 to the *previous* term, then *divide* by 3." Applying this to the third term gives us (20 + 4) ÷ 3 = 8 for the second term. Therefore, the sum of the second term and the fourth term is 8 + 56 = 64.

11. **B** **Advanced Mathematics (factoring polynomials) MEDIUM-HARD**

We can find the x-intercepts of $y=g(x)$ by factoring:
$g(x)=x^2+6x+8=(x+2)(x+4)$
The Zero Product Property shows that the graph of $y=g(x)$ has x-intercepts at (−2, 0) and (−4, 0). If the graph of $y=f(x)$ shares these same x-intercepts, it must also share the same factors associated with those x-intercepts, $(x+2)$ and $(x+4)$. If we define the third x-intercept of g as $x=k$, then f should factor like this:
$f(x)=x^3+2x^2-16x-32=(x+2)(x+4)(x-k)$
Now we can find the value of k by just noticing that the product of the constant terms in the factors must equal the constant term in the original polynomial, which is −32:
$(2)(4)(-k)=-32$
Simplify: $-8k=-32$
Divide by −8: $k=4$
Therefore, the third x-intercept of $y=f(x)$ is (4, 0).

12. **D** **Algebra (simplifying rational expressions) MEDIUM**

Substitute for x and y, then factor and cancel:

$$\frac{10x}{y}=\frac{10(a^2-b^2)}{2(a-b)}=\frac{10\cancel{(a-b)}(a+b)}{2\cancel{(a-b)}}=\frac{10(a+b)}{2}=5a+5b$$

13. **A** **Advanced Mathematics (analysis of parabolas) MEDIUM-HARD**

The graph of $y=g(x)$ is a parabola with a vertex at (2, 0). If this parabola is shifted up one unit, it will have a vertex at (2, 1). Since all of the equations in the choices are written in vertex form ($y=(x-k)^2+k$, where (h, k) is the vertex), it should be easy to see that the correct equation is $y=(x-2)^2+1$.

14. **B** **Additional Topics (volume) MEDIUM**

Since we want the volume of the beam in cubic feet, it's a good idea to convert all measures to feet. The edges of the cross-shaped base are all 6 inches, or 1/2 foot, in measure; therefore, each of the five squares on the base has an area of $(1/2)^2=1/4$ square foot, so the total area of the base is $5(1/4)=5/4$ square feet. The length of the beam is 72 inches, or 72 ÷ 12 = 6 feet. Since the volume of a prism is equal to the area of the base times the height, the volume is (5/4)(6) = 30/4 = 7.5 cubic feet.

15. **C** **Additional Topics (exponentials and radicals) HARD**

Recall that analyzing exponentials is easier if we have a common base. Notice that $64=2^6$ and $4=2^2$.
Original equation: $4^{4x}=64a^4$
Substitute $64=2^6$ and $4=2^2$: $(2^2)^{4x}=2^6a^4$
Simplify: $2^{8x}=2^6a^4$
Raise both sides to the 1/4 power: $2^{2x}=2^{3/2}a$
Substitute second equation, $2^{2x}=na$: $na=2^{3/2}a$
Divide by a: $n=2^{3/2}=(2^3)^{1/2}=\sqrt{2^3}=\sqrt{8}$

16. **3 or 1/2 or .5** **Advanced Mathematics (quadratic systems) MEDIUM**

There are several ways to tackle this question, but perhaps the simplest is to solve the first equation for t and then plug into the second equation to find x.
$t^2=5t$
Subtract $5t$: $t^2-5t=0$
Factor: $t(t-5)=0$
Use the Zero Product Property: $t=0$ or $t=5$
Plug $t=0$ into the second equation: $0=2x-1$
Add 1: $1=2x$
Divide by 2: $1/2=x$
Plug $t=5$ into the second equation: $5=2x-1$
Add 1: $6=2x$
Divide by 2: $3=x$
Therefore, x can equal either 1/2 or 3.

17. **13** **Algebra (word problems) MEDIUM**

Let n equal the number of 5-point questions that Alliyah answered correctly. If she answered 20 questions correctly overall, then she must have answered $20-n$ of the 3-point questions correctly. Therefore, she earned $5n+3(20-n)$ points in total: $5n+3(20-n)=86$

Distribute: $5n + 60 - 3n = 86$
Combine like terms: $2n + 60 = 86$
Subtract 60: $2n = 26$
Divide by 2: $n = 13$

18. **9** **Algebra (solving equations) MEDIUM-HARD**

$$\frac{x}{x^2-9} - \frac{1}{x+3} = \frac{1}{4x-12}$$

Factor the denominators:

$$\frac{x}{(x-3)(x+3)} - \frac{1}{x+3} = \frac{1}{4(x-3)}$$

Multiply by the common denominator, $4(x-3)(x+3)$:

$$4\cancel{(x+3)}\cancel{(x-3)}\left(\frac{x}{\cancel{(x-3)}\cancel{(x+3)}}\right) - 4\cancel{(x+3)}(x-3)\left(\frac{1}{\cancel{x+3}}\right)$$

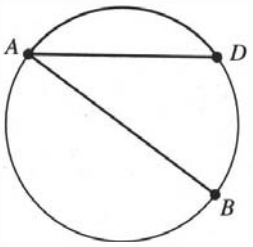
$$= -4(x+3)\cancel{(x-3)}\left(\frac{1}{\cancel{4(x-3)}}\right)$$

Simplify: $4x - 4(x-3) = x + 3$
Distribute: $4x - 4x + 12 = x + 3$
Simplify: $12 = x + 3$
Subtract 3: $9 = x$

19. **3** **Algebra (systems of equations) MEDIUM**

Given equations: $m^2 + 5 = x$ and $9y^2 = x - 5$
Substitute $m^2 + 5 = x$ into the second equation: $9y^2 = (m^2 + 5) - 5$
Simplify: $9y^2 = m^2$
Divide by y^2: $9 = \frac{m^2}{y^2}$
Take the square root: $3 = \left|\frac{m}{y}\right|$

20. **6** **Additional Topics (arcs and circles) HARD**



The circumference of a circle is equal to its diameter times π; therefore, the circumference of the circle is $\frac{20}{\pi} \times \pi = 20$. Since $<BAD$ is an inscribed angle with measure of $\frac{\pi}{5}$ radians, its intercepted arc DB has a measure $2 \times \frac{\pi}{5} = \frac{2\pi}{5}$. Since arc ADB is a semicircle, arc AD has a radian measure of $\pi - \frac{2\pi}{5} = \frac{3\pi}{5}$. We can find its length now based on the fact that the length of any arc in a circle is proportional to its radian measure:

$$\frac{\frac{3\pi}{5}}{2\pi} = \frac{x}{20}$$

Cross-multiply: $12\pi = 2x\pi$
Divide by 2π: $6 = x$

Section 4: Math (Calculator)

1. **B** **Algebra (analysis of lines) EASY**

One strategy you could use is to match up the key features of the linear equations with the features of the graph and work by process of elimination. The graph has an x-intercept of 4, a y-intercept of 2, and a slope of $-1/2$. Choice A has the wrong slope (1/2). Choice C has the correct y-intercept (2) but the wrong slope (–2), and choice D has both the wrong slope (2) and the wrong y-intercept (4).

2. **C** **Algebra (representing relationships) EASY**

Let x be the maximum number of shares Gretchen can purchase. Therefore, $\$3.19x$ is the total cost of those shares, and $\$(3.19x + 75)$ must be less than or equal to $\$4{,}000$: $3.19x + 75 \le 4{,}000$
Subtract 75: $3.19x \le 3{,}925$
Divide by 3.19: $x \le 1{,}230.41$
Therefore, Gretchen can purchase a maximum of 1,230 shares and stay within her budget.

3. **A** **Additional Topics (3-D geometry) EASY**

If a sphere is inscribed in a cube, then the diameter of the sphere is equal to the length of one edge of the cube. If each edge of the cube has length x inches and the cube has a volume of 64 cubic inches: $V = x^3 = 64$
Take the cube root: $x = 4$
This means that the diameter of the sphere is also 4 inches, so the radius of the sphere is $4 \div 2 = 2$ inches.

4. **A** **Advanced Mathematics (functions) EASY**

If $x = 1$ is a zero of the function, then $f(1) = 0$:
$f(1) = (1)^3 - (1)^2 + p(1) - 3 = 0$
Simplify: $1 - 1 + p - 3 = 0$
Simplify: $p - 3 = 0$
Add 3: $p = 3$

5. **C** **Problem Solving and Data Analysis (counting) EASY**

One good way to count all of the possible pairs is to use the Method of Exhaustion: start by "exhausting" all such

pairs that include the number 1; then move on to all of the remaining pairs that include the number 2, et cetera:
$1+2=3$ $1+4=5$ $2+3=5$ $2+5=7$ $3+4=7$ $4+5=9$

6. **C** **Problem Solving and Data Analysis (multi-variable graphs) EASY**

The solid line shows the average daily low temperatures. In October, the average low temperature varies from about 76°F to about 73°F, so the average low temperature for October is closest to 75°F.

7. **B** **Problem Solving and Data Analysis (multi-variable graphs) MEDIUM**

You might notice that the difference between the average high temperature and the average low temperature is between 12°F and 18°F throughout the year. Therefore, in the month when the average high temperature is 80°F, we should expect the average low temperature to be between 80°F – 18°F = 62°F and 80°F – 12°F = 68°F. The only choice within this range is (C) 63°F. Specifically, the months in which the average high temperature is closest to 80°F are February and October, and the graph shows that the average low temperatures for these months are both around 63°F.

8. **B** **Algebra (inequalities) EASY**

Original inequality: $-3x - 5y < 4y - 6$
Add $5y$: $-3x < 9y - 6$
Divide by –3 and "flip" the inequality: $x > -3y + 2$

9. **A** **Algebra (rates) MEDIUM**

We can up proportions to determine how many minutes it would take for each to squeeze 40 lemons.

Anton's machine: $\frac{75 \text{ lemons}}{60 \text{ minutes}} = \frac{40 \text{ lemons}}{x \text{ minutes}}$
Cross-multiply: $75x = 2{,}400$
Divide by 75: $x = 32$ minutes

Luigi's machine: $\frac{80 \text{ lemons}}{60 \text{ minutes}} = \frac{40 \text{ lemons}}{y \text{ minutes}}$
Cross multiply: $80x = 2{,}400$
Divide by 80: $y = 30$ minutes

Therefore, Anton's machine takes 2 minutes longer than it takes Luigi's machine.

10. **B** **Problem Solving and Data Analysis (histograms) MEDIUM**

According to the histogram, the proportion of households that own 4 or more dogs is approximately 0.07 + 0.04 + 0.04 + 0.02 = 0.17, or 17% of the households.

11. **C** **Problem Solving and Data Analysis (central tendency) MEDIUM**

The median is the number below which 50% of the data fall and above which 50% of the data fall. According to the histogram, about 36% of the households have no dogs, and another 24% have 1 dog. Because 36% + 24% = 60%, the median number of dogs is 1. The overall shape of the histogram is "skewed right," which means that there are more numbers that are much larger than the median than much lower than the median. In these cases, the average is always larger than the median.

To calculate the average, you can multiply each proportion of households times the number of dogs that proportion owns and add the results. Average = 0.36(0) + 0.24(1) + 0.14(2) + 0.09(3) + 0.07(4) + 0.04(5) + 0.04(6) + 0.02(7) = 1.65 pets per household.

12. **B** **Problem Solving and Data Analysis (percentages) EASY-MEDIUM**

Since the total number of applicants was 46,815, one-third of those, or 46,815 ÷ 3 = 15,605 were admitted. Of these, 100% – 40% = 60% will not enroll, and 0.60(15,605) = 9,363.

13. **D** **Data Analysis (scatter plots) MEDIUM**

To find the rate of increase per decade, it is best to choose two points that are 10 years apart, such as 1980 and 1990. In 1980, the atmospheric concentration of carbon dioxide was approximately 339 parts per million, and in 1990, it was approximately 354 parts per million, for a rate of 354 – 339 = 15 parts per million per decade, which is closest to choice D.

14. **D** **Additional Topics (logic) MEDIUM**

For many logic questions, it helps to work by process of elimination.

A) This choice violates the rule that **exactly one of the four numbers is even**. It has two: 22 and 66.

B) This choice violates the rule that **exactly one of the four numbers is prime**. It has two: 17 and 11.

C) This choice violates the rule that **exactly one of the four numbers is even**. It has no even numbers.

D) This violates none of the rules.

15. **C** **Problem Solving and Data Analysis (central tendency) MEDIUM**

The final score for each student is the higher of the two, so the average final score for these 6 students is (48 + 40 + 34 + 38 + 36 + 26) ÷ 6 = 37.

16. **B** **Problem Solving and Data Analysis (averages) MEDIUM**

The sum of the final scores for the original 6 students is 48 + 40 + 34 + 38 + 36 + 26 = 222. If the average of the

scores for all seven students is 35, then the sum of the seven scores must be $35 \times 7 = 245$. Therefore, the score for the seventh student must by $245 - 222 = 23$.

17. **D** **Advanced Mathematics (graphs of quadratic functions) MEDIUM-HARD**

Original equation: $y - 4 = k(x-1)^2$
Add 4: $y = k(x-1)^2 + 4$
This is the "vertex form" of a quadratic function, which reveals that the graph is a parabola with vertex at (1, 4). The fact that k is a constant less than zero tells us that this parabola is "open down." The only choice that satisfies both of these criteria is D.

18. **A** **Advanced Mathematics (functions) MEDIUM-HARD**

If the graph of f contains the point $(-2, a)$, then $f(-2) = a$:
$f(-2) = (-2)^2 - (-2) - 2 = a$
Simplify: $4 + 2 - 2 = a$
Simplify: $4 = a$
The graph of g must also contain the point (–2, 4), so
$g(-2) = 4$: $g(-2) = k(-2) - 6 = 4$
Simplify: $-2k - 6 = 4$
Add 6: $-2k = 10$
Divide by –2: $k = -5$

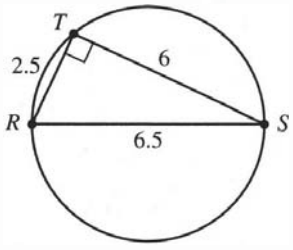
19. **C** **Data Analysis (calculations) MEDIUM**

In 2006, the receipts per attendee was \$5.5 billion ÷ 1.25 billion people = \$4.40. In 2011, the receipts per attendee was \$7.7 billion ÷ 1.40 billion people = \$5.50, for an increase of \$5.50 – \$4.40 = \$1.10 per person.

20. **A** **Algebra (word problems) MEDIUM-HARD**

If the current price per gallon for unleaded gasoline is \$2.10, then the current price for a gallon of diesel fuel is 30% higher: (1.3)(\$2.10) = \$2.73. In m months, the price of unleaded gas will be $\$(2.10 + 0.16m)$ and the price of diesel will be $\$(2.73 + 0.09m)$. We want to know when these prices are equal: $2.10 + 0.16m = 2.73 + 0.09m$
Subtract $0.09m$ and 2.10: $0.07m = 0.63$
Divide by 0.07: $m = 9$

21. **A** **Additional Topics (Circles and Triangles) MEDIUM-HARD**



Because RS is a diameter, it divides the circle into two semicircles. Since angle RTS is an inscribed angle that intercepts an arc of 180°, angle RTS must be a 90° angle. The circumference is equal to π times the diameter, so the diameter must be 13/2 = 6.5. We can now use the Pythagorean Theorem to find TS: $TS^2 + 2.5^2 = 6.5^2$
Simplify: $TS^2 + 6.25 = 42.25$
Subtract 6.25: $TS^2 = 36$
Take the square root: $TS = 6$
Now we may use RT and TS as the bases of the triangle to calculate the area of the triangle: (2.5)(6) ÷ 2 = 7.5.

22. **B** **Algebra (slopes of lines) MEDIUM-HARD**

Recall that slopes of perpendicular lines are opposite reciprocals of each other.
Slope of line l: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{d-b}{c-a}$
Therefore, the slope of line m is its opposite reciprocal:
$$-\frac{c-a}{d-b} = \frac{a-c}{d-b}$$
Substitute for all unknowns in terms of k and simplify:
$$\frac{a-c}{d-b} = \frac{3k-k}{(4k-5)-(6k+5)} = \frac{2k}{-2k-10} = -\frac{k}{k+5}$$

23. **A** **Algebra (exponentials) MEDIUM**

Original equation: $\frac{3x}{y^2} = \frac{24y^{-2}}{x^{-2}}$
Cross-multiply: $(3x)(x^{-2}) = (24y^{-2})(y^2)$
Simplify: $3x^{-1} = 24y^0$
Multiply by x (and remember that $y^0 = 1$): $3 = 24x$
Divide by 24: $\frac{3}{24} = \frac{1}{8} = x$

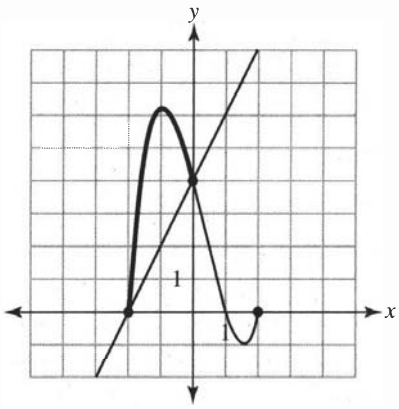
24. **D** **Additional Topics (circles) MEDIUM-HARD**

The large circle has a diameter of 6 centimeters, so its radius is 6 ÷ 2 = 3 centimeters, and its area is $\pi(3)^2 = 9\pi$ square centimeters. Each small circle has a diameter of 2 centimeters, so its radius is 2 ÷ 2 = 1 centimeter, and its area is $\pi(1)^2 = \pi$ square centimeters. Since there are three of them, their total area is 3π square centimeters. This means that the shaded region has an area of $9\pi - 3\pi = 6\pi$, which is $6\pi/9\pi = 2/3$ of the total.

25. **D** **Algebra (solving equations) MEDIUM-HARD**

Original equation: $18 - 2k^2 = 16k$
Add $2k^2$ and subtract 18: $0 = 2k^2 + 16k - 18$
Divide by 2: $0 = k^2 + 8k - 9$
Factor: $0 = (k+9)(k-1)$
Solve using the Zero Product Property: $k = -9$ or 1
Since the problem tells us that k must be negative, $k = -9$ and therefore, $k^2 = 81$.

26. **A** **Advanced Mathematics (analyzing functions) HARD**

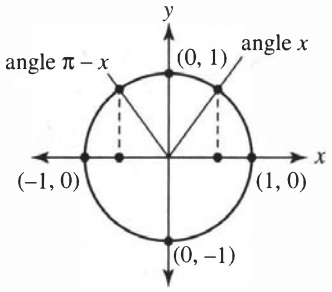


If we draw the line $y = 2x + 4$, then all of the points above this line are the points where $y > 2x + 4$. The part of the function that lies above the line, as the graph shows, are the points corresponding to the values of x between -2 and 0.

27. **B** **Additional Topics (logic) MEDIUM-HARD**

Since Stephanie's test is before at least one test (Brian's test), it can't be the last test, so her test must be on either Monday or Tuesday. Since Stephanie's test is after at least one test (the U.S. History test), it cannot be the first test, so it must be on either Tuesday or Wednesday. Therefore, Stephanie's test must be on Tuesday, the U.S. History test is on Monday, and Brian's test is on Wednesday. If Brian is not taking the Biology test, and he is not taking the U.S. History test (because it is on Monday), he must be taking the Calculus test. Therefore, Daphne must be taking the U.S. History test on Monday, Stephanie is taking the Biology test on Tuesday, and Brian is taking the Calculus test on Wednesday.

28. **B** **Advanced Mathematics (trigonometry) HARD**



It first helps to notice that x and $\pi - x$ are supplements; that is, they have a sum of π radians or 180°. Since x is in the first quadrant, where the tangent is positive, its supplement is in the second quadrant, where the tangent is negative. These two angles have the same reference angle, so their tangents are opposites.

29. **A** **Problem Solving and Data Analysis (rates) MEDIUM**

The period from 1911 to 1936 is a 25-year span, and the total decrease in the world record time over that period is $11.5 - 18.8 = -7.3$ seconds. Therefore, the rate of change is $-7.3 \div 25 = -0.292$ seconds per year.

30. **C** **Data Analysis (drawing comparisons) MEDIUM-HARD**

The change from 1936 to 1973 was $10.8 - 11.5 = -0.7$ seconds. In 1925, the winning time was 12.4. A change of -0.7 would take the winning time to 11.7, which was the record in 1934.

31. **8** **Algebra (linear relationships) EASY**

We can simply use the slope formula to solve this one:

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{27 - (-5)}{a - 0} = \frac{32}{a}$$

If the slope of this line is 4: $\frac{32}{a} = 4$

Multiply by a: $32 = 4a$

Divide by 4: $8 = a$

32. **1.5** **Data Analysis (ratios) MEDIUM**

Let x = the size of the model tree in inches. The scale of 1 inch to 64 feet allows us to set up a proportion:

$$\frac{1}{64} = \frac{x}{96}$$

Cross-multiply; $64x = 96$

Divide by 64: $x = 1.5$

33. **9** **Advanced Mathematics (solving quadratics) MEDIUM**

Original equation: $t^2 + 5t = 14$

Subtract 14: $t^2 + 5t - 14 = 0$

Factor: $(t + 7)(t - 2) = 0$

Solve with the Zero Product Property: $t = -7$ or $t = 2$

Since a is the lesser of the two solutions, then $a = -7$ and $b = 2$, so $b - a = 2 - (-7) = 9$.

34. **2480** **Problem Solving and Data Analysis (percents) MEDIUM**

Let n represent the number of registered voters in Curtsville. If 25% of these are registered as Republicans, then there are $0.25n$ registered Republicans in Curtsville. If 60% of these voted, then $(0.60)(0.25n)$ registered Republicans voted: $(0.60)(0.25n) = 372$

Simplify: $0.15n = 372$

Divide by 0.15: $n = 2480$

35. **4** **Algebra (inequalities) MEDIUM-HARD**

$$2 < \left| \frac{1}{5} - \frac{2}{5}n \right| < 5$$

Consider that the argument to the absolute value could be positive or negative:

$$2<\frac{1}{3}-\frac{2}{3}n<3 \text{ or } -3<\frac{1}{3}-\frac{2}{3}n<-2$$

Multiply by 3: $6 < 1 - 2n < 9$ or $-9 < 1 - 2n < -6$

Subtract 1: $5 < -2n < 8$ or $-10 < -2n < -7$

Divide by –2 and "flip" the inequalities:

$$-2.5 > n > -4 \text{ or } 5 > n > 3.5$$

The only integer that satisfies the first inequality is $n = -3$, and the only integer that satisfies the second inequality is $n = 4$. Since the question asks only for the positive integer value, the answer is $n = 4$.

36. **15** **Additional Topics (rational inequalities) MEDIUM**

$$\frac{1}{\sqrt{50}}-\frac{2}{m}>0$$

Multiply by $m\sqrt{50}$ (the common denominator):

$$m-2\sqrt{50}>0$$

Add $2\sqrt{50}$: $m>2\sqrt{50}\approx 14.1$

So the least positive integer that satisfies this inequality is $m = 15$.

37. **3/5 or .6** **Problem Solving and Data Analysis (probability) MEDIUM**

If the first head appeared, on average, after 2.5 tosses, then the best estimate for the probability of landing heads is $1 \div 2.5 = 0.4$. Recall that the probability of the coin landing heads plus the probability of it landing tails must equal 1; therefore, the probability of the coin landing tails is $1 - 0.4 = 0.6$ or 3/5.

38. **4/3 or 1.33** **Advanced Mathematics (probability) HARD**

If the coin land heads an average of 3.75 times out of 5, then the best estimate for the probability of the coin landing heads is $3.75 \div 5 = 0.75$. If this value for p is then used in the formula for Method 2, where y represents the number of the roll on which the first head appears, we have:

$$0.75=\frac{1}{y}$$

Multiply by y: $0.75y = 1$

Divide by 0.75: $y = 1.33$ or $4/3$

3 3

Math Test—No Calculator

25 MINUTES, 20 QUESTIONS

Turn to Section 3 of your answer sheet to answer the questions in this section.

DIRECTIONS

For questions 1–15, solve each problem, choose the best answer from the choices provided, and fill in the corresponding circle on your answer sheet. **For questions 16–20**, solve the problem and enter your answer in the grid on the answer sheet. Please refer to the directions before question 16 on how to enter your answers in the grid. You may use any available space in your test booklet for scratch work.

NOTES

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3. Figures provided in this test are drawn to scale unless otherwise indicated.
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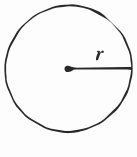
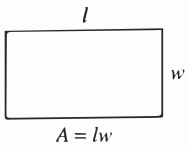
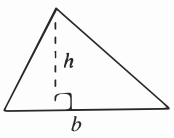
REFERENCE



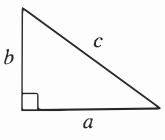
$A = \pi r^2$
$C = 2\pi r$



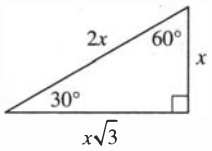
$A = lw$

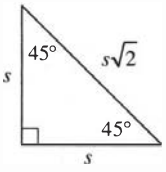


$A = \frac{1}{2}bh$

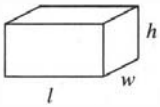


$c^2 = a^2 + b^2$

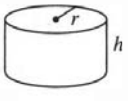




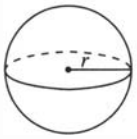
Special Right Triangles



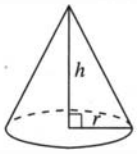
$V = lwh$



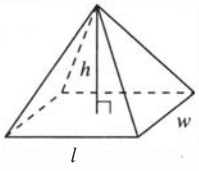
$V = \pi r^2 h$



$V = \frac{4}{3}\pi r^3$

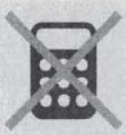


$V = \frac{1}{3}\pi r^2 h$



$V = \frac{1}{3}lwh$

The number of degrees of arc in a circle is 360.
The number of radians of arc in a circle is 2π.
The sum of the measures in degrees of the angles of a triangle is 180.

3  3

1

If $\frac{7}{3}a = \frac{3}{4}$, what is the value of a?

A) $\frac{9}{28}$

B) $\frac{6}{11}$

C) $\frac{7}{4}$

D) $\frac{28}{9}$

2

If $3x + a = x + b$, what is the value of x in terms of a and b?

A) $\frac{b-a}{2}$

B) $\frac{b-a}{3}$

C) $\frac{a+b}{3}$

D) $\frac{a-b}{2}$

3

Laura's car averages 27 miles per gallon for highway driving and 21 miles per gallon for city driving. Which of the following equations relates the minimum number of gallons, x, her car needs to travel 10 miles on the highway and b miles in the city?

A) $x = \frac{27}{10} + \frac{21}{b}$

B) $x = \frac{10}{21} + \frac{b}{27}$

C) $x = \frac{10}{27} + \frac{b}{21}$

D) $x = 270 + 21b$

4

Which of the following is equal to $\sqrt[3]{x^7}$ for all values of x?

A) $x^{\frac{3}{7}}$

B) $x^{\frac{7}{3}}$

C) x^{10}

D) x^{21}

5

A line in the xy-plane has a slope of $-\frac{3}{4}$ and passes through the point (8, 4). What is the y-intercept of this line?

A) (0, −6)

B) (0, −2)

C) (0, 6)

D) (0, 10)

6

$$(ab^2c^2 + abc - a^2b^2c) - (a^2b^2c - ab^2c^2 + abc)$$

Which of the following is equivalent to the expression above?

A) $2abc - 2a^2b^2c$

B) $2abc + 2a^2b^2c$

C) $2ab^2c^2 - 2a^2b^2c$

D) $2ab^2c^2 + 2a^2b^2c$

7

In the xy-plane, an isosceles triangle has two vertices on the x-axis and one vertex on the y-axis at (0, 4). The perimeter of the triangle is 16 units. Which of the following could be another vertex of the triangle?

A) (0, 3)

B) (−3, 0)

C) (4, 0)

D) (0, −4)

CONTINUE

3

3

8

If $a = x^2 - 1$ and $b = (x - 1)^2$, which of the following is equal to $\frac{a-b}{2}$ for all values of x?

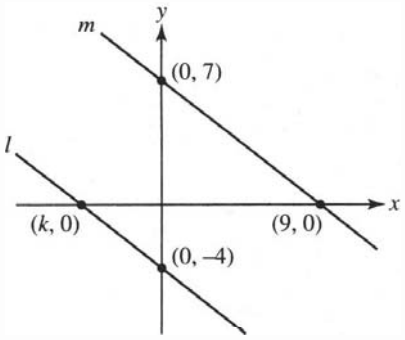
A) 0

B) 2

C) x

D) $x - 1$

9



In the xy-plane above, line l is parallel to line m. What is the value of k?

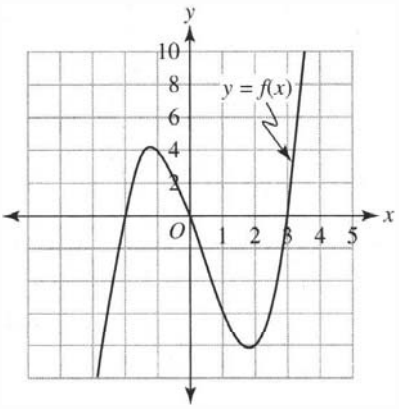
A) $-\frac{36}{7}$

B) $-\frac{32}{7}$

C) $-\frac{37}{9}$

D) $-\frac{28}{9}$

10



$f(x) = x^3 - x^2 - 6x$

The graph in the xy-plane of the function $y = f(x)$ is shown above. If the graph of $y = -2x - 4$ were drawn on the same axes, which of the following would not be a point of intersection of the two graphs?

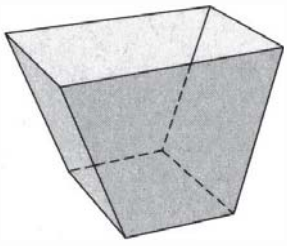
A) $(-2, 0)$

B) $(0, 0)$

C) $(1, -6)$

D) $(2, -8)$

11



A planter is built with a 12-inch by 12-inch square base and a 12-inch by 18-inch rectangular top so that the front and back faces of the planter are parallel trapezoids. The planter is 2 feet high. If this planter is filled with soil that costs $4 per cubic foot, what is the cost of soil required to fill the planter? (1 foot = 12 inches)

A) $8

B) $10

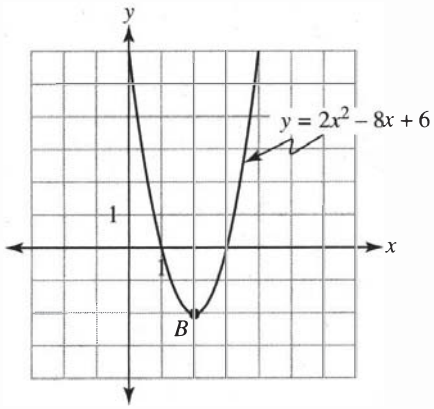
C) $12

D) $14

CONTINUE

3 3

12



Which of the following equivalent forms of the equation graphed in the xy-plane above shows the coordinates of vertex B as constants in the equation?

A) $y = 2(x^2 - 4x + 3)$

B) $y = 2(x - 1)(x - 3)$

C) $y = 2(x - 2)^2 - 2$

D) $y = 2(x^2 - 4x) + 6$

13

The nth term, a_n, of a sequence is given by the formula $a_n = a_1 r^{n-1}$ where r is a constant. If $a_1 = \frac{1}{4}$ and $r = 2$, for what value of n is $a_n = 32$?

A) 5

B) 6

C) 7

D) 8

14

For a polynomial $p(x)$, the value of $p(2) = 1$ and $p(4) = -1$. Which of the following must be true about $p(x)$?

A) $x - 4$ is a factor of $p(x)$.

B) $x - 1$ is a factor of $p(x)$.

C) $p(x)$ has a zero between $x = 2$ and $x = 4$.

D) The remainder when $p(x)$ is divided by $x - 1$ is 2.

15

$$\frac{x^2+1}{x+1} = x - 1 + \frac{b}{x+1}$$

If the equation above is true for all values of x, what is the value of b?

A) -2

B) 0

C) 1

D) 2

CONTINUE

3 3

16

If $x > 0$ and $(x-3)^2 = 49$, what is the value of x?

17

Fernanda needs to buy a total of 24 pens and pencils. Pens cost 50 cents each, and pencils cost 5 cents each. If she does not want to spend more than \$5 total for the writing utensils, what is the maximum number of pens she can buy?

18

If (1, 2) is the midpoint of the segment with endpoints at $(-2, -2)$ and (a, b), what is the value of $a + b$?

19

What is the least positive integer n that satisfies the inequality $\frac{1}{100} - \frac{9}{n^2} > 0$?

20

$$ax + bx = 8$$

$$\frac{1}{5}x + \frac{2}{3}y = 30$$

If the system above has no solutions, what is the value of $\frac{a}{b}$?

STOP

If you finish before time is called, you may check your work on this section only. Do not turn to any other section of the test.

4 4

Math Test—Calculator

55 MINUTES, 38 QUESTIONS

Turn to Section 4 of your answer sheet to answer the questions in this section.

DIRECTIONS

For questions 1–30, solve each problem, choose the best answer from the choices provided, and fill in the corresponding circle on your answer sheet. **For questions 31–38**, solve the problem and enter your answer in the grid on the answer sheet. Please refer to the directions before question 31 on how to enter your answers in the grid. You may use any available space in your test booklet for scratch work.

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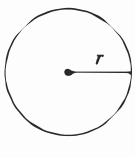
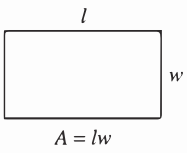
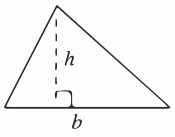


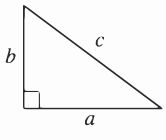
$A = \pi r^2$
$C = 2\pi r$



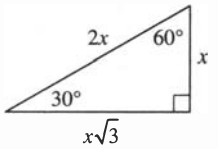
$A = lw$

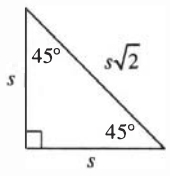


$A = \frac{1}{2}bh$

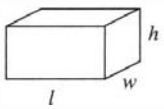


$c^2 = a^2 + b^2$

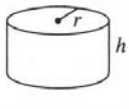




Special Right Triangles

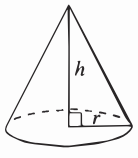


$V = lwh$

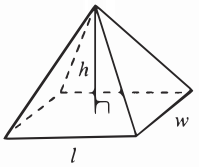


$V = \pi r^2 h$

$V = \frac{4}{3}\pi r^3$



$V = \frac{1}{3}\pi r^2 h$



$V = \frac{1}{3}lwh$

The number of degrees of arc in a circle is 360.
The number of radians of arc in a circle is 2π.
The sum of the measures in degrees of the angles of a triangle is 180.

CONTINUE

4 4

1

$$f(x) = 3x + c$$

In the function above, c is a constant. If $f(4) = 17$, what is the value of c?

A) 5

B) 15

C) 28

D) 29

2

A routine motor vehicle check determines that approximately 5 out of every 200 vehicles inspected have an expired emissions sticker. At this rate, approximately how many vehicles with expired emissions stickers should there be if 5,000 vehicles are inspected?

A) 100

B) 125

C) 150

D) 250

3

If $\frac{1}{3}m + \frac{1}{12}p = 12$, what is the value of $4m + p$?

A) 36

B) 48

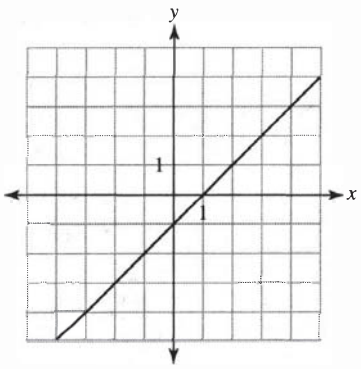
C) 72

D) 144

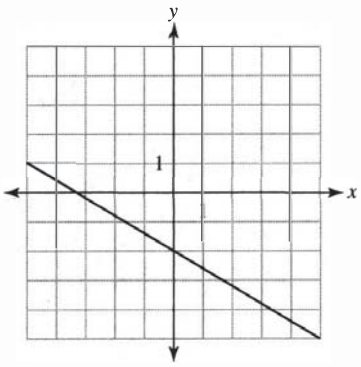
4

If k is a constant greater than zero, which of the following could be the graph of $y = k(x - k)$ in the xy-plane?

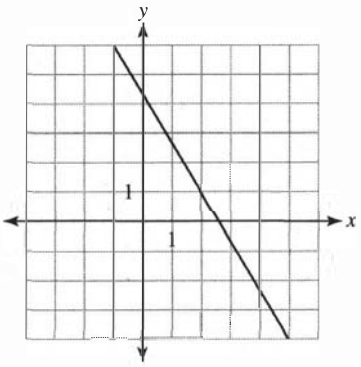
A)



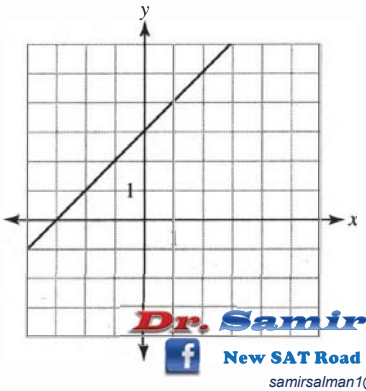
B)



C)



D)



CONTINUE

4 4

5

Alissa received a $20 gift card for her birthday, which she was able to use on her favorite video streaming site. The site charges an $8 one-time membership fee. Each episode of Alissa's favorite TV series costs $1.99. Which of the following expressions represents the amount remaining, in dollars, on her gift card after she watches x episodes of the series?

A) $\$1.99(x - \$20) + \$8$

B) $\$28 - \$1.99x$

C) $\$1.99x + \$28$

D) $\$12 - \$1.99x$

6

x	2	3	4	5	6
$f(x)$	5	4	3	2	1
$g(x)$	2	6	5	4	3

The table above shows some values of the functions $g(x)$ and $f(x)$ for particular values of x. If b is a number such that $g(f(b)) = 5$, which of the following could be a value of b?

A) 3

B) 4

C) 5

D) 6

Questions 7 and 8 refer to the following information.

Kate teaches a 5-session online course for a company that pays its instructors a base amount of $200 per 5-session course plus a fixed amount of money for each student who enrolls. When 50 students enroll, she is paid $725 to teach the course.

7

How much is Kate paid for each student who enrolls in her course?

A) $9.50

B) $10.50

C) $12.50

D) $14.50

8

Next year, the company will raise the base amount paid to each instructor for each course by 25%, but will not change fixed amount paid per enrolled student. If 60 students enroll in Kate's course next year, how much will the company pay her to teach the course?

A) $855.00

B) $880.00

C) $987.50

D) $1,012.50

4 4

9

Keith can read p pages in 20 minutes. At this rate, how many minutes will it take him to read 70 pages?

A) $\frac{7p}{2}$

B) $\frac{2p}{7}$

C) $\frac{p}{1,400}$

D) $\frac{1,400}{p}$

10

Jason buys a new smartphone with a monthly data plan. The phone costs $355 and the data plan costs $40 per month. For each month that he pays his data bill on time, he receives a $10 rebate toward the cost of his phone. If Jason pays his bill on time every month but cancels his contract after 18 months, how much will he have paid for the phone and data plan in total?

A) $895

B) $1,075

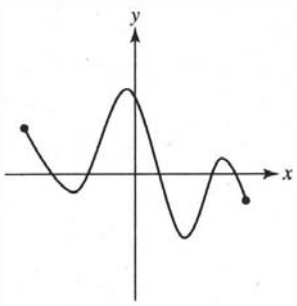
C) $1,135

D) $1,315

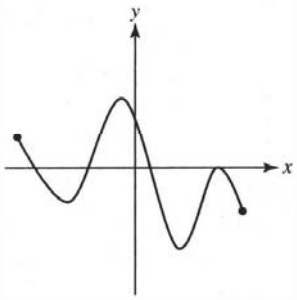
11

The function g has four distinct zeros. Which of the following could represent the complete graph of g in the xy-plane?

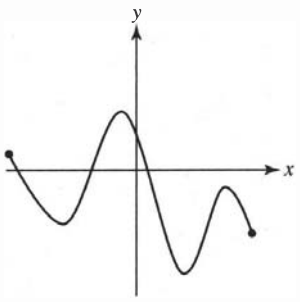
A)



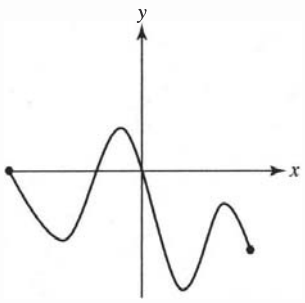
B)



C)



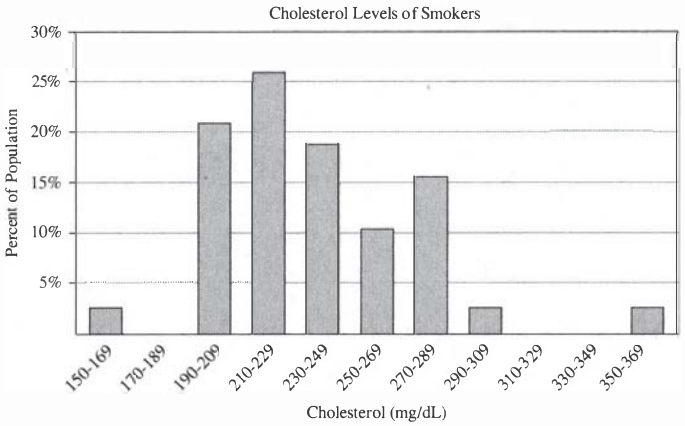
D)

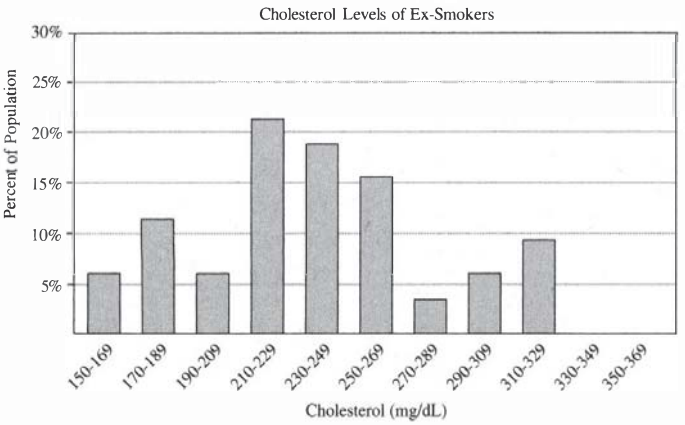


CONTINUE

4 4

Questions 12 and 13 refer to the following information.





The graphs above show the results of a study conducted to test the hypothesis that quitting smoking will reduce the chances that a person will have high cholesterol levels. The study measured the overall cholesterol level of 500 current smokers and 500 people who had been habitual smokers but who had quit at least two years previously. For the sake of this study, "high" cholesterol is defined as overall levels higher than 269 mg/dL.

12

According to the data, approximately how many of the current smokers in the study had cholesterol levels below 210 mg/dL?

A) 48

B) 72

C) 120

D) 160

13

Do these graphs provide strong evidence in support of the tested hypothesis?

A) Yes, because the average cholesterol level for ex-smokers is much less than the average cholesterol level for current smokers.

B) Yes, because far fewer ex-smokers than current smokers have "high" cholesterol levels.

C) No, because the average cholesterol level for ex-smokers is slightly higher than the average cholesterol level for current smokers.

D) No, because the percentage of ex-smokers with "high" cholesterol levels is not significantly different than the percentage of current smokers with "high" cholesterol levels.

14

$$x^2 + y^2 = 80$$
$$y = 2x$$

When the equations above are graphed in the xy-plane, they form a circle and a line containing a secant of the circle. If one of the points of intersection of the two graphs is (a, b), what is the value of ab?

A) -32

B) 8

C) 32

D) 64

CONTINUE

4 4

15

Which of the following is the equation of a line that, when graphed in the xy-plane, has a slope of 2 and contains the point (1, 3)?

A) $2x+y=5$

B) $2x-y=5$

C) $2x+y=-1$

D) $2x-y=-1$

16

The chirping rate of crickets varies linearly with outdoor temperature. One evening, Nicki counted 44 cricket chirps in one 15-second span, and noted that the outdoor temperature was 29°C. On another evening, she counted 32 chirps in 15 seconds, and noted that the temperature was 22°C. Which equation can be used to find the temperature, T, in degrees Celsius, given the number of cricket chirps, c, that are counted in a 15-second span?

A) $T=\frac{12}{7}c-\frac{230}{7}$

B) $T=\frac{12}{7}c-\frac{160}{7}$

C) $T=\frac{7}{12}c+\frac{115}{6}$

D) $T=\frac{7}{12}c+\frac{10}{3}$

17

$$x-2y=10$$

Which of the following is the equation of a line that, when graphed in the xy-plane, is perpendicular to the line described by the equation above?

A) $3x+6y=18$

B) $3x-6y=18$

C) $6x+3y=18$

D) $6x-3y=18$

18

$$2x-\frac{5}{x}=\frac{1}{3}$$

The equation above has two solutions. What is the sum of these two solutions?

A) $\frac{1}{6}$

B) $\frac{2}{5}$

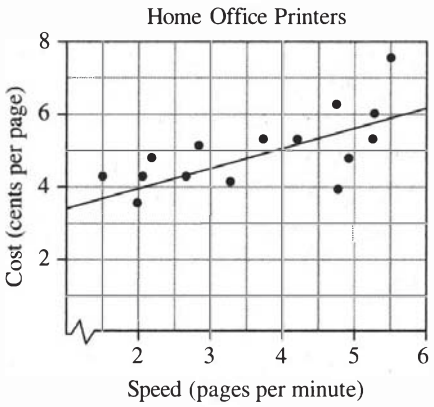
C) $\frac{3}{2}$

D) $\frac{5}{3}$

CONTINUE

4 4

Questions 19 and 20 refer to the following information.



The scatter plot above shows the print speed and cost per printed page for 15 home office printers and the line of best fit for the data.

19

The line of best fit for the data is given by the equation $c = 0.58s + 2.74$, where s is the printing cost, in cents per page, and s is the printing speed, in pages per minute. According to this equation, what is the best estimate for the speed, in pages per minute, of a printer that prints 20 pages per dollar?

A) 3.70 pages per minute
B) 3.90 pages per minute
C) 4.00 pages per minute
D) 4.20 pages per minute

20

It takes the slowest printer in this group k minutes longer to print a 100-page document than it take the fastest printer in this group to print the same document. Which of the following is closest to k?

A) 42
B) 48
C) 54
D) 72

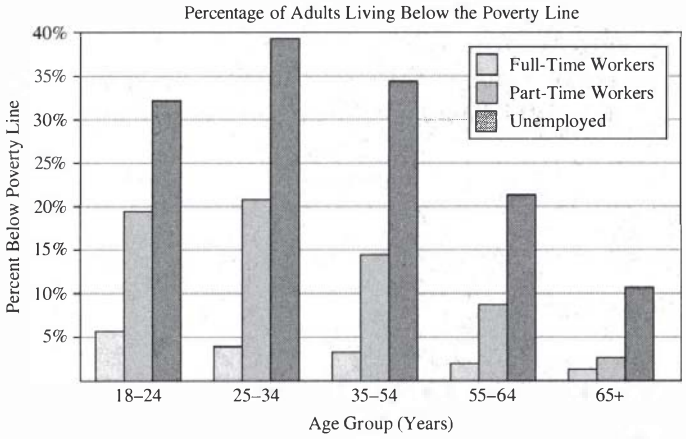
21

In a survey of 50 people who attended a certain movie, 41 of the people said that they purchased snacks at the concession stand, and 32 people said that they enjoyed the movie. What is the least possible number of people surveyed who could have both purchased snacks and enjoyed the movie?

A) 9
B) 18
C) 22
D) 23

4 4

22



The graph above shows the percentage of 15 groups of adults, by age group and employment level, that live below the poverty line. The data were gathered from a survey of 1,000 randomly chosen adults.

Which of the following statistics about the 1,000 surveyed adults can be most accurately determined from the graph?

A) The percentage of unemployed adults ages 18 and above who live below the poverty line.

B) The percentage of people ages 65 and above who live below the poverty line

C) The percentage of unemployed adults ages between 25 and 34 years of age who live below the poverty line

D) The percentage of adults ages 55 to 64 who work part-time

23

$$P = P_0 r^t$$

A researcher at a bird conservancy wants to model the population of finches in an urban area by using the equation above, where P is the population of finches in the area and t is the number of years that have passed since the study began. The researcher determines that the finch population in this area at the beginning of the study is 1,200 and that this population is declining at a rate of 7% per year. What value should the researcher use for r?

A) -0.07

B) 0.93

C) 1.07

D) 1.93

24

In a particular city, the monthly cost of premium cable television service is normally 15% higher than the monthly cost of satellite television service. To attract new customers, each provider offers a discounted rate for the first two years. The cable provider discounts its rate by $10 per month, and the satellite service discounts its rate by 5%. At those sale rates, the monthly cost of the two services is the same. What is the regular (nonsale) monthly cost of premium cable television service?

A) $47.50

B) $50.00

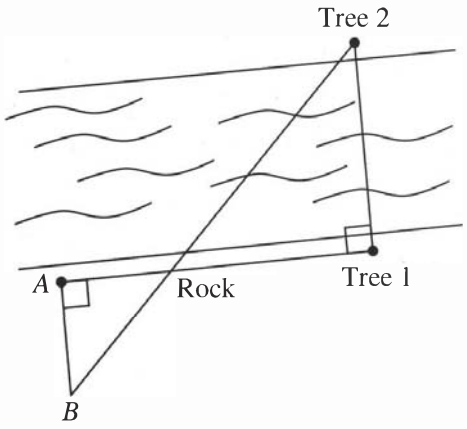
C) $57.50

D) $66.25

CONTINUE

4 4

25



Note: Figure not drawn to scale.

Jessica wants to determine the width of a river, measured between two large trees, one on each bank of the river, as shown above. Standing at Tree 1, and looking at Tree 2, she turns 90° and walks along the river bank. When she reaches a point 60 yards from Tree 1, she marks the spot with a large rock and then continues along the same line until she reaches point A, which is 96 yards away from Tree 1. From point A she turns 90° away from the river and walks until she reaches point B where she can sight along a line through the marker rock to the tree on the other bank. If points A and B are 171 feet apart, what is the width of the river? (1 yard = 3 feet)

A) 38 yards

B) 95 yards

C) 114 yards

D) 285 yards

26

If $\frac{a}{2b} > 1$, and $a < 0$, which of the following statements must be true?

A) $a > 2b + 1$

B) $a < 2b + 1$

C) $a > -2b + 1$

D) $a = 2b + 1$

27

If the equation $y = (x - 3)(x - 7)$ is graphed in the xy-plane, it forms a parabola with a vertex at (a, b). What is the value of b?

A) −8

B) −4

C) −2

D) 4

28

$$V = \frac{k}{P}$$

At a fixed temperature, the volume of a sample of gas, V, in liters, is related to the applied pressure, P, in atmospheres, by the equation above, where k is a constant. A sample of gas at a fixed temperature has a volume of 6 liters. What will be the volume of this sample if the applied pressure is increased by 50%?

A) 2 liters

B) 3 liters

C) 4 liters

D) 5 liters

29

Which of the following expressions is equal to $\sqrt{-18} - \sqrt{-8}$? (Note: $i^2 = -1$)

A) $-i\sqrt{2}$

B) $-\sqrt{2}$

C) $i\sqrt{2}$

D) $3i\sqrt{2}$

30

A triangle has angles with measures 90°, a°, and b°. Which of the following must be true?

A) $\sin a° = \cos b°$

B) $\sin a° = \tan b°$

C) $\sin a° = \cos (90 - b)°$

D) $\sin a° = \sin (90 - a)°$

CONTINUE

4 4

31

Any temperature in degrees Fahrenheit, F, can be converted to a temperature in degrees Celsius, C, by the formula $C = \frac{5}{9}(F - 32)$. What temperature, in degrees Fahrenheit, is equivalent to 30° Celsius?

32

Abby makes blankets by stitching together small crocheted squares. She can stitch together between 8 and 15 of these squares in an hour. If Abby wants to make a blanket that requires 1,170 squares, and she can devote three hours each day to stitching squares, what is the minimum number of days she needs to complete the blanket?

33

Mrs. Battle's current monthly salary is $3,000. Each month, her employer deducts 5% from her monthly salary and transfers that money into her pension plan. Her employer then contributes an additional amount equal to 3% of Mrs. Battle's monthly salary to her pension plan. Next year, Mrs. Battle will receive a 4% salary increase. By how much, in dollars, will the total monthly contribution to her pension plan increase after her raise takes effect? (Ignore the $ sign when gridding.)

34

If the measure of angle A is $\frac{7\pi}{10}$ radians and the measure of angle A is four times the measure of angle B, what is the measure of angle B, in degrees?

35

What is the product of all possible solutions to the equation $(p - 1)(p + 2) = -8$?

36

$$x^3 + 14x^2 + bx + 60$$

If $x + 2$ is a factor of the polynomial above, what is the value of b?

CONTINUE

4 4

Questions 37 and 38 refer to the following information.

2012 House of Representatives Election			
District	Total Votes Cast for All Candidates	Incumbent's Campaign Expenditures (× $1,000)	% of Votes Received by Incumbent
1	360,000	420	48%
2	380,000	418	48%
3	420,000	580	50%
4	480,000	526	52%
5	510,000	730	55%
6	661,000	1,200	60%

The table above shows data for the 2012 campaigns of the incumbents in 6 Congressional districts.

37

How much campaign money, in dollars, did the District 5 incumbent spend for each vote received? (Round your answer to the nearest cent, and ignore the $ sign when gridding.)

38

A political consultant uses the data in this table to analyze the relationship between campaign expenditures of an incumbent's campaign and the total number of votes that candidate received. She calculates that the line of best fit relating the total number of votes received, v, to the total campaign expenses in dollars, x, is given by $v = kx + 72{,}600$, where k is a positive constant. If the data for District 6 match this model perfectly, what is the value of k?

STOP

If you finish before time is called, you may check your work on this section only. Do not turn to any other section of the test.

SAT PRACTICE TEST 3 ANSWER KEY

Section 1: Reading

1. D
2. B
3. B
4. D
5. D
6. C
7. A
8. C
9. A
10. B
11. D
12. B
13. D
14. B
15. D
16. C
17. C
18. A
19. C
20. A
21. D
22. A
23. A
24. B
25. C
26. C
27. C
28. D
29. C
30. B
31. C
32. A
33. A
34. B
35. C
36. D
37. D
38. B
39. A
40. D
41. C
42. B
43. B
44. C
45. C
46. A
47. D
48. D
49. A
50. A
51. D
52. D

Total Reading Points (Section 1)

Section 2: Writing and Language

1. B
2. B
3. A
4. A
5. B
6. B
7. D
8. A
9. D
10. C
11. C
12. D
13. B
14. C
15. D
16. C
17. C
18. C
19. B
20. A
21. D
22. B
23. D
24. C
25. D
26. A
27. B
28. D
29. A
30. B
31. B
32. A
33. C
34. C
35. C
36. B
37. A
38. A
39. C
40. A
41. D
42. A
43. D
44. A

Total Writing and Language Points (Section 2)

Section 3: Math (No Calculator)

1. A
2. A
3. C
4. B
5. D
6. C
7. B
8. D
9. A
10. B
11. B
12. C
13. D
14. C
15. D
16. 10
17. 8
18. 10
19. 31
20. 3/10 or .3

Section 4: Math (Calculator)

1. A
2. B
3. D
4. A
5. D
6. A
7. B
8. B
9. D
10. A
11. B
12. C
13. D
14. C
15. D
16. D
17. C
18. A
19. B
20. B
21. C
22. C
23. B
24. C
25. B
26. B
27. B
28. C
29. C
30. A
31. 86
32. 26
33. 9.60
34. 31.5
35. 6
36. 54
37. 2.6
38. .27

Total Math Points (Section 3 + Section 4)

42. **A** **Cohesiveness**

This sentence refers to the questions about van Gogh that were discussed in the previous paragraph. These were questions about van Gogh's *intent* (*Does the painting symbolize turmoil or tranquility?*) and *state of mind* (*perhaps it indicates the hope of a heavenly journey*).

43. **D** **Verb Tense/Verb Aspect/ Subject-Verb Agreement**

This verb indicates that the current status of van Gogh and "Starry Night" is a consequence of previous events, so it requires the "present perfect" (or "present consequential") form, *have lost*. Although choice C is also in the "present consequential," it does not agree with the plural subject, *compelling painting and its enigmatic painter*.

44. **A** **Cohesiveness/Conclusions**

The original sentence effectively concludes the passage and ties into the first paragraph by echoing its themes of torment (*his brief, tortured lifetime*) and poverty (*sold only one painting*).

Section 3: Math (No Calculator)

1. **A** **Algebra (linear equations) EASY**

Original equation: $\frac{7}{3}a=\frac{3}{4}$

Multiply by $\frac{3}{7}$: $a=\frac{3}{4}\times\frac{3}{7}=\frac{9}{28}$

2. **A** **Algebra (solving equations) EASY**

Original equation: $3x+a=x+b$

Subtract x: $2x+a=b$

Subtract a: $2x=b-a$

Divide by 2: $x=\frac{b-a}{2}$

3. **C** **Algebra (expressing relationships) EASY**

Because Laura's car averages 27 miles per gallon for highway driving, she needs 10/27 gallons to travel 10 miles on the highway. (Notice that "dimensional analysis" helps here: when "miles" are divided by "miles/gallon," the quotient yields "gallons," which is the unit we want.) Similarly, because her car averages 21 miles per gallon for city driving, she needs $b/21$ gallons to travel b miles in the city.

4. **B** **Advanced Mathematics (exponentials) EASY**

Recall that $\sqrt[n]{x}=x^{\frac{1}{n}}$ and $(x^m)^n=x^{mn}$: $\sqrt[3]{x^7}=(x^7)^{\frac{1}{3}}=x^{\frac{7}{3}}$

5. **D** **Additional Topics (analyzing linear equations and graphs) EASY**

We can solve this problem in several ways, but perhaps the simplest is to write the equation in "point-slope" form (since we are given the slope or the line and a point on the line) and convert it to "slope-intercept" form.

Point-slope form given slope $=-\frac{3}{4}$ and line contains (8, 4): $(y-4)=-\frac{3}{4}(x-8)$

Distribute: $y-4=-\frac{3}{4}x+6$

Add 4: $y=-\frac{3}{4}x+10$

Therefore, the y-intercept of the graph is at the point (0, 10).

6. **C** **Advanced Mathematics (simplifying polynomial expressions) MEDIUM**

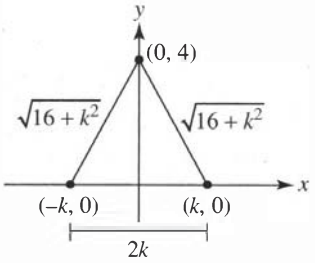
Original expression: $(ab^2c^2+abc-a^2b^2c)-(a^2b^2c-ab^2c^2+abc)$

Distribute: $ab^2c^2+abc-a^2b^2c-a^2b^2c+ab^2c^2-abc$

Collect like terms: $(ab^2c^2+ab^2c^2)+(abc-abc)+(-a^2b^2c-a^2b^2c)$

Simplify: $2ab^2c^2-2a^2b^2c$

7. **B** **Advanced Mathematics (functions) MEDIUM**



If we consider the point (0, 4) to be the vertex where the two congruent sides meet, then because of the symmetry of an isosceles triangle, the two other vertices must be at $(-k, 0)$ and $(k, 0)$. Now, if you want, you can test the coordinates of the points in the choices to see which one yields a triangle with a perimeter of 16 units.

Alternately, if you want to flex your algebra and analytical geometry muscles, you can "solve." The base of the triangle has a length of $2k$, as shown above, and we can then use the Pythagorean Theorem to find the lengths of the two congruent sides in terms of k. Since $4^2+k^2=(\text{side})^2$, we can see that the two congruent sides have length $\sqrt{16+k^2}$, as shown above. Since the perimeter is 16: $2k+2\sqrt{16+k^2}=16$

Divide by 2: $k+\sqrt{16+k^2}=8$

Subtract k: $\sqrt{16+k^2}=8-k$

Square both sides (and FOIL on the right side): $16+k^2=64-16k+k^2$

Subtract k^2+16: $0=48-16k$

Add $16k$: $16k = 48$
Divide by 16: $k = 3$
Therefore, the remaining two vertices are (3, 0) and (–3, 0).

8. **D** **Advanced Mathematics (quadratic systems) MEDIUM**

Substitute and simplify:

$$\frac{a-b}{2}=\frac{(x^2-1)-(x-1)^2}{2}=\frac{x^2-1-(x^2-2x+1)}{2}$$

$$=\frac{x^2-1-x^2+2x-1}{2}=\frac{2x-2}{2}=x-1$$

9. **A** **Algebra (linear graphs) MEDIUM**

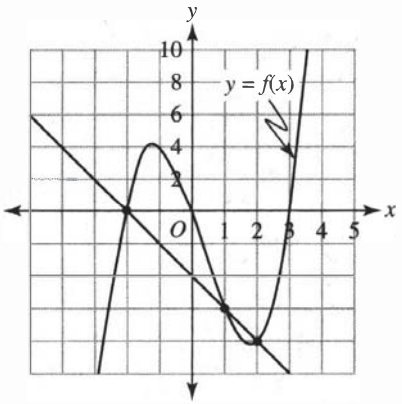
We can calculate the slope of line m using the slope formula: $\text{slope}=\frac{y_2-y_1}{x_2-x_1}=\frac{0-7}{9-0}=-\frac{7}{9}$

Since line l must have the same slope:

$$\frac{-4-0}{0-k}=\frac{-4}{-k}=\frac{4}{k}=-\frac{7}{9}$$

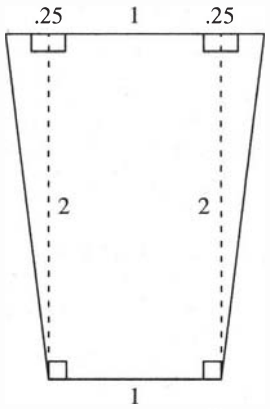
Cross-multiply: $-36 = 7k$
Divide by 7: $-\frac{36}{7}=k$

10. **B** **Advanced Mathematics (non-linear systems) MEDIUM-HARD**



We can simply graph the line $y = -2x - 4$ on the same xy-plane and note the intersection points. One tricky thing here is that the vertical (y-axis) scale is different from the horizontal (x-axis scale). Notice that the vertical lines are only one unit apart (so the x-axis scale is 1 unit), but the horizontal lines are two units apart (so the y-axis scale is 2 units). Therefore, the line with a slope of –2 and a y-intercept of –4 looks like the one graphed above. Just reading the coordinates of the points of intersection shows that those points are at (–2, 0), (1, –6), and (2, –8). You should also be able to confirm that all three of these ordered pairs satisfy both equations.

11. **B** **Additional Topics (volume) MEDIUM-HARD**



The planter is in the form of a prism with trapezoidal bases. The volume of any prism is equal to the area of the base times the height. To find the area of the trapezoidal bases, we can use the formula for the area of a trapezoid, $A_{\text{trapezoid}}=\frac{b_1+b_2}{2}\times h$, after we convert all lengths to feet:

$$A_{\text{trapezoid}}=\frac{1+1.5}{2}\times 2=2.5 \text{ square feet}$$

(Alternately, you can look at the trapezoid as being composed of a rectangle and two right triangles, as shown in the diagram above, with units converted to feet, and calculate the area as the sum of those three areas.)

Since the height of this prism (the "depth" of the planter) is 1 foot, the total volume of the planter is (2.5)(1) = 2.5 cubic feet. Since the soil costs \$4 per cubic foot, the total cost of soil is \$4(2.5) = \$10.

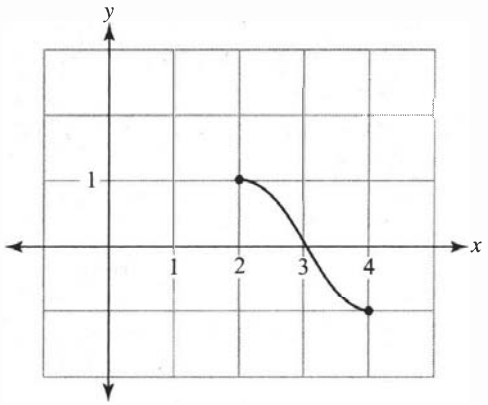
12. **C** **Advanced Mathematics (forms of quadratic functions) MEDIUM-HARD**

The parabola shown has a vertex, B, at (–2, 2). The equation in choice C is in "vertex form" ($y = a(x - h)^2 + k$), so it shows these coordinates directly as constants.

13. **D** **Advanced Mathematics (sequence problems) MEDIUM-HARD**

If $a_1=\frac{1}{4}$ and $r = 2$, then the first 10 terms of the sequence are $\frac{1}{4}, \frac{1}{2}$, 1, 2, 4, 8, 16, 32, 64, and 128. Since the 8th of these terms is 32, $n = 8$.

14. **C** **Advanced Mathematics (graphs of polynomials) MEDIUM-HARD**



All graphs of polynomial functions are continuous; that is, they can be sketched in the xy-plane without lifting your pencil from the page. If we plot the two points of the function that are given, (2, 1) and (4, –1), and draw a curve connecting the two, it is clear that the graph must cross the x-axis somewhere between $x = 2$ and $x = 4$. Choice A is incorrect because it implies that $p(4) = 0$, which is clearly not true. Choice B is incorrect because it implies that $p(1) = 0$, which may or may not be true. Choice D is incorrect because it implies that $p(1) = 2$, which may or may not be true.

15. **D** **Advanced Mathematics (analyzing polynomial functions) HARD**

Although you might prefer to answer this question by actually performing "long division" on the rational expression on the left of the equation (in which case b represents the remainder), you may find it to be a bit more straightforward to just multiply both sides of the equation by the common denominator and solve for b.

Original equation: $\frac{x^2+1}{x+1} = x - 1 + \frac{b}{x+1}$

Multiply both sides by $x + 1$: $x^2 + 1 = (x-1)(x+1) + b$

FOIL on the right side: $x^2 + 1 = x^2 - 1 + b$

Subtract x^2: $1 = -1 + b$

Add 1: $2 = b$

16. **10** **Algebra (solving equations) EASY**

Original equation: $(x-3)^2 = 49$

Take the square root (remember there are two!): $x - 3 = \pm 7$

Add 3: $x = 10$ or -4

Since $x > 0$, the correct answer is $x = 10$.

17. **8** **Algebra (solving inequalities) MEDIUM**

Since the question asks for the maximum number of pens Fernanda can buy, let's define p as that number. Since she buys a total of 24 pens and pencils, she must buy $24 - p$ pencils. Since each pen costs 50 cents, she spends $50p$ cents on pens, and since each pencil costs 5 cents, she spends $5(24 - p) = 120 - 5p$ cents on pencils. Since she can't spend more than 5 dollars, or 500 cents:

$50p + 120 - 5p \le 500$

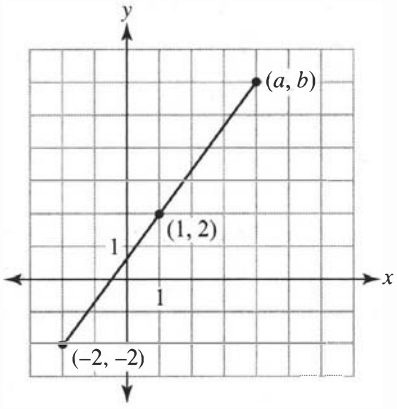
Simplify: $45p + 120 \le 500$

Subtract 120: $45p \le 380$

Divide by 45: $p \le 8.444\ldots$

Since p must be a whole number, its greatest possible value is 8.

18. **10** **Algebra (graphs of lines) MEDIUM**



It's important to read this question carefully. Notice that we are *given* the midpoint, so we aren't trying to find it. We are trying to find the other *endpoint*. One way to find it is to sketch a graph of the points, as shown above. If you count carefully, it should be easy to see that the other endpoint must be at (4, 6). Alternately, you can use the midpoint formula, midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$:

$$(1,2) = \left(\frac{-2+a}{2}, \frac{-2+b}{2}\right)$$

Find a by equating the x-coordinates: $1 = \frac{-2+a}{2}$

Multiply by 2: $2 = -2 + a$

Add 2: $4 = a$

Find b by equating the y-coordinates: $2 = \frac{-2+b}{2}$

Multiply by 2: $4 = -2 + b$

Add 2: $6 = b$

Therefore, $a + b = 4 + 6 = 10$.

19. **31** **Algebra (solving inequalities) MEDIUM-HARD**

Original inequality: $\frac{1}{100} - \frac{9}{n^2} > 0$

Notice that $100n^2$ must be positive, so multiplying by $100n^2$ doesn't require "flipping:" $n^2 - 900 > 0$

Add 900: $n^2 > 900$

Take square root (assuming n is positive): $n > 30$

Since n must be greater than 30, the least positive integer solution is $n = 31$.

20. **3/10 or .3** **Algebra (linear systems) HARD**

If this system has no solutions, then the equations represent parallel lines in the xy-plane. This means that their slopes must be equal. Recall that if a line is expressed in "standard form," $ax + by = c$, that its slope is $-a/b$. (You can also prove this by converting this equation to "slope-intercept form."). Therefore, if the two slopes are the same:

$$-\frac{a}{b} = -\frac{\frac{1}{5}}{\frac{2}{3}} = -\frac{1}{5} \times \frac{3}{2} = -\frac{3}{10}$$

Multiply by –1: $\frac{a}{b} = \frac{3}{10}$

Section 4: Math (Calculator)

1. **A** **Advanced Mathematics (functions) EASY**

If $f(4) = 17$: $3(4) + c = 17$
Simplify: $12 + c = 17$
Subtract 12: $c = 5$

2. **B** **Problem Solving and Data Analysis (proportions) EASY**

The phrase "at this rate" always suggests a proportion. The number of cars with expired emissions stickers can be calculated by setting up and solving this proportion:

$$\frac{5}{200} = \frac{x}{5000}$$

Cross-multiply: $25{,}000 = 200x$
Divide by 200: $125 = x$

3. **D** **Algebra (solving equations) EASY**

The simplest strategy here is to look closely at what the question is asking for and find the simplest way to get there. The question asks for the value of $4m + p$. Notice that this is precisely the expression we get if we multiply both sides of the original equation by its common denominator, 12:

Original equation: $\frac{1}{3}m + \frac{1}{12}p = 12$
Multiply both sides by 12: $4m + p = 144$

4. **A** **Algebra (graphs of linear equations) EASY**

Notice that we can get the given equation into "slope-intercept" form by just distributing:
Original equation: $y = kx - k^2$
This means that this line has a slope of k and a y-intercept of $-k^2$. If k is positive, then the slope is positive and the y-intercept is negative. Only choice A gives a graph of a line with a positive slope and a negative y-intercept. Notice that this is the graph for $k = 1$.

5. **D** **Algebra/Problem Solving (expressing linear relationships) EASY**

Since each episode of the series costs \$1.99, it costs Alissa $\$1.99x$ to watch x episodes. The total remaining on her gift card after the \$8 membership fee and the payment for x episodes is $\$20 - \$8 - \$1.99x = \$12 - \$1.99x$.

6. **A** **Advanced Mathematics (compositions of functions) EASY-MEDIUM**

The expression $g(f(b))$ means the result when an input b is put through the function f, and then this result is put through the function g. If $g(f(b)) = 5$, then $f(b)$ must be a number that, when put into g, yields an output of 5. The table indicates that the only input to g that yields an output of 5 is 4; therefore, $f(b) = 4$. This means that b is a number that, when put into f, yields an output of 4. The table shows that 3 is the only input to f that yields an output of 4.

7. **B** **Algebra (analyzing linear relationships in context) EASY**

Let x represent the amount that Kate is paid per student enrolled in her course. The total amount that Kate will be paid for this course when 50 students are enrolled is then
$\$200 + \$50x$: $200 + 50x = 725$
Subtract 200: $50x = 525$
Divide by 50: $x = 10.50$

8. **B** **Algebra (analyzing linear relationships) EASY**

If the base amount is increased by 25%, it becomes (1.25)(\$200) = \$250. If 60 students enroll for next year's course, the total she will receive is $\$250 + 60(\$10.50) = \$880$.

9. **D** **Problem Solving and Data Analysis (proportions) EASY-MEDIUM**

Since the answer choices contain unknowns, one way to approach this question is to pick a convenient value for p to make the question easier to think about. Notice that if we choose $p = 20$, that is, if Keith reads 20 pages in 20 minutes, then clearly he reads 1 page per minute, and therefore should take 70 minutes to read 70 pages. If we plug in $p = 20$ to the choices, we get

A) $7(20)/2 = 70$
B) $2(20)/7 = 5.7$
C) $20/1{,}400 = 0.014$
D) $1{,}400/20 = 70$

Which, of course, eliminates choices B and C, but doesn't help us choose between A and D. (Note: This is a good reminder NOT to be too hasty in choosing an answer when plugging in. Just because a choice gives you the value you are looking for in that particular case doesn't mean that it will *always* give the right answer.)
Of course, we could plug in a different value for p to eliminate one of the remaining choices, but let's use this opportunity to look at the algebraic method, which is to simply treat the problem as a conversion:

$$70 \text{ pages} \times \frac{20 \text{ minutes}}{p \text{ pages}} = \frac{1{,}400}{p} \text{ minutes}$$

Notice that "pages" unit cancels top and bottom, and we get the proper units of "minutes."

10. **A** **Algebra (translating quantitative information) MEDIUM**

Jason pays \$355 for the phone itself, and then pays \$40 each month for the next 18 months, for a total of $\$355 + 18(\$40) = \$355 + \$720 = \$1{,}075$. However, each month he receives a \$10 rebate, so the total is reduced by $18(\$10) = \$180$, and $\$1{,}075 - \$180 = \$895$.

11. **B** **Advanced Mathematics (graphs of polynomials) MEDIUM**

The zeros of a function correspond to the points at which the graph of that function in the xy-plane cross the x-axis. If the function has four distinct zeros, its graph must touch the x-axis in only four points, which is true only of the graph in choice B.

12. **C** **Problem Solving and Data Analysis (interpreting histograms) MEDIUM**

The question asks about current smokers, so the relevant information is in the top graph. The first three bars on the left correspond to those cholesterol levels below 210 mg/dL, and these bars account for a total of about 3% + 0% + 21% = 24% of the smokers in the study. Since there were 500 smokers in this study, this accounts for about (0.24)(500) = 120 smokers.

13. **D** **Problem Solving and Data Analysis (drawing conclusions) MEDIUM-HARD**

The hypothesis being tested is *that quitting smoking will reduce the chances that a person will have high cholesterol levels.* This theory would be supported by evidence that those who have recently quit smoking have a significantly smaller probability of having high cholesterol levels than do current smokers. However, the data do not show that. In fact, the proportions that have cholesterol levels above 269 are about the same for both groups: 16% + 3% + 0% + 0% + 3% = 22% for smokers, and 4% + 6% + 9% + 0% + 0% = 22% for ex-smokers.

Although choice C might seem to be a reasonable response, this fact is not relevant to the *stated hypothesis*, which is about the *chances of having high cholesterol levels* rather than about *average* cholesterol levels.

14. **C** **Advanced Mathematics (non-linear systems) MEDIUM**

This system can be solved most easily by substitution:

$$x^2 + y^2 = 80$$

Substitute the second equation, $y = 2x$, into the first:

$$x^2 + (2x)^2 = 80$$

Simplify: $x^2 + 4x^2 = 80$

Combine like terms: $5x^2 = 80$

Divide by 5: $x^2 = 16$

Take the square root: $x = \pm 4$

Since the question does not specify a particular point of intersection, we can choose $x = 4$, and plug into the second equation find the corresponding value of y:

$$y = 2(4) = 8$$

Therefore, $a = 4$ and $b = 8$, so $ab = 32$. (Notice that if we chose the solution $x = -4$, the corresponding value of y would be -8, so the product ab would still be 32.)

15. **D** **Algebra (graphing lines) EASY**

Since the equations are all given in standard $(ax + by = c)$ form, we can find the slope of each line with the slope formula $= -a/b$. This gives us slopes of (A) $-2/1 = -2$, (B) $-2/-1 = 2$, (C) $-2/1 = -2$, and (D) $-2/-1 = 2$. Since the line must have a slope of 2, we can eliminate choices A and C. Then we can substitute $x = 1$ and $y = 3$ to see which equation is satisfied by the point (1, 3). Only choice D works, because $2(1) - 3 = -1$.

16. **D** **Algebra (modeling linear relationships) MEDIUM**

We are given two ordered-pair solutions: (44 chirps, 29°C) and (32 chirps, 22°C). All of the choices are linear equation in slope-intercept form, so to find the right linear equation, we can first find the slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{29-22}{44-32} = \frac{7}{12}$. Notice that this eliminates choices A and B. We can then find the value of the y-intercept by first plugging in one of the ordered pairs into the point-slope form of the line:

$$T - 22 = \frac{7}{12}(c - 32)$$

Distribute: $T - 22 = \frac{7}{12}c - \frac{56}{3}$

Add 22 (or $\frac{66}{3}$) to both sides:

$$T = \frac{7}{12}c - \frac{56}{3} + \frac{66}{3} = \frac{7}{12}c + \frac{10}{3}$$

17. **C** **Algebra (linear equations) MEDIUM**

The slope of a line in standard $(ax + by = c)$ form is $-a/b$, so the slope of the given line is $(-1)/(-2) = 1/2$. Perpendicular lines have slopes that are opposite reciprocals, so we are looking for the equation of a line with a slope of $-2/1 = -2$. Since all of the choices are linear equations in standard form, we can find their slopes in the same way we found the slope of the first line.

A) slope $= -3/6 = -1/2$
B) slope $= 3/(-6) = 1/2$
C) slope $= -6/3 = -2$
D) slope $= -6/(-3) = 2$

18. **A** **Advanced Mathematics (solving quadratics) MEDIUM-HARD**

Although it may not look like it, this is a quadratic equation. We can see this by simply multiplying both sides of the equation by the common denominator, $3x$.

Original equation: $2x - \frac{5}{x} = \frac{1}{3}$

Multiply both sides by $3x$: $6x^2 - 15 = x$

Subtract x: $6x^2 - x - 15 = 0$

At this point we could solve this quadratic by factoring or by using the quadratic formula, but there is actually a one-step solution. Divide by 6: $x^2 - \frac{1}{6}x - \frac{15}{6} = 0$

This gives us a lead coefficient of 1 and puts the quadratic in the form $x^2 + bx + c = 0$. Any quadratic in this form has two solutions (which may be the same number repeated, or two non-real numbers). The sum of these solutions is always $-b$ and their product is always c. Since the question asks only for the sum of these solutions, this sum is just the opposite of the x coefficient: $\frac{1}{6}$.

If you prefer to go the whole nine yards, you can solve the quadratic by factoring:

$$6x^2 - x - 15 = (3x - 5)(2x + 3) = 0$$

So, by the Zero Product Property, the solutions are $x = \frac{5}{3}$ and $x = -\frac{3}{2}$, and their sum is $\frac{5}{3} + \left(-\frac{3}{2}\right) = \frac{10}{6} - \frac{9}{6} = \frac{1}{6}$.

19. **B** **Problem Solving and Data Analysis (lines of best fit) MEDIUM-HARD**

The line of best fit, like the scatter plot itself, relates *cost in cents per page* and *speed in pages per minute.* The quantity we're given, however, is neither of those, so it must be converted. If a printer can print 20 pages per dollar, it can print 20 pages per 100 cents, or 20/100 = 1/5 of a *page per cent*. The cost in *cents per page* is just the reciprocal of this rate, so this printer can print at a cost of 5/1 = 5 *cents per page.* This corresponds to the c in the equation that represents the line of best fit:

$c = 0.58s + 2.74$

Substitute $c = 5$: $5 = 0.58s + 2.74$

Subtract 2.74: $2.26 = 0.58s$

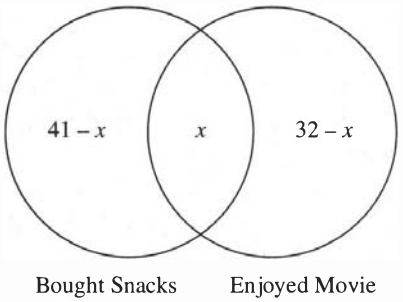
Divide by 0.58: $3.90 = s$

Therefore, the best estimate for the speed of the printer is 3.90 pages per minute.

20. **B** **Problem Solving and Data Analysis (scatter plots) MEDIUM-HARD**

The slowest printer in this group is the printer represented by the leftmost dot in the scatter plot, and the fastest printer is represented by the rightmost dot. The leftmost dot corresponds to a speed of 1.5 pages per minute, and the rightmost dot corresponds to a speed of 5.5 pages per minute. (Notice that the horizontal scale does not have a "zero" on the left. This fact is indicated by the jagged portion on the left end of the horizontal axis. The rest of the axis should make it clear that the tick marks are 0.5 apart, and since the leftmost dot is one tick mark to the left of 2, its horizontal value is 2.0 – 0.5 = 1.5.) The faster printer takes (100 pages)/(5.5 pages per minute) = 18.2 minutes to print a 100-page document, while the slowest printer takes (100 pages)/(1.5 pages per minute) = 66.7 minutes to print the same document, for a difference of 66.7 – 18.2 = 48.5 minutes, which is closest to choice B, 48 minutes.

21. **C** **Problem Solving and Data Analysis (analysis of sets) MEDIUM-HARD**



A Venn diagram is helpful in examining this situation. Look carefully at the diagram above and notice how it represents the situation this question describes. The circle on the left represents everyone who bought snacks and the circle on the right represents everyone who enjoyed the movie. Let's let x represent the number of people who *both* bought snacks *and* enjoyed the movie: the overlap between the two sets. Since 41 people in total bought snacks, and x of them also liked the movie, then $41 - x$ of them bought snacks but did *not* like the movie. Similarly, since 32 people in total liked the movie, and x of them also bought snacks, then $32 - x$ people enjoyed the movie but did *not* buy snacks. Since these three sets can account for no more than 50 people (the total number surveyed):

$(41 - x) + (x) + (32 - x) \leq 50$

Simplify: $72 - x \leq 50$

Add x and subtract 50: $22 \leq x$

Since x can be no less than 22, this is the least possible number of people surveyed who could have both purchased snacks and enjoyed the movie.

22. **C** **Problem Solving and Data Analysis (interpreting graphs) MEDIUM-HARD**

This graph shows data for a group of 1,000 people who were broken into 15 subsets, as indicated by the 15 separate bars. Notice, however, that the graph does not show how *many* people are in each set, but only the *percentage* of the people in each set who live below the poverty line. The quantity in choice A cannot be determined from the graph, because the set of *unemployed adults ages 18 and above* is represented by 5 separate bars, and since we don't know how to "weigh" each bar (because we don't know how many people belong in each subset), we cannot calculate an overall percentage. Likewise, the quantity in choice B cannot be determined, because the set of *adults ages 65 and over* is represented by 3 bars. The quantity in choice D cannot be determined, because the 11th bar from the left represents the *percentage of surveyed part-time workers ages 55 to 64 who live below the poverty line,* and NOT *the percentage of adults ages 55 to 64 who work part-time.* The quantity in choice C, however, can be determined from this graph because it is the quantity represented by the 6th bar from the left.

23. **B** **Advanced Mathematics (exponential growth) MEDIUM**

We know that $P = 1{,}200$ when $t = 0$, so:

$$1{,}200 = P_0 r^0 = P_0$$

Therefore, the equation has the form $P = 1{,}200r$. After one year, the bird population decreases by 7%, so the population is $1{,}200 - 0.07(1{,}200) = 1116$. Therefore:

$$1116 = 1{,}200r^1$$

Divide by 1,200: $0.93 = r$

A simpler strategy is simply to recognize that r represents the common ratio of one year's population to the previous year's population. Since each year, the population is decreasing by 7%, it is $100\% - 7\% = 93\% = 0.93$ of the previous year's population.

24. **C** **Problem Solving and Data Analysis (percents) MEDIUM-HARD**

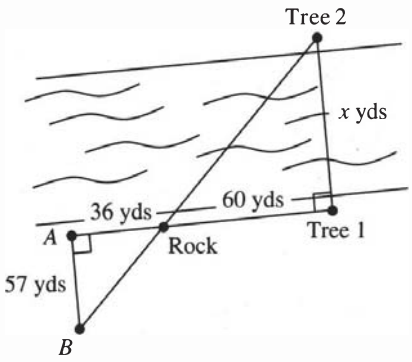
Let $\$x$ represent the monthly cost of satellite TV. Since premium cable service costs 15% more, its monthly cost is $\$x + \$0.15x = \$1.15x$. After the \$10 monthly discount, the monthly cost for premium cable becomes $\$1.15x - \$10$. After a 5% discount, the monthly cost of satellite TV is $\$x - \$.05x = \$0.95x$. If the two services now cost the same: $1.15x - 10 = 0.95x$

Subtract $0.95x$ and add 10: $0.20x = 10$

Divide by 0.20: $x = 50$

This means that the regular monthly cost of satellite TV is \$50. Since premium cable service normally costs 15% more, its monthly cost is $(1.15)(\$50) = \$57.50$.

25. **B** **Additional Topics (similar triangles) MEDIUM-HARD**



It is important to mark up the diagram and confirm the measures in the diagram with the information in the problem. For consistency, we should indicate all measurements in yards. Let x represent the width of the river, in yards. The distance from Tree 1 to the Rock is 60 yards. The distance from point A to Tree 1 is 96 yards, so the distance from the rock to point A is 36 yards. The distance from A to B is $171 \div 3 = 57$ yards. To find the value of x, we must use the fact that the two triangles are similar because two of their corresponding angles are equal.

Now we can set up a proportion: $\frac{57}{36} = \frac{x}{60}$

Multiply by 60 and simplify:

$$\frac{(60)(57)}{36} = \frac{3420}{36} = 95 = x$$

26. **B** **Algebra (inequalities) HARD**

Original inequality: $\frac{a}{2b} > 1$

This means, of course, that $\frac{a}{2b}$ is positive, which means that the numerator and denominator of this fraction must have the same sign. Since we are told that a is negative, $2b$ must also be negative. Therefore, multiplying both sides of the inequality by $2b$ requires "flipping" the inequality: $a < 2b$

Now, since $2b$ must be less than $2b + 1$:

$$a < 2b < 2b + 1$$

Therefore, by the Transitive Law of Inequality:

$$a < 2b + 1$$

Alternately, we can often solve "must be true" questions by process of elimination. That is, we can find counterexamples for the four wrong answers. We simply need to start choosing values for a and b that satisfy the conditions of the problem. For instance, $a = -6$ and $b = -2$ satisfy both inequalities (which you should confirm for yourself now). Next, we notice that these solutions are *not* solutions of any of the statements in A, C, or D.

27. **B** **Advanced Mathematics (graphing parabolas) MEDIUM-HARD**

There are several ways to approach this problem, but the best strategy is usually to start with the information that is handed to us. Since the quadratic is expressed in factored form, it is easy to see that the graph has zeroes at $x = 3$ and $x = 7$. The question asks about the vertex of this parabola, so we should remember that the x-coordinate of the vertex of a parabola is always the average of the zeroes. (This is because the vertical line through the vertex is the axis of symmetry for the parabola.) Therefore, the x-coordinate of the vertex is $(3 + 7) \div 2 = 5$. We can plug this directly back into the equation: $y = (5 - 3)(5 - 7) = (2)(-2) = -4$. Therefore, the vertex of the parabola is $(5, -4)$.

28. **C** **Problem Solving and Data Analysis (variation) MEDIUM-HARD**

The question doesn't specify the applied pressure for the original sample of gas, so although we could solve the problem by leaving P as an unknown, it's not a bad idea to pick a convenient value for P, such as $P = 2$ atmospheres.

Given formula: $V = \frac{k}{P}$

If $P = 2$ when $V = 6$: $6 = \frac{k}{2}$

Multiply by 2: $12 = k$

Since k is a constant, it doesn't change even when the other quantities change. If P increases by 50%, it becomes

(1.5)(2) = 3 atmospheres. We can substitute these values into the original formula to find the new value for V.

$$V = \frac{12}{3} = 4 \text{ liters}$$

29. **C** **Special Topics (complex numbers) HARD**

Original expression: $\sqrt{-18} - \sqrt{-8}$
Factor out $\sqrt{-1}$: $\sqrt{-1}\sqrt{18} - \sqrt{-1}\sqrt{8}$
Substitute $\sqrt{-1} = i$: $i\sqrt{18} - i\sqrt{8}$

Factor the perfect square from each radicand:

$$i\sqrt{9}\sqrt{2} - i\sqrt{4}\sqrt{2}$$

Simplify: $3i\sqrt{2} - 2i\sqrt{2}$
Combine like terms: $i\sqrt{2}$

30. **A** **Special Topics (trigonometry) MEDIUM-HARD**

As with most geometry and trigonometry questions, it is helpful to draw a diagram. Since the angles in a triangle must always have a sum of 180°, $a + b + 90 = 180$, and so $a + b = 90$. We can choose values for a and b that have a sum of 90, and see which statement must be true. For instance, if we choose $a = 30°$ and $b = 60°$, only choice A yields a true statement. Alternately we can recall the Co-function Identity: any trigonometric ratio for an angle is equal to the co-trigonometric ratio of its complement. That is: $\sin(x°) = \cos(90° - x°)$ for all x, etc.

31. **86** **Algebra (solving equations) EASY**

We simply need to substitute $C = 30$ into the equation and solve for F:

$$30 = \frac{5}{9}(F - 32)$$

Multiply by $\frac{9}{5}$: $54 = F - 32$
Add 32: $86 = F$

32. **26** **Problem Solving and Data Analysis (rates) EASY-MEDIUM**

Since we want to minimize the number of days that Abby needs to complete the blanket, she must stitch as many squares as she can per day. Since she can stitch a maximum of 15 squares per hour, and the blanket requires 1,170 squares, it will take her 1,170 ÷ 15 = 78 hours in total. Since she can work only 3 hours per day, it will take her 78 ÷ 3 = 26 days minimum.

33. **9.60** **Algebra (percent change) MEDIUM**

The monthly contributions to Mrs. Battle's pension plan total 8% of her monthly salary (5% from Mrs. Battle's personal deduction plus 3% from her employer). Mrs. Battle's current monthly salary is $3,000, so the monthly contributions to her pension total (0.08)($3,000) = $240. When she receives a 4% raise, her new salary will be (1.04)($3,000) = $3,120 so monthly contributions to her pension will be (0.08)($3,120) = $249.60, which is a monthly increase of $249.60 − $240 = $9.60.

34. **31.5** **Additional Topics (angles) MEDIUM**

Recall that π radians is equivalent to 180°. Therefore, an angle of $\frac{7\pi}{10}$ radians has a measure of $\frac{7\pi}{10}$ radians $\times \frac{180°}{\pi \text{ radians}} = 126°$. Since this measure is 4 times the measure of angle B, the measure of angle B must be 126° ÷ 4 = 31.5°.

35. **6** **Algebra (solving equations) MEDIUM-HARD**

Original equation: $(p - 1)(p + 2) = -8$
FOIL: $p^2 + 2p - p - 2 = -8$
Simplify and add 8: $p^2 + p + 6 = 0$

Now we have the equation in quadratic form with a leading coefficient of 1 (that is, it resembles the form $x^2 + bx + c = 0$). It turns out that this quadratic is not easily factorable, but we can find the solutions using the quadratic formula: $p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Unfortunately, this will yield complex solutions because the discriminant is negative: $b^2 - 4ac = (1)^2 - 4(1)(6) = -23$. Fortunately, however, there is a simple way to find the answer we are looking for. The question doesn't ask for the individual solutions, but rather the *product* of these solutions. If a quadratic equation is in the form $x^2 + bx + c = 0$, then the sum of the solutions is always $-b$ and the product of these solutions is always c. Since $c = 6$ in our equation, this must be the product of the two solutions.

If you prefer to do it the hard way (which, of course, you shouldn't), multiplying the two results given by the quadratic formula will yield the same answer:

$$\left(\frac{-1+\sqrt{1^2-4(1)(6)}}{2}\right)\left(\frac{-1-\sqrt{1^2-4(1)(6)}}{2}\right)$$

$$= \left(\frac{-1+i\sqrt{23}}{2}\right)\left(\frac{-1-i\sqrt{23}}{2}\right)$$

$$= \frac{1^2 - i^2(\sqrt{23})^2}{4} = \frac{1+23}{4} = 6$$

36. **54** **Additional Topics (analyzing polynomials) MEDIUM-HARD**

The Factor Theorem states that any polynomial with a factor of $(x - h)$ must equal 0 when $x = h$. (This is because when $x = h$, the factor of $(x - h)$ becomes $(h - h) = 0$, and anything times 0 equals 0.) Therefore, if $x + 2$ is a factor of $x^3 + 14x^2 + bx + 60$, then this polynomial must equal 0 when $x = -2$:

$$(-2)^3 + 14(-2)^2 + b(-2) + 60 = 0$$

Simplify: $-8 + 56 - 2b + 60 = 0$
Combine like terms: $108 - 2b = 0$
Add $2b$: $108 = 2b$
Divide by 2: $54 = b$

37. **2.60** **Data Analysis (interpretation of data) MEDIUM-HARD**

The District 5 incumbent spent 730 × \$1,000 = \$730,000 and received 55% of the 510,000 votes (0.55 × 510,000 = 280,500 votes). So if the incumbent spent \$730,000 for 280,500 votes:

\$730,000 ÷ 280,500 votes = \$2.60 per vote

38. **.27** **Data Analysis (line of best fit) HARD**

We are told that the line of best fit is given by the equation $v = kx + 72{,}600$. If the data in District 6 matches this model perfectly, we can plug the values from District 6 in to solve for k. In District 6 there were 661,000 votes cast and the incumbent received 60% of those votes. So v = 661,000 × 0.6 = 396,600 votes. The total expenditures in District 6 were \$1,200,000:

Now plug those in and solve:

$$396{,}600 = k(1{,}200{,}000) + 72{,}600$$

Subtract 72,600: $324{,}000 = 1{,}200{,}000k$

Divide by 1,200,000: $.27 = k$

3 3

Math Test—No Calculator

25 MINUTES, 20 QUESTIONS

Turn to Section 3 of your answer sheet to answer the questions in this section.

DIRECTIONS

For questions 1–15, solve each problem, choose the best answer from the choices provided, and fill in the corresponding circle on your answer sheet. **For questions 16–20**, solve the problem and enter your answer in the grid on the answer sheet. Please refer to the directions before question 16 on how to enter your answers in the grid. You may use any available space in your test booklet for scratch work.

NOTES

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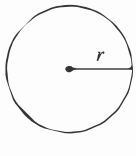
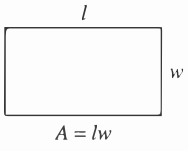
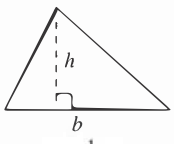
REFERENCE



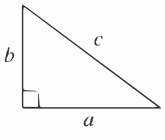
$A = \pi r^2$
$C = 2\pi r$



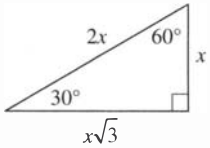
$A = lw$

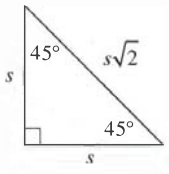


$A = \frac{1}{2}bh$

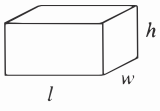


$c^2 = a^2 + b^2$

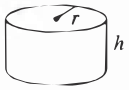




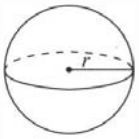
Special Right Triangles



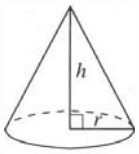
$V = lwh$



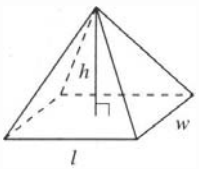
$V = \pi r^2 h$



$V = \frac{4}{3}\pi r^3$

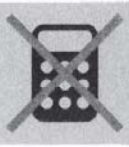


$V = \frac{1}{3}\pi r^2 h$



$V = \frac{1}{3}lwh$

The number of degrees of arc in a circle is 360.
The number of radians of arc in a circle is 2π.
The sum of the measures in degrees of the angles of a triangle is 180.

3  3

1

If $\frac{2}{x+2}=\frac{1}{5}$ what is the value of x?

A) 3

B) 5

C) 8

D) 10

2

If a is 4 times as large as b and $a-b=6$, what is the value of a?

A) 12

B) 8

C) 6

D) 2

3

$$-3(a-1)^2+2(a-1)$$

Which of the following is equivalent to the expression above?

A) $-a^2+2a-1$

B) $-3a^2+2a$

C) $-3a^2+5a-2$

D) $-3a^2+8a-5$

4

If $f(x)=2-2x$, which of the following is equivalent to $f(-2x)$?

A) $2-2x^2$

B) $2-4x$

C) $2+4x$

D) $2+2x^2$

5

$$\frac{x}{y}=3$$

$$y=3(x+2)$$

If (x, y) is a solution to the system of equations above, what is the value of y?

A) $-\frac{9}{4}$

B) $-\frac{6}{5}$

C) $-\frac{3}{4}$

D) $\frac{3}{4}$

6

Which of the following expressions cannot have a positive value for any real value of x?

A) $1-|4+x^2|$

B) $4-|1+x^2|$

C) $1-|4-x^2|$

D) $4-|1-x^2|$

7

Rey wants to run a total of 30 miles in 6 days, starting on Monday and finishing on Saturday. Which of the following training plans will accomplish this goal?

A) Running 3.5 miles on Monday, and increasing his daily mileage by 0.5 miles

B) Running 2.5 miles on Monday, and increasing his daily mileage by 1 mile

C) Running 7 miles on Monday, and decreasing his daily mileage by 1 mile

D) Running 6 miles on Monday, and decreasing his daily mileage by 0.5 miles

CONTINUE

3

3

8

$$\sqrt{x^2+16}=x+2$$

What is the complete solution set to the equation above?

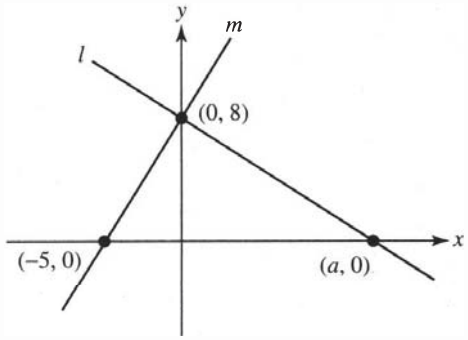
A) {−3, 3}

B) {−5, 3}

C) {3}

D) {5}

9



In the xy-plane above, if line l is perpendicular to line m, what is the value of a?

A) 12.2

B) 12.8

C) 13.2

D) 13.8

10

$$B=\frac{x+W}{y+W}$$

Given the formula above, which choice gives the formula for W in terms of x, y, and B?

A) $W=\frac{x-B}{y-B}$

B) $W=\frac{x-By}{B-1}$

C) $W=\frac{x-y}{B-x}$

D) $W=\frac{x-By}{B}$

11

$$a=2.5t+3$$
$$b=5.5t+2$$

In the equations above, a and b represent the distances that particles A and B, respectively, are from the origin at time t. How far is point B from the origin when it is twice as far from the origin as point A?

A) 2

B) 8

C) 23

D) 46

12

If $x > 1$, which of the following is equivalent to $\frac{1+\frac{1}{x+1}}{1-\frac{1}{x-1}}$?

A) $\frac{(x-1)(x+2)}{(x+1)(x-2)}$

B) $\frac{(x-1)(x-2)}{(x+1)(x+2)}$

C) $\frac{(x+1)(x-2)}{(x-1)(x+2)}$

D) $\frac{(x+1)(x+2)}{(x-1)(x-2)}$

13

If $p(x)$ is a quadratic function and $p(-1) = p(7) = 2$, which of the following could be the coordinates of the vertex of the parabola formed when $y = p(x)$ is graphed in the xy-plane?

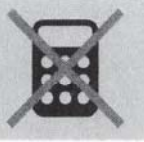
A) (0, 8)

B) (4, 3)

C) (6, 7)

D) (3, 9)

CONTINUE

3  3

14

If $(3x + a)(4x + b) = 12x^2 + cx + 3$ for all values of x, and a and b are negative integers, what are the two possible values for c?

A) -15 and -13

B) -15 and -10

C) -13 and -9

D) -10 and -9

15

If $a - 3b = 5$, what is the value of $\frac{3^a}{27^b}$?

A) 3^{-5}

B) 27^{-5}

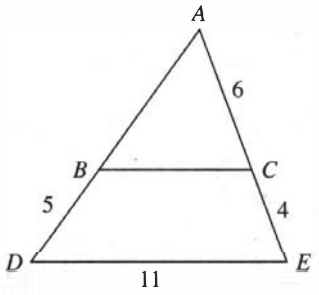
C) 3^5

D) The value cannot be determined from the information given.

CONTINUE

3 3

16



In triangle DAE above, segment BC is parallel to segment DE. What is the length of segment AB?

17

If $x > 0$ and $x^2 - 12 = 4x$, what is the value of x?

18

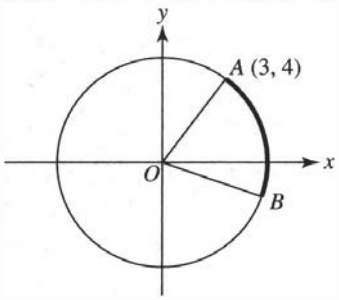
Paul plays a video game that awards points for breaking two kinds of codes: access codes and classified codes. Players are awarded 20 points for breaking each access code and 75 points for breaking each classified code. If Paul has broken 12 codes and has earned 735 points, how many classified codes did Paul break?

19

$$2x^2(4x^2 - 5) = 18$$

If $x > 0$, what is the solution to the equation above?

20



In the xy-plane above, point O is the center of the circle. If arc AB has a length of 2π, what is the measure of $\angle AOB$ in degrees? (Ignore the degree symbol when gridding.)

STOP

If you finish before time is called, you may check your work on this section only. Do not turn to any other section of the test.

4  4

Math Test—Calculator

55 MINUTES, 38 QUESTIONS

Turn to Section 4 of your answer sheet to answer the questions in this section.

DIRECTIONS

For questions 1–30, solve each problem, choose the best answer from the choices provided, and fill in the corresponding circle on your answer sheet. **For questions 31–38**, solve the problem and enter your answer in the grid on the answer sheet. Please refer to the directions before question 31 on how to enter your answers in the grid. You may use any available space in your test booklet for scratch work.

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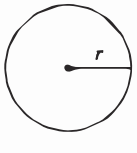
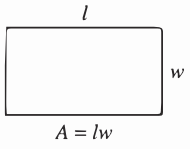
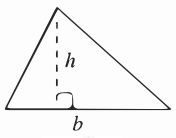


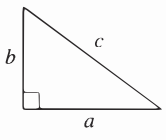
$A = \pi r^2$
$C = 2\pi r$



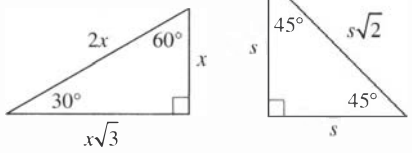
$A = lw$



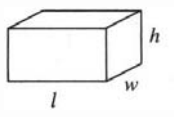
$A = \frac{1}{2}bh$



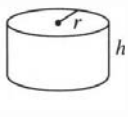
$c^2 = a^2 + b^2$



Special Right Triangles

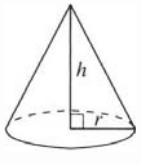


$V = lwh$

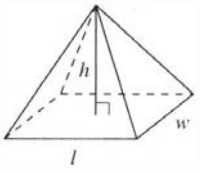


$V = \pi r^2 h$

$V = \frac{4}{3}\pi r^3$



$V = \frac{1}{3}\pi r^2 h$



$V = \frac{1}{3}lwh$

The number of degrees of arc in a circle is 360.
The number of radians of arc in a circle is 2π.
The sum of the measures in degrees of the angles of a triangle is 180.

4 4

1

Matt has published a book that is available as a paperback and as an e-book. He earns $1.12 in royalties for each paperback that is sold and $0.58 in royalties for each e-book that is sold. Which of the following represents the amount of royalties, in dollars, Matt earns if x paperbacks and y e-books are sold?

A) $0.58x + 1.12y$

B) $0.58x - 1.12y$

C) $1.12x + 0.58y$

D) $1.12x - 0.58y$

2

The average of three numbers is 8, and the average of two other numbers is 6. What is the average of all five numbers?

A) 7.0

B) 7.2

C) 7.5

D) 7.6

3

If $f(x) = x^2 + c$, and $f(2) = 9$, what is the value of $f(4)$?

A) 5

B) 16

C) 21

D) 25

4

$$y = ax + b$$
$$y = cx + d$$

In the equations above, a, b, c, and d are nonzero constants. When the equations above are graphed in the xy-plane, they form two perpendicular lines. Which of the following statements must be true?

A) $ac = -1$

B) $a + c = 0$

C) $ac = 1$

D) $ac = 0$

5

Which equation below represents a parabola in the xy-plane that passes through the origin and the point $(4, 0)$?

A) $y = 2x^2 + 4$

B) $y = -4x^2 + 4$

C) $y = x^2 - 16$

D) $y = 2x^2 - 8x$

6

x	1	3	5
$h(x)$	-2	a	3

The table above show some values of the linear function h for particular values of x. What is the value of a?

A) -0.5

B) 0.5

C) 1.5

D) 2.5

CONTINUE

4 4

Questions 7 and 8 refer to the following information.

The population, P, of bison at a preserve is estimated by the equation $P = 240 + 21t$ where t is the number of years that have elapsed since the preservation program began on March 30, 2006.

7

According to the given equation, on which date is the bison population predicted to reach 660?

A) March 30, 2022

B) March 30, 2024

C) March 30, 2026

D) March 30, 2028

8

On March 30, 2014, the preserve conducted a new survey and determined that the bison population was 420. The naturalist in charge of the program decided to adjust the population equation so that it would give the correct population for March 30, 2014. If the naturalist adjusts only the coefficient of t in the equation, what should be the new coefficient of t?

A) 20.5

B) 21.5

C) 22.5

D) 23.5

9

If $\frac{x\sqrt{2}-4+5x}{2}=0$, what is the value of x?

A) 0

B) $\frac{\sqrt{2}+5}{2}$

C) $\frac{\sqrt{2}+5}{4}$

D) $\frac{4}{\sqrt{2}+5}$

10

Which of the following equations indicates the total value, V, of an investment fund after t years if the fund begins with a \$2,400 initial investment and earns a 10% annual interest rate that is compounded monthly? (Assume that no withdrawals are made.)

A) $V = \$2,400(1+0.1)^{12t}$

B) $V = \$2,400\left(1+\frac{0.1}{12}\right)^{t}$

C) $V = \$2,400\left(1+\frac{0.1}{12}\right)^{12t}$

D) $V = \$2,400\left(\frac{1.1}{12}\right)^{12t}$

CONTINUE

4 4

11

A researcher is studying the eating habits of all adults in a large city and is interested particularly in how often those adults eat fast food rather than prepare their own meals. The researcher asked 350 adult customers at a major chain restaurant about how often they ate fast food and how often they prepared their own meals. Of these respondents, 120 were unmarried. Which of the following changes in the survey method would best improve the reliability of the results?

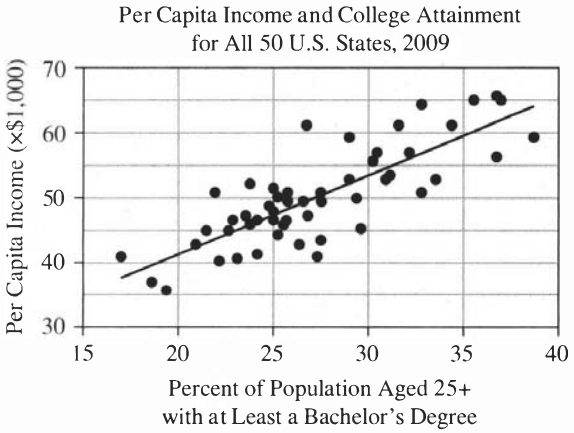
A) Giving the survey to a larger sample group at the restaurant

B) Conducting the survey at a farmer's market rather than at a chain restaurant

C) Excluding the results from the unmarried respondents

D) Giving the survey to a group of adults selected at random from public records

Questions 12 and 13 refer to the following information.



The scatter plot above shows the relationship between per capita annual income and the percentage of adults with college degrees for all 50 U.S. states in 2009.

12

According to the line of best fit, in 2009, approximately what percent of the adults ages 25 and above have college degrees in a state with a per capita annual income of $50,000?

A) 21%

B) 27%

C) 34%

D) 37%

13

According to the scatter plot, the per capita annual income for the state with the highest per capita income is approximately what percent greater than the per capita income for the state with the lowest per capita income?

A) 30% greater

B) 36% greater

C) 53% greater

D) 83% greater

14

$$x^2 + (y-3)^2 = 49$$

$$y = b$$

If the equations above are graphed in the xy-plane, they form a circle and a line. If the line is tangent to the circle, which of the following could be a value of b?

A) -7

B) -4

C) 3

D) 4

CONTINUE

4 4

15

The gross domestic product of Country A is 80% greater than the gross domestic product of Country B. If the gross domestic product of Country A is $720 billion, what is the gross domestic product of Country B?

A) $640 billion

B) $576 billion

C) $400 billion

D) $144 billion

16

	Top 10% of class	Bottom 90% of class
Took at least one AP course	42	312
Took no AP courses	3	93

The table above shows number of students in the top 10% and the bottom 90% of a recent graduating class from Madison Regional High School who did or did not enroll in any AP courses during their high school careers. If a student who graduated in the top 10% of this class is chosen at random, what is the probability that he or she did not take any AP courses?

A) $\frac{1}{32}$

B) $\frac{1}{15}$

C) $\frac{1}{14}$

D) $\frac{13}{15}$

17

Mars has an approximately circular orbit around the Sun. If this orbit has a radius of 142 million miles, and Mars makes this orbit once every 687 days, which of the following is closest to the average speed of Mars, in miles per hour, as it orbits the Sun?

A) 5,000

B) 54,000

C) 108,000

D) 1,300,000

18

$$\left(\frac{1}{2}\right)^{\frac{t}{k}}$$

The expression above shows the fraction of the original mass of a radioactive substance that remains after t years if that substance has a half-life of k years. If substance A has a half-life of 3 years and substance B has a half-life of 7 years, and there is initially 1 kilogram of each substance, which of the following expressions shows how many more kilograms of substance B is left than substance A after t years?

A) $\left(\frac{1}{2}\right)^{\frac{t}{7-3}}$

B) $\left(\frac{1}{2}\right)^{\frac{t}{3}-\frac{t}{7}}$

C) $\left(\frac{1}{2}\right)^{\frac{t}{3}}-\left(\frac{1}{2}\right)^{\frac{t}{7}}$

D) $\left(\frac{1}{2}\right)^{\frac{t}{7}}-\left(\frac{1}{2}\right)^{\frac{t}{3}}$

CONTINUE

4  4

Questions 19 and 20 refer to the following information.

Allison is considering opening an online boutique to sell her designer boots. She needs to hire a company to design her website, to host the site on its secure servers, and to provide her employees with regular technical support in using and updating the website. The website design is a one-time expense, but fees for hosting and support are charged monthly. The table below shows what each of three companies charges for those three services.

Company	Cost for Website Design Services	Monthly Cost for Web Hosting	Monthly Cost for Technical Support
A	$5,000	$30	$360
B	$3,300	$35	$320
C	$4,200	$25	$300

19

How much more money would it cost Allison to hire Company A to design, host, and support her website for 2 years than it would cost her to hire Company B to provide the same services for the same amount of time?

A) $1,235

B) $1,620

C) $2,540

D) $3,275

20

Allison has decided that she will choose either Company B or Company C to provide all three services. Which of the following describes all the values of m, where m is the number of months for which the total cost of hiring Company B is less than the total cost of hiring Company C?

A) $m > 30$

B) $m < 30$

C) $m > 90$

D) $m < 90$

21

If $m = 2^k$, which of the following is equivalent to $\left(\frac{8}{m}\right)^2$?

A) 2^{6-2k}

B) 2^{2k-6}

C) $2^{\frac{6}{k}}$

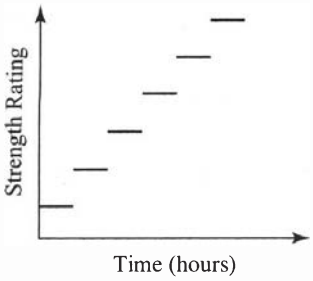
D) $2^{\frac{k}{6}}$

4 4

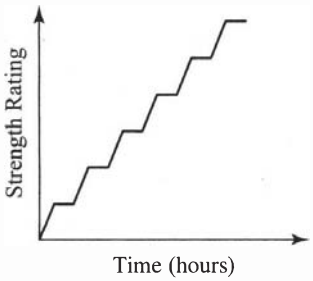
22

In a particular video game, the "strength rating" of a player is an integer from 1 to 6 based on the number of consecutive hours the player has remained in the game without being eliminated. Every player's strength rating increases by 1 unit for every whole hour, or part thereof, that the player remains in the game, up to 6 hours. Which of the graph shows the strength rating of a player as a function of the time the player remains in the game?

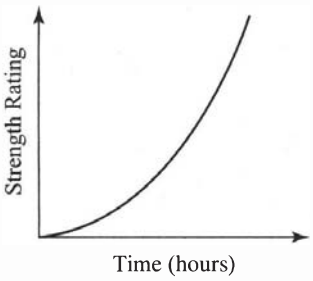
A)



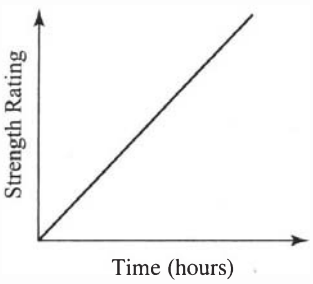
B)



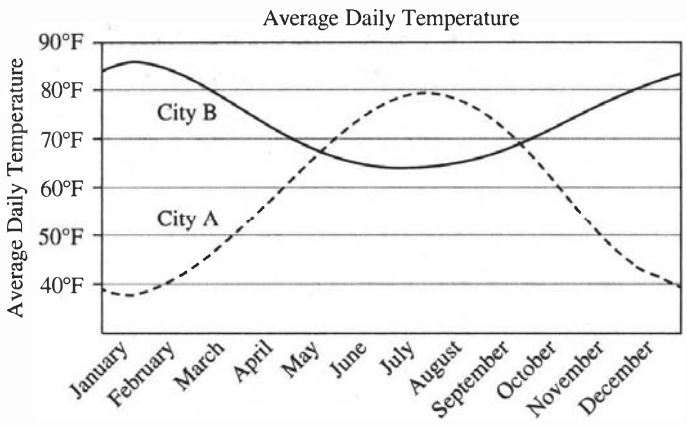
C)



D)



23



The graph above shows the average daily temperature for City A (which is in the Northern Hemisphere) and City B (which is in the Southern Hemisphere) over the course of a year. Which of the following is the most accurate statement about this information?

A) The average annual temperature for City A is greater than the average annual temperature for City B.

B) The standard deviation of the average daily temperatures for City A is greater than the standard deviation of the average daily temperatures for City B.

C) The coolest average daily temperature for City B is warmer than the warmest average daily temperature for City A.

D) The warmest average daily temperature for City B is approximately the average annual temperature for City A.

24

In the equation $y = A(x - 1)(x - k)$, A and k are non-zero constants. When this equation is graphed in the xy-plane, it forms a parabola with a vertex at $(4, 5)$. What is the value of k?

A) 3

B) 5

C) 7

D) 9

CONTINUE

4 4

25

A researcher wants to conduct a study in which 50% of the subjects are male and 50% of the subjects are female. Currently, the ratio of males to females is 5:7. If there are 420 total subjects currently in the study, how many more male subjects are needed?

A) 70

B) 120

C) 168

D) 175

26

Danielle is reading a 800-page book and has already read 120 pages. If she reads 16 pages a day for the next 15 days, what percentage of the book will she have read after those 15 days?

A) 40%

B) 45%

C) 48%

D) 54%

27

Anna's car has an efficiency of p miles per gallon of gasoline when it travels at a highway speed of k miles per hour. Which of the following represents the number of hours Anna can travel on 10 gallons of gasoline if she maintains a steady speed of k miles per hour?

A) $\frac{10p}{k}$

B) $\frac{10k}{p}$

C) $\frac{10}{pk}$

D) $10pk$

28

$$y = a$$
$$y = bx^2 + 5$$

In the system of equations above, a and b are constants. For which of the following values of a and b does the system of equations have no real solutions?

A) $a = 2, b = 4$

B) $a = -1, b = -1$

C) $a = 6, b = 1$

D) $a = 2, b = -1$

29

Which of the following expressions is equal to $(1 - i)^3$? (Note: $i^2 = -1$)

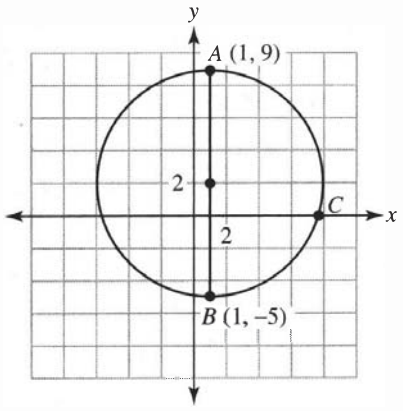
A) $2 + 2i$

B) $2 - 2i$

C) $-2 + 2i$

D) $-2 - 2i$

30



In the xy-plane above, segment AB is a diameter of the circle and point C is one of the two x-intercepts of the circle. What are the coordinates of point C?

A) $(\sqrt{45}, 0)$

B) $(\sqrt{46}, 0)$

C) $(1+\sqrt{45}, 0)$

D) $(2+\sqrt{46}, 0)$

CONTINUE

4 4

31

If the depth of the water table at a particular farm increases at a rate of 0.15 meters per year, how many years will it take for the depth of the water table to increase by 3 meters?

32

In the xy-plane, the point (2, 14) lies on the graph of $y = 3x + b$. What is the value of b?

33

Food	Rent	Clothing	Entertainment
$350	$950	$240	$260

The table above shows Dana's monthly expenses broken down into four categories. If she constructs a pie graph with four sectors representing each of the four monthly expense categories, what should be the degree measure of the central angle of the sector representing clothing expenses? (Ignore the degree symbol when gridding.)

34

$$f(x) = 2 - 4x$$
$$g(x) = 5x - 2$$

If the functions f and g are defined by the equations above, for what value of m does $f(g(m)) = 0$?

35

$$2x^3 - 3x^2 + 4x - 6 = 0$$

For what real value of x is the equation above true?

36

What is the smallest positive value of c such that $2 \sin(4c^\circ) = \sqrt{3}$?

CONTINUE

4 4

Questions 37 and 38 refer to the following information.

$$d=12-9\cos\left(\frac{2\pi-(t+9)}{365}\right)$$

The day length in Fairbanks, Alaska—that is, the number of hours per day that the sun is above the horizon—can be modeled by the equation above, where d is the day length, in hours, and t is the number of the day of the year, where t =1 corresponds to January 1st and t = 365 corresponds to December 31st.

37

In Fairbanks, Alaska, the longest day length of the year is how many hours longer than the shortest day length of the year?

38

What integer value of t corresponds to the shortest day length of the year in Fairbanks, Alaska?

STOP

If you finish before time is called, you may check your work on this section only. Do not turn to any other section of the test.

SAT PRACTICE TEST 4 ANSWER KEY

Section 1: Reading

1. B
2. A
3. C
4. B
5. B
6. C
7. B
8. A
9. C
10. A
11. A
12. D
13. B
14. C
15. A
16. D
17. D
18. D
19. C
20. B
21. B
22. C
23. C
24. A
25. A
26. C
27. D
28. D
29. B
30. C
31. A
32. D
33. C
34. A
35. B
36. C
37. A
38. C
39. B
40. D
41. D
42. A
43. A
44. C
45. A
46. B
47. C
48. C
49. B
50. C
51. D
52. A

Total Reading Points (Section 1)

Section 2: Writing and Language

1. B
2. D
3. C
4. A
5. C
6. B
7. A
8. D
9. B
10. D
11. C
12. A
13. B
14. B
15. C
16. A
17. C
18. B
19. C
20. A
21. C
22. B
23. D
24. C
25. C
26. A
27. D
28. D
29. B
30. B
31. C
32. D
33. D
34. C
35. A
36. D
37. B
38. B
39. A
40. C
41. C
42. C
43. A
44. D

Total Writing and Language Points (Section 2)

Section 3: Math (No Calculator)

1. C
2. B
3. D
4. C
5. C
6. A
7. B
8. C
9. B
10. B
11. D
12. A
13. D
14. A
15. C
16. 7.5 or 15/2
17. 6
18. 9
19. 1.5 or 3/2
20. 72

Section 4: Math (Calculator)

1. C
2. B
3. C
4. A
5. D
6. B
7. C
8. C
9. D
10. C
11. D
12. B
13. D
14. B
15. C
16. B
17. B
18. D
19. C
20. B
21. A
22. A
23. B
24. C
25. A
26. B
27. A
28. A
29. D
30. C
31. 20
32. 8
33. 48
34. .5 or 1/2
35. 1.5 or 3/2
36. 15
37. 18
38. 356

Total Math Points (Section 3 + Section 4)

because the sentence provides no surprising fact. Choice D is incorrect because the sentence does not contrast with a previous sentence.

44. **D** **Idiom**

The context makes it clear that the monument is *to* Daniel Freeman and the Homestead Act in general, but the monument is *on* the site of the Daniel Freeman homestead.

Section 3: Math (No Calculator)

1. **C** **Algebra (linear equations) EASY**

Original equation: $\frac{2}{x+2}=\frac{1}{5}$
Cross-multiply: $x+2=10$
Subtract 2: $x=8$

2. **B** **Algebra (solving equations) EASY**

Given equation: $a-b=6$
Since the question asks us for the value of a, it's a good idea to express b in terms of a, so let's solve for b.
Add b: $a=6+b$
Subtract 6: $a-6=b$
If a is 4 times as large as b: $a=4b$
Substitute $b=a-6$: $a=4(a-6)$
Distribute: $a=4a-24$
Subtract $4a$: $-3a=-24$
Divide by -3: $a=8$

3. **D** **Algebra (rewriting expressions) EASY**

Original expression: $-3(a-1)^2+2(a-1)$
FOIL: $-3(a^2-2a+1)+2(a-1)$
Distribute: $-3a^2+6a-3+2a-2$
Collect like terms: $-3a^2+8a-5$

4. **C** **Advanced Mathematics (functions) EASY**

Definition of function: $f(x)=2-2x$
Use $-2x$ as the input: $f(-2x)=2-2(-2x)$
Simplify: $f(-2x)=2+4x$

5. **C** **Algebra (linear systems) EASY**

Since the question asks us to find the value of y, it makes sense to find the value of x in terms of y and substitute.

First equation: $\frac{x}{y}=3$
Multiply by y: $x=3y$
Second equation: $y=3(x+2)$
Substitute $x=3y$: $y=3(3y+2)$
Distribute: $y=9y+6$
Subtract $9y$: $-8y=6$
Divide by -8: $y=\frac{6}{-8}=-\frac{3}{4}$

6. **A** **Algebra (absolute value and numerical reasoning) MEDIUM**

Each choice is the difference between a positive number and an absolute value, and this difference is positive if and only if the first number in the difference is greater than the absolute value. In choice A, $|4+x^2|$ can never be smaller than 1 (in fact, the smallest it can be is 4, when $x = 0$). Therefore, the expression in choice A is always negative. All of the others can have positive values, at least for certain values of x. Choice B $(4 - |1 + x^2|)$ is positive when $x = 0$. Choice C $(1 - |4 - x^2|)$ is positive when $x = 2$. Choice D $(4 - |1 - x^2|)$ is positive when $x = 0$.

7. **B** **Algebra (linear relations and numerical reasoning) EASY**

Finding the total weekly mileage for each choice requires adding the terms in an arithmetic sequence.
A) $3.5+4.0+4.5+5.0+5.5+6.0=28.5$
B) $2.5+3.5+4.5+5.5+6.5+7.5=30$
C) $7+6+5+4+3+2=27$
D) $6.0+5.5+5.0+4.5+4.0+3.5=28.5$
Therefore, the only plan that gets Rey to his 30-mile goal is plan B.

8. **C** **Advanced Mathematics (rational equations) MEDIUM**

Original equation: $\sqrt{x^2+16}=x+2$
Square both sides: $x^2+16=(x+2)^2$
FOIL: $x^2+16=x^2+4x+4$
Subtract x^2 and 4: $12=4x$
Divide by 4: $3=x$

9. **B** **Additional Topics (perpendicular lines) MEDIUM**

First we can calculate the slope of line m using the slope formula: $\text{slope}=\frac{y_2-y_1}{x_2-x_1}=\frac{8-0}{0-(-5)}=\frac{8}{5}$
Since perpendicular lines in the xy-plane have slopes that are negative reciprocals of each other, line l must have a slope of $-5/8$: $\frac{0-8}{a-0}=-\frac{5}{8}$
Simplify: $\frac{-8}{a}=-\frac{5}{8}$
Cross-multiply: $64=5a$
Divide by 5: $12.8=a$

10. **B** **Algebra (solving equations) MEDIUM-HARD**

Original equation: $B=\frac{x+W}{y+W}$
Multiply by $y+W$: $B(y+W)=x+W$
Distribute: $By+BW=x+W$
Now, to solve for W, we must gather all terms that include W on one side, so we should subtract W and subtract By:
$BW-W=x-By$

Factor W from both terms on left side: $W(B-1)=x-By$

Divide both sides by $B-1$: $W=\dfrac{x-By}{B-1}$

11. **D** **Algebra (linear parametric equations) MEDIUM**

If point B is twice as far from the origin as point A, then $b=2a$. Substituting $a=2.5t+3$ and $b=5.5t+2$ gives

$$5.5t+2=2(2.5t+3)$$

Distribute: $5.5t+2=5t+6$

Subtract $5t$ and 2: $0.5t=4$

Multiply by 2: $t=8$

Be careful—don't pick B yet, because the question does not ask for the value of t, but rather for *how far point B is from the origin.* This means we must plug this value for t into the equation that lets us solve for b:

$$b=5.5(8)+2=44+2=46$$

12. **A** **Advanced Mathematics (simplifying rationals) MEDIUM**

Original expression: $\dfrac{1+\dfrac{1}{x+1}}{1-\dfrac{1}{x-1}}$

Multiply numerator and denominator by $(x+1)(x-1)$:

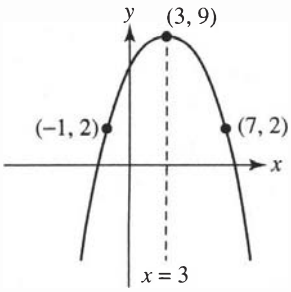
$$\frac{(x+1)(x-1)\left(1+\dfrac{1}{x+1}\right)}{(x+1)(x-1)\left(1-\dfrac{1}{x-1}\right)}$$

Distribute: $\dfrac{(x+1)(x-1)+(x-1)}{(x+1)(x-1)-(x+1)}$

Factor numerator and denominator in terms of common factors: $\dfrac{(x-1)((x+1)+1)}{(x+1)((x-1)-1)}$

Simplify: $\dfrac{(x-1)(x+2)}{(x+1)(x-2)}$

13. **D** **Advanced Mathematics (graphing parabolas) MEDIUM-HARD**



The problem gives us two points on the parabola: (−1, 2) and (7, 2). Since these two points have the same y-coordinate, they must be reflections of each other over the parabola's axis of symmetry, which is a vertical line at the average of the x-coordinates of the two points. Since the average of their x-coordinates is $(-1+7)/2=3$, the axis of symmetry is $x=3$. The vertex of the parabola must be on this axis, and choice D (3, 9) provides the only option that satisfies this condition.

14. **A** **Advanced Mathematics (quadratic equations) MEDIUM-HARD**

If the equation is true for all values of x—that is, it is an *identity*—then the quadratics on either side of the equation must have identical coefficients. We can FOIL the left-hand side of the equation so that the sides match up:

$$(3x+a)(4x+b)=12x^2+cx+3$$

FOIL: $12x^2+(3b+4a)x+ab=12x^2+cx+3$

Clearly, the first coefficients of the quadratics are the same on both sides, but the others must be identical, too, so $3b+4a=c$ and $ab=3$. This second equation will help us find a and b. Since they are negative integers, the only possible solutions are $a=-1$ and $b=-3$, or $a=-3$ and $b=-1$. Plugging in these two solutions into the other equation gives us the two possible values of $c=3(-1)+4(-3)=-15$ and $c=3(-3)+4(-1)=-13$.

15. **C** **Advanced Mathematics (analyzing polynomial functions) HARD**

When working with exponential expressions, it usually helps to express the exponentials in terms of the same base, if possible. Since $27=3^3$, both the numerator and denominator can be expressed in base 3.

Original expression: $\dfrac{3^a}{27^b}$

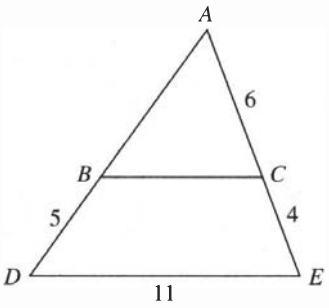
Substitute $27=3^3$: $\dfrac{3^a}{(3^3)^b}$

Use identity $(x^m)^n=x^{mn}$: $\dfrac{3^a}{3^{3b}}$

Use identity $x^m/x^n=x^{m-n}$: 3^{a-3b}

Substitute $a-3b=5$: 3^5

16. **7.5 or 15/2** **Additional Topics (similarity) EASY**



If segment BC is parallel to segment DE, then $\angle ABC$ and $\angle ADE$ are congruent because they form a corresponding pair. Since triangles ABC and ADE also share angle A, they are similar by the AA (angle-angle)

similarity theorem. This means that all of the corresponding sides are proportional: $\frac{AB}{AD}=\frac{AC}{AE}$

Let's define x as the length of AB and substitute the lengths into this equation: $\frac{x}{5+x}=\frac{6}{6+4}$

Cross-multiply: $10x=30+6x$
Subtract $6x$: $4x=30$
Divide by 4: $x=7.5$

17. **6** **Advanced Mathematics (solving quadratics) EASY-MEDIUM**

This is simply a quadratic equation, so it can be solved by factoring or by using the quadratic formula. In this case, the quadratic is fairly easy to factor once it's in the right form. $x^2-12=4x$
Subtract $4x$: $x^2-4x-12=0$
Factor: $(x-6)(x+2)=0$
Solve with the Zero Product Property: $x=6$ or $x=-2$
Since $x>0$, the solution is $x=6$.
Alternately, we can solve this with the quadratic formula:

$$x=\frac{4\pm\sqrt{(-4)^2-4(1)(-12)}}{2}=\frac{4\pm\sqrt{16+48}}{2}$$

$$=\frac{4\pm\sqrt{64}}{2}=\frac{4\pm8}{2}=-2 \text{ or } 6$$

18. **9** **Algebra (linear systems and numerical reasoning) MEDIUM**

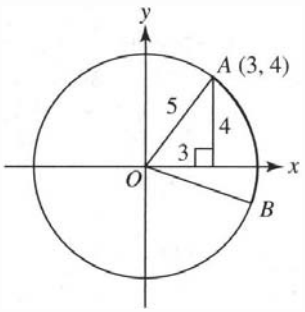
Let's let m represent the number of classified codes that Paul has broken. Since the question tells us that Paul has broken 12 codes in all, he must have broken $12-m$ access codes. Since access codes are worth 20 points and classified codes are worth 75 points, and Paul earned 735 points altogether, $75m+20(12-m)=735$
Distribute: $75m+240-20m=735$
Combine like terms: $55m+240=735$
Subtract 240: $55m=495$
Divide by 55: $m=9$

19. **1.5 or 3/2** **Advanced Mathematics (solving polynomials) MEDIUM-HARD**

Original equation: $2x^2(4x^2-5)=18$
Distribute: $8x^4-10x^2=18$
Subtract 18: $8x^4-10x-18=0$
Divide by 2: $4x^4-5x-9=0$
Although this is a 4th-degree polynomial, it is "quadratic in x^2," which means that it becomes a normal quadratic equation if we make the substitution $y=x^2$:
$4(x^2)^2-5x^2-9=0$
Substitute $y=x^2$: $4y^2-5y-9=0$
Factor: $(4y-9)(y+1)=0$
Solve using the Zero Product Property:
$4y-9=0$ or $y+1=0$; therefore, $y=9/4$ or $y=-1$
Substitute $x^2=y$: $x^2=9/4$ or $x^2=-1$

Since x is a real number, $x^2=-1$ has no solutions. And if x must be positive, then the only solution to $x^2=9/4$ is $x=3/2$ or 1.5

20. **72** **Additional Topics (arcs) HARD**



Since we know the coordinates of the center of the circle as well as a point on the circle, we can calculate the radius of the circle using the distance formula: $r=\sqrt{(3-0)^2+(4-0)^2}=\sqrt{9+16}=\sqrt{25}=5$ (or we can just notice that dropping the perpendicular from (3, 4) to the x-axis creates a 3-4-5 right triangle, as shown above). This means that the circumference of the circle is $2\pi r=2\pi(5)=10\pi$. Since arc AB has a length of 2π, it represents $2\pi/10\pi=1/5$ of the entire circumference. Since the circumference of the circle has a degree measure of 360°, the measure of the central angle AOB is $(1/5)(360°)=72°$.

Section 4: Math (Calculator)

1. **C** **Algebra (representing quantities) EASY**

If Matt sells x paperbacks and earns \$1.12 each in royalties, he earns $\$1.12x$ in paperback royalties altogether. If he sells y e-books and earns \$0.58 each in royalties, he earns $\$0.58y$ in e-book royalties altogether, for a total of $\$(1.12x+0.58y)$ in royalties.

2. **B** **Problem Solving and Data Analysis (averages) EASY**

When analyzing problems that deal with averages, it often helps to recall that the sum of any set of numbers equals their average times the number of numbers (sum = average × #). Therefore, if three numbers have an average of 8, their sum is 8 × 3 = 24, and if two numbers have an average of 6, their sum is 6 × 2 = 12. Therefore, the sum of all five numbers is 24 + 12 = 36, and so their average is 36 ÷ 5 = 7.2.

3. **C** **Advanced Mathematics (analyzing functions) EASY**

We can calculate the value of $f(4)$ as long as we know the value of c in the definition of f, so this is our first task.
Original definition of function: $f(x)=x^2+c$
Substitute $f(2)=9$: $f(2)=(2)^2+c=9$

Subtract 4: $c = 5$
Therefore, $f(x) = x^2 + 5$, and so $f(4) = (4)^2 + 5 = 16 + 5 = 21$.

4. **A** **Additional Topics (perpendicular lines) EASY**

If perpendicular lines are graphed in the xy-plane, and if neither line is vertical, then their slopes are opposite reciprocals. Neither line is vertical, because a and c are nonzero. The slopes of the two lines are a and c, respectively, and if these are opposite reciprocals, then $ac = -1$.

5. **D** **Advanced Mathematics (parabolas) EASY**

Since each equation is in the form of a quadratic function ($y = ax^2 + bx + c$), then each represents the graph of a parabola. The only question is which one is satisfied by the ordered pairs (0, 0) and (4, 0). We could simply plug these values into each equation and see which one is true for both points. We could also notice that both of these points are x-intercepts, so they both correspond to a factor of the quadratic, and so the quadratic must include $(x - 0)$ and $(x - 4)$ as factors. Notice that choice D factors to $y = 2x^2 - 8x = 2(x)(x - 4)$.

6. **B** **Algebra (analyzing linear relationships in context) EASY**

We can use the two known ordered pairs to calculate the slope of the line: $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-2)}{5-1} = \frac{5}{4}$

Then we can use this equation again to solve for a:

$$\frac{3-a}{5-3} = \frac{3-a}{2} = \frac{5}{4}$$

Cross-multiply: $12 - 4a = 10$
Subtract 12: $-4a = -2$
Divide by -4: $a = 0.5$

7. **C** **Algebra (linear relationships in context) EASY**

First, we must calculate the value of t that corresponds to a population of 660 bison: $660 = 240 + 21t$
Subtract 240: $420 = 21t$
Divide by 21: $20 = t$
Since t is defined as the number of years that have elapsed since the preservation program began on March 30, 2006, the bison population is predicted to reach 660 precisely 20 years later, on March 30, 2026.

8. **C** **Algebra (linear relationships in context) MEDIUM**

The new survey determined that on March 30, 2014, the bison population was 420. By the definition of the variable, this corresponds to $P = 420$ when $t = 8$. The naturalist wants to use this to adjust the t coefficient in the population equation. Let's call this new coefficient m. This means that the new equation is $P = 240 + mt$. If we plug in our new values for P and t we get: $420 = 240 + m(8)$
Simplify: $420 = 240 + 8m$
Subtract 240: $180 = 8m$
Divide by 8: $22.5 = m$

9. **D** **Advanced Mathematics (radical equations) MEDIUM**

Original equation: $\frac{x\sqrt{2} - 4 + 5x}{2} = 0$
Multiply by 2: $x\sqrt{2} - 4 + 5x = 0$
Add 4: $x\sqrt{2} + 5x = 4$
Factor x from the left side: $x(\sqrt{2} + 5) = 4$
Divide by $\sqrt{2} + 5$: $x = \frac{4}{\sqrt{2} + 5}$

10. **C** **Advanced Mathematics (exponential relationships) MEDIUM**

If the annual interest rate of the fund is 10% (or 0.1), and it is compounded monthly, the monthly rate is (0.1)/12. To increase a quantity by (0.1)/12, we must multiply it by 1 + (0.1)/12, which represents 100% plus the 1/12 of 10% interest. Over the course of t years, the fund will have had $12t$ compounding periods, since there are 12 months per year. This means that the total value of the fund at the end of t years is $V = \$2{,}400\left(1 + \frac{0.1}{12}\right)^{12t}$.

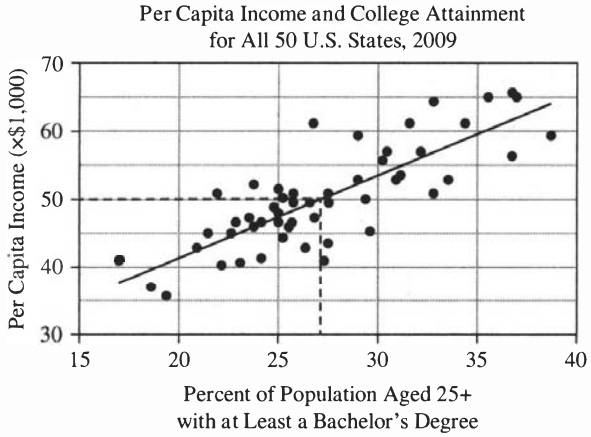
11. **D** **Problem Solving and Data Analysis (data-gathering methods) MEDIUM**

The researcher is interested in the eating habits of *all adults* in a large city, but the adults who eat at a major chain restaurant are not likely to represent the eating habits of urban adults in general. Those adults who make all of their own meals and do not eat fast food are very unlikely to be found in a major chain restaurant. A better survey method, then, would be to utilize a more randomized sampling method.

Choice A, giving the survey to a larger sample group at the restaurant, would not address the original sampling bias. Choice B, conducting the survey at a farmer's market rather than at a chain restaurant, would merely tilt the sampling bias toward those who make their own meals, but would not provide a less biased sample. Choice C, excluding the results from the unmarried respondents, would not address any relevant bias in the sample. The only choice that would introduce a more reliable sample is D, giving the survey to a group of adults selected at random from public records.

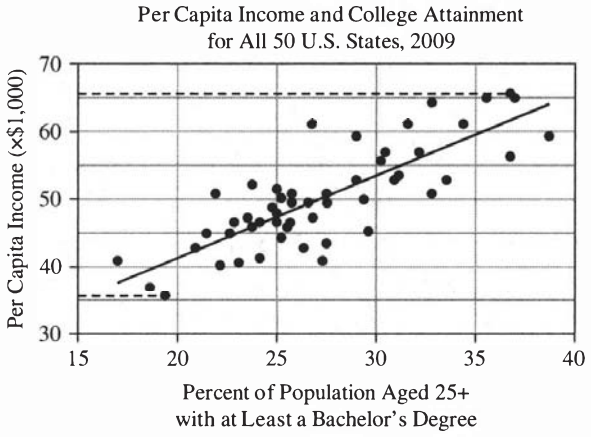
(Of course, a careful experimenter should always make sure that these public records do not contain other biases. For instance, if they are public records of homeowners rather than renters, they would likely contain a disproportional number of wealthy people who tend not to eat fast food.)

12. **B** **Problem Solving and Data Analysis (scatter plots) MEDIUM**



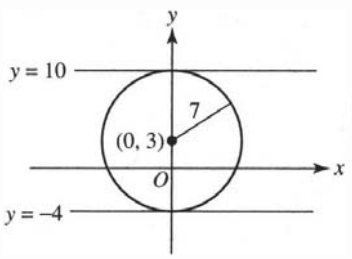
As the dotted lines above indicate, the line of best fit appears to contain the point (27%, $50,000), which means that a state in which 27% of the adult population have college degrees is expected to have a per capita income of $50,000.

13. **D** **Problem Solving and Data Analysis (scatter plots) MEDIUM**



As the dotted lines above indicate, the lowest per capita income indicated on the scatter plot is about $36,000 and the highest is about $66,000. The greater value is (66,000 − 36,000)/36,000 = 30,000/36,000 = 0.83 or 83% greater than the smaller value.

14. **B** **Advanced Mathematics (nonlinear systems) MEDIUM**



The equation $x^2 + (y - 3)^2 = 49$ represents a circle with center (0, 3) and a radius of $\sqrt{49} = 7$, as shown above. The equation $y = b$ represents a horizontal line. If this horizontal line is tangent to the circle, then it is one of the two shown in the diagram above, either 7 units above the center of the circle, at $y = 3 + 7 = 10$, or 7 units below the center of the circle, at $y = 3 - 7 = -4$.

15. **C** **Problem Solving and Data Analysis (percent change) MEDIUM**

Let's define b as the gross domestic product of Country B. If the gross domestic product of Country A is $720 billion, and this is 80% greater than the gross domestic product of Country B, then 720 billion = $1.80b$. Dividing both sides by 1.80 gives $b = 400$ billion.

16. **B** **Problem Solving and Data Analysis (tables and probabilities) MEDIUM**

The table shows that there are a total of 42 + 3 = 45 students in the top 10% of the graduating class, and that 3 of these students took no AP courses. This means that 3/45 = 1/15 of the students in the top 10% of the class took no AP courses.

17. **B** **Algebra (conversions) MEDIUM**

This is a fairly straightforward conversion problem. First, we must determine the distance that Mars travels when it makes one full orbit around the Sun. This orbit is a circle with radius 142 million miles, so it has a length equal to the circumference of this circle, or 2π(142 million) = 892 million miles. Next we convert this to a speed in miles per hour by using the fact that this orbit takes 687 days and there are 24 hours in a day.

$$\frac{892{,}000{,}000 \text{ miles}}{1 \text{ orbit}} \times \frac{1 \text{ orbit}}{687 \text{ days}} \times \frac{1 \text{ day}}{24 \text{ hours}}$$

$$= 54{,}113 \text{ miles per hour}$$

Notice that the units cancel correctly, and that the units match up on both sides of the equation.

18. **D** **Advanced Mathematics (exponential decay) MEDIUM-HARD**

According to the equation, the fraction of substance A that remains after t years is $\left(\frac{1}{2}\right)^{\frac{t}{3}}$. Since this is a fraction of 1 kilogram, it also represents the number of kilograms that remain of substance A after t years. Similarly, $\left(\frac{1}{2}\right)^{\frac{t}{7}}$ represents the amount of substance B that remains after t years. Because substance B has a longer half life than substance A has, there will always be more of substance B remaining than substance A; therefore, the expression $\left(\frac{1}{2}\right)^{\frac{t}{7}} - \left(\frac{1}{2}\right)^{\frac{t}{3}}$ tells us how much more of substance B remains than substance A. Choice C

gives the opposite of this difference, and so it yields a negative value.

19. **C** **Problem Solving and Data Analysis (data in context) MEDIUM-HARD**

To hire Company A to provide all three services for 24 months would cost \$5,000 + \$30(24) + \$360(24) = \$14,360. (Keep in mind that the website design fee is a one-time fee, but the other two are monthly fees.) Similarly, the cost for hiring Company B for the same amount of time would be \$3,300 + \$35(24) + \$320(24) = \$11,820. The difference between the costs is \$14,360 − \$11,820 = \$2,540.

20. **B** **Problem Solving and Data Analysis (scatter plots) MEDIUM-HARD**

The cost for hiring Company B to provide all three services is \$3,300 + \$35m + \$320m = \$3,300 + \$355m, and the cost of hiring Company C to provide the same services for the same length of time is \$4,200 + \$25m + \$300m = \$4,200 + \$325m. If the cost of hiring Company B is less than that of hiring Company C, then

$3{,}300 + 355m < 4{,}200 + 325m$

Subtract 3,300: $355m < 900 + 325m$

Subtract 325m: $30m < 900$

Divide by 30: $m < 30$

21. **A** **Advanced Mathematics (exponentials) MEDIUM-HARD**

The answer choices suggest that this expression can be simplified to an exponential with base 2. Therefore, it's important to notice that $8 = 2^3$.

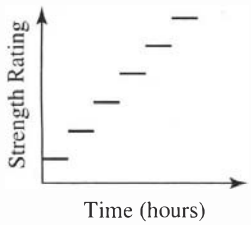
Original expression: $\left(\frac{8}{m}\right)^2$

Substitute $8 = 2^3$ and $m = 2^k$: $\left(\frac{2^3}{2^k}\right)^2$

Use the identity $x^m/x^n = x^{m-n}$: $(2^{3-k})^2$

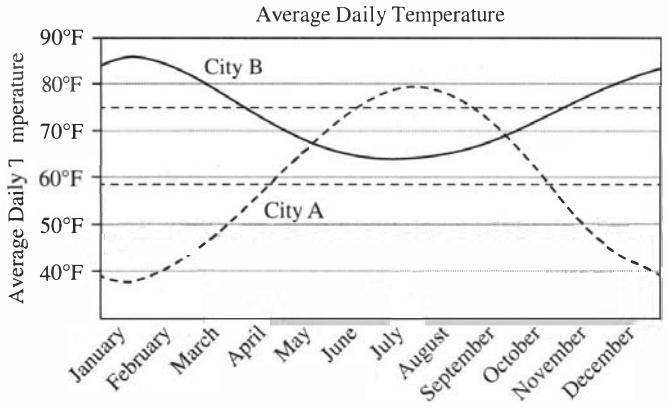
Use the identity $(x^m)^n = x^{mn}$: 2^{6-2k}

22. **A** **Problem Solving and Data Analysis (interpreting graphs) MEDIUM-HARD**



The problem makes it clear that the strength rating of a player can only have integer values: 1, 2, 3, 4, 5, and 6. Therefore, the graph cannot be a "continuous" one, but rather one that makes abrupt jumps whenever the player's strength rating increases. Choice A is the only one that shows a graph with such abrupt and discontinuous jumps. Choice B looks somewhat close, but it must be incorrect because it shows a continuous function.

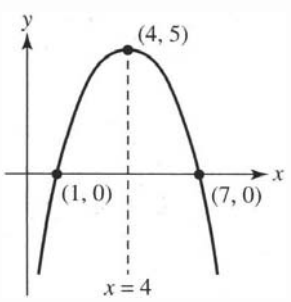
23. **B** **Data Analysis and Problem Solving (graphical statistics) MEDIUM**



It helps to draw the "midlines" for both curves—the line halfway between the annual extremes—as shown above. These midlines show the average annual temperature for the two cities: about 58° for City A and about 75° for City B. Choice A is incorrect because the average annual temperature for City A (58°) is not greater than the average temperature for City B (75°). Choice B is correct because the "spread" of temperatures (which is what standard deviation measures) is clearly greater for City A than City B: the temperatures for City A have a spread of over 40° (from about 38° to about 80°), but the temperatures for City B have a spread of only about 20° (from about 65° to about 85°). Choice C is incorrect because the coolest average daily temperature for City B (about 65°) is not warmer than the warmest average daily temperature for City A (about 80°). Choice D is incorrect because the warmest average daily temperature for City B (about 85°) is not approximately the average annual temperature for City A (about 58°).

24. **C** **Advanced Mathematics (analyzing quadratics) MEDIUM-HARD**

The problem gives us two points on the parabola: the vertex at (4, 5) and the zero at (1, 0), which we can see by using the Zero Product Property on one of the factors, $(x-1)$, of the equation. The equation tells us that the parabola has x-intercepts at $x = 1$ and $x = k$, and we need to find the value of k. Graphing this information and using the symmetry of parabolas helps us to find k:



Since the line of symmetry is a vertical line passing through the vertex, it must be the line $x = 4$, as shown in the figure above. The x-intercepts must be symmetric to this line, and since the intercept $x = 1$ is 3 units to the left of this line, the other x-intercept must be 3 units to the right, at $x = 7$, and so $k = 7$.

25. **A** **Problem Solving and Data Analysis (ratios) MEDIUM-HARD**

If the ratio of males to females is 5:7, then the total consists of $5 + 7 = 12$ parts. Since there are 420 total subjects in the study, then each part is $420 \div 12 = 35$ subjects, so there are $5(35) = 175$ males and $7(35) = 245$ females. Since we want the number of males and females to be equal, we need $245 - 175 = 70$ more males.

26. **B** **Problem Solving and Data Analysis (percentages and rates) HARD**

If Danielle has already read 120 pages, she has $800 - 120 = 680$ pages left to read. If she reads 16 pages a day for 15 days, she will read an additional $(16)(15) = 240$ pages, for a total of $120 + 240 = 360$ pages. This is $360/800 = 0.45$ or 45% of the entire book.

27. **A** **Problem Solving and Data Analysis (rates) MEDIUM-HARD**

We can use the given rates as conversion factors to convert 10 gallons into the number of hours she can drive:

$$10 \text{ gallons} \times \frac{p \text{ miles}}{1 \text{ gallon}} \times \frac{1 \text{ hour}}{k \text{ miles}} = \frac{10p}{k} \text{ hours}$$

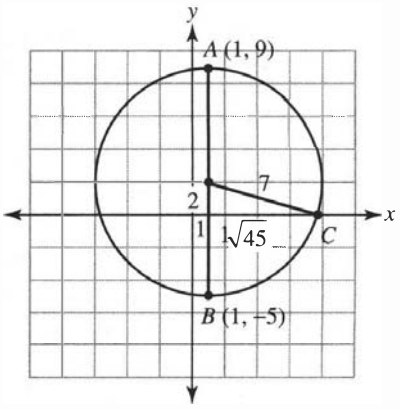
28. **A** **Advanced Mathematics (nonlinear systems) HARD**

The graph of $y = a$ is a horizontal line, and the graph of $y = bx^2 + 5$ is a parabola with a vertex at (0, 5). If the system has no real solutions, then these two graphs cannot intersect. This can happen in two ways: if the horizontal line is above the vertex and the parabola is "open down," or if the horizontal line is below the vertex and the parabola is "open up." Choice A yields a horizontal line that is below the vertex of an "open up" parabola.

29. **D** **Special Topics (complex numbers) MEDIUM-HARD**

Original expression:	$(1-i)^3$
Expand:	$(1-i)(1-i)(1-i)$
FOIL first two factors:	$(1-i-i+i^2)(1-i)$
Combine like terms and substitute $i^2 = -1$:	$(-2i)(1-i)$
Distribute:	$-2i + 2i^2$
Substitute $i^2 = -1$:	$-2i - 2 = -2 - 2i$

30. **C** **Special Topics (circles) HARD**



First let's draw the radius from the center of the circle to point C. Since AB is the diameter and its length is $9 - (-5) = 14$, the radius of the circle is $14/2 = 7$. Notice that this radius is the hypotenuse of the right triangle with shorter leg of 2. (Notice that the scale on the coordinate plane is *2 units* per line.) We can use the Pythagorean Theorem to find the longer leg of this triangle: $2^2 + b^2 = 7^2$. This gives us $b = \sqrt{45}$. Since point C is this distance to the right of the line $x = 1$ (the line that contains AB), it has coordinates $(1 + \sqrt{45}, 0)$.

31. **20** **Algebra (word problems) EASY**

$$3 \text{ meters} \times \frac{1 \text{ year}}{0.15 \text{ meters}} = 20 \text{ years}$$

32. **8** **Algebra (lines in the coordinate plane) EASY**

Given equation:	$y = 3x + b$
Substitute $x = 2$ and $y = 14$:	$14 = 3(2) + b$
Simplify:	$14 = 6 + b$
Subtract 6:	$8 = b$

33. **48** **Problem Solving and Data Analysis (pie graphs) MEDIUM**

Since the total expenditures are $350 + 950 + 240 + 260 = 1{,}800$ dollars, clothing expenses constitute $240/1{,}800 = 2/15$ of the total. Therefore, on a pie graph, the clothing expenses sector would have a central angle of $(2/15)(360°) = 48°$.

34. **.5 or 1/2** **Advanced Mathematics (compositions) MEDIUM**

Given functions:	$f(x) = 2 - 4x$
	$g(x) = 5x - 2$
Substitute to find the expression for $f(g(m))$:	$f(g(m)) = f(5m - 2) = 2 - 4(5m - 2)$
Distribute and simplify:	$f(g(m)) = 10 - 20m$
Set $f(g(m)) = 0$:	$0 = 10 - 20m$
Add $20m$:	$20m = 10$
Divide by 20:	$m = 0.5$

35. **1.5 or 3/2** **Advanced Mathematics (solving polynomials) MEDIUM-HARD**

Original equation: $2x^3 - 3x^2 + 4x - 6 = 0$

The expression on the left can be factored by grouping:

$$x^2(2x-3) + 2(2x-3) = (x^2+2)(2x-3) = 0$$

Only the second of these factors can equal 0, so:

$$2x - 3 = 0$$

Add 3 and divide by 2: $x = 1.5$

36. **15** **Additional Topics (trigonometry) MEDIUM-HARD**

Original equation: $2\sin(4c^\circ) = \sqrt{3}$

Divide by 2: $\sin(4c^\circ) = \frac{\sqrt{3}}{2}$

The smallest positive angle with a sine of $\frac{\sqrt{3}}{2}$ is 60°.

Therefore $4c = 60$

Divide by 4: $c = 15$

37. **18** **Data Analysis (trigonometric formulas) MEDIUM-HARD**

If the formula $d = 12 - 9\cos\left(\frac{2-(t+9)}{365}\right)$ is graphed in the xy-plane, where d is represented by the y-axis and t is represented by the x-axis, the resulting graph is a sinusoid with an amplitude of 9. This means that the longest day length of the year is 9 hours longer than the average day length and the shortest day length of the year is 9 hours shorter than the average day length. Therefore the difference between the longest and shortest day length is 18 hours.

38. **356** **Data Analysis (trigonometric formulas) MEDIUM-HARD**

The function $d = 12 - 9\cos\left(\frac{2\pi(t+9)}{365}\right)$ has its minimum when $9\cos\left(\frac{2\pi(t+9)}{365}\right)$ has a maximum. Since $y = \cos x$ has its first positive maximum at $x = 2\pi$: $\frac{2\pi(t+9)}{365} = 2\pi$

Distribute and multiply both sides by 365:

$$2\pi t + 18\pi = 730\pi$$

Subtract 18π: $2\pi t = 712\pi$

Divide by 2π: $t = 356$

3 3

Math Test—No Calculator

25 MINUTES, 20 QUESTIONS

Turn to Section 3 of your answer sheet to answer the questions in this section.

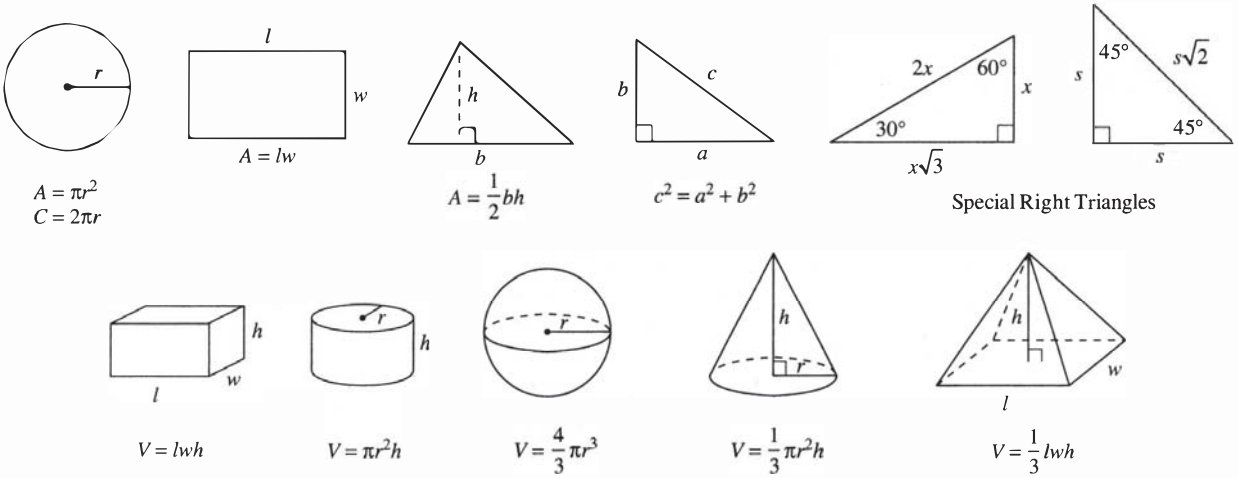
DIRECTIONS

For questions 1–15, solve each problem, choose the best answer from the choices provided, and fill in the corresponding circle on your answer sheet. **For questions 16–20**, solve the problem and enter your answer in the grid on the answer sheet. Please refer to the directions before question 16 on how to enter your answers in the grid. You may use any available space in your test booklet for scratch work.

NOTES

1. The use of a calculator **is not permitted**.
2. All variables and expressions used represent real numbers unless otherwise indicated.
3. Figures provided in this test are drawn to scale unless otherwise indicated.
4. All figures lie in a plane unless otherwise indicated.
5. Unless otherwise indicated, the domain of a given function f is the set of all real numbers x for which $f(x)$ is a real number.

REFERENCE



The number of degrees of arc in a circle is 360.
The number of radians of arc in a circle is 2π.
The sum of the measures in degrees of the angles of a triangle is 180.

CONTINUE

3 3

1

A grocery store pledges to completely eliminate its use of plastic bags in exactly three years. To meet this goal, it will reduce consumption by 100 bags per month. How many plastic bags per month is it using currently?

A) 300

B) 1,200

C) 2,400

D) 3,600

2

Which choice is equal to $\sqrt{81}+\sqrt{36}$?

A) $\sqrt{117}$

B) 12

C) 14

D) 15

3

A car rental company rents cars for $25 per day. An optional navigation system can be rented for an additional $10 per day. Renters are charged for gas at a rate of 10 cents per mile. If Carla rents a car with a navigation system for three days and travels x miles in that time, which formula gives the cost of her rental, C, in dollars?

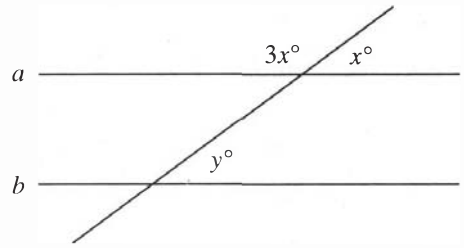
A) $C=0.10x+105$

B) $C=0.10x+75$

C) $C=10x+105$

D) $C=10x+85$

4



If lines a and b are parallel in the figure above, what is the value of $x+y$?

A) 45

B) 90

C) 105

D) 135

5

$$\frac{1}{2}(2x-6)=-2(-2x-7.5)$$

What is the solution to the equation above?

A) -6

B) -4

C) 4

D) 6

6

Which choice is equivalent to $\dfrac{x^{-\frac{1}{3}}y^3}{y^{-2}}$?

A) $\dfrac{y}{\sqrt[3]{x}}$

B) $\dfrac{y}{x^3}$

C) $\dfrac{y^5}{\sqrt[3]{x}}$

D) $\dfrac{y^5}{x^3}$

CONTINUE

3 3

7

Which factored expression below is equivalent to $x(2x-4)(-x+3)$?

A) $-2x(x-6)(x+1)$

B) $x(x+4)(x-3)$

C) $-2x(x-2)(x-3)$

D) $-x(2x-4)(x+3)$

8

$$y=-\frac{1}{2}x-4$$

$$x=2y+2$$

If the two equations above are graphed in the xy-plane, what is the relationship between the lines?

A) The two lines are perpendicular.

B) The two lines are distinct and parallel.

C) The two lines intersect but are not perpendicular.

D) The two lines are identical.

9

The equation $p=7x-125$ can be used by the owner of a smoothie cart to calculate p, the total profit in dollars in a given day if x smoothies are sold. What does the number 125 represent in this equation?

A) The daily cost, in dollars, to run the smoothie cart

B) The average income from smoothies sold for any day of the week

C) The average numbers of smoothies sold

D) The total income from smoothies sold the previous day

10

What is the remainder when x^2-5x+6 is divided by $x-4$?

A) 0

B) 1

C) 2

D) 3

11

The function g has the property that $g(-x)=-g(x)$ for all real values of x. If $g(-2)=-8$, which of the following could define g?

A) $g(x)=x^3-x$

B) $g(x)=x^3$

C) $g(x)=x^2-12$

D) $g(x)=x^2+6x$

12

What is the sum of all values of r that satisfy the equation $3r^2-12r+7=0$?

A) -4

B) 0

C) 2

D) 4

13

If $i^x i^y=1$ and $i=\sqrt{-1}$, which of the following could be the values of x and y?

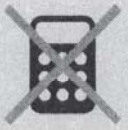
A) $x=0, y=1$

B) $x=1, y=2$

C) $x=2, y=6$

D) $x=3, y=6$

CONTINUE

3  3

14

Ibrahim must collect and catalog different specimens of flowers and leaves for a botany project. If he gathers and catalogs flower specimens at a rate of 4 per hour and leaf specimens at a rate of 2 every 20 minutes, and he must collect a total of f flowers and l leaves for the project, which expression represents the total number of minutes it will take him to complete the project?

A) $4f+2l$

B) $60\left(\frac{f}{4}+\frac{l}{2}\right)$

C) $\frac{f}{4}+\frac{l}{6}$

D) $60\left(\frac{f}{4}+\frac{l}{6}\right)$

15

$$y=-6x+10$$

$$y=x^2-7x+10$$

If (x, y) is a solution to the system above, which choice is a possible value of $x+y$?

A) 1

B) 4

C) 5

D) 12

CONTINUE

3 3

16

In a carnival ring-toss game, a player must earn 100 points to win. Successful short tosses earn 5 points each and successful long tosses earn 15 points each. Mike plays the ring-toss game and wins, making at least four long tosses and missing none of his tosses, how many tosses could he have made in total?

17

If $a = b + 4$, what is the value of $a^2 - 2ab + b^2$?

18

If $w = 3\sqrt{5}$ and $2w - \sqrt{20} = \sqrt{8y}$, what is the value of y?

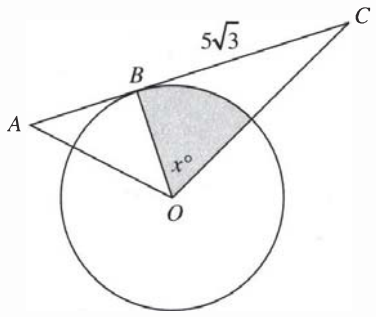
19

$$mx + ny = 3$$

$$34x + 17y = 51$$

In the system of equations above, m and n are constants. If the system has infinitely many solutions, what is the value of $m + n$?

20



Note: Figure not drawn to scale.

In the figure above, point O is the center of the circle and segment AC is tangent to the circle at point B such that $BC = 5\sqrt{3}$. If $\sin(x°) = \frac{\sqrt{3}}{2}$ and the area of the shaded sector is $k\pi$, what is the value of k ?

STOP

If you finish before time is called, you may check your work on this section only. Do not turn to any other section of the test.

4 4

Math Test—Calculator

55 MINUTES, 38 QUESTIONS

Turn to Section 4 of your answer sheet to answer the questions in this section.

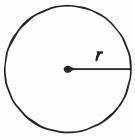
DIRECTIONS

For questions 1–30, solve each problem, choose the best answer from the choices provided, and fill in the corresponding circle on your answer sheet. **For questions 31–38**, solve the problem and enter your answer in the grid on the answer sheet. Please refer to the directions before question 31 on how to enter your answers in the grid. You may use any available space in your test booklet for scratch work.

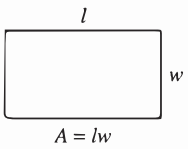
NOTES

1. The use of a calculator is **permitted**.
2. All variables and expressions used represent real numbers unless otherwise indicated.
3. Figures provided in this test are drawn to scale unless otherwise indicated.
4. All figures lie in a plane unless otherwise indicated.
5. Unless otherwise indicated, the domain of a given function f is the set of all real numbers for which $f(x)$ is a real number.

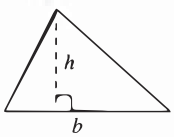
REFERENCE



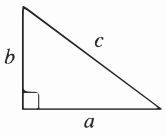
$A = \pi r^2$
$C = 2\pi r$



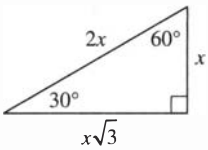
$A = lw$

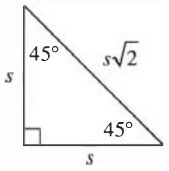


$A = \frac{1}{2}bh$

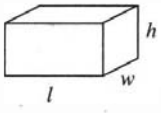


$c^2 = a^2 + b^2$

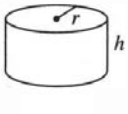




Special Right Triangles

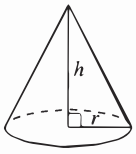


$V = lwh$

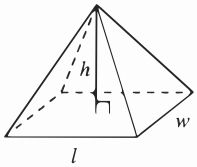


$V = \pi r^2 h$

$V = \frac{4}{3}\pi r^3$



$V = \frac{1}{3}\pi r^2 h$



$V = \frac{1}{3}lwh$

The number of degrees of arc in a circle is 360.
The number of radians of arc in a circle is 2π.
The sum of the measures in degrees of the angles of a triangle is 180.

CONTINUE

4  4

1

When the equation $3x - 2y = 12$ is graphed in the xy-plane, what is the y-intercept?

A) -6

B) -4

C) 6

D) 12

2

If $4n \geq 9$, what is the least possible value of $4n+1$?

A) 13

B) 10

C) 8

D) 5

3

$$4c+7d=29$$

$$2c+3d=13$$

What is the value of c that corresponds to the solution of the system above?

A) 2

B) 3

C) 4

D) 8

4

Priscilla is stocking her garden store with soil and fertilizer for the spring. She plans to buy 2 bags of fertilizer for every 6 bags of soil. If she buys a total of 132 bags of soil, how many bags of fertilizer did she buy?

A) 22

B) 44

C) 68

D) 396

5

At Wide Awake coffee shop, a customer can buy a small coffee for \$2.50 and any number of additions, such as soy milk, whipped cream, or flavored syrup, for \$0.30 each. Which expression represents the cost, in dollars, of a small coffee with n additions?

A) $0.3n$

B) $0.3n + 2.5$

C) $2.8n$

D) $2.5n + 0.3n$

6

If $g(x)=2x-5$ and $f(x)=-x+3$, which of the following represents $f(g(x))$?

A) $-2x-2$

B) $-2x-1$

C) $-2x+1$

D) $-2x+8$

CONTINUE

4  4

7

Which of the following is the simplified form of $(3x^2-4x+2)-(4x^2-3x-1)$?

A) $-x^2-x+3$

B) $-x^2-7x+1$

C) x^2-7x+3

D) $-x^2-x+1$

8

If x is the degree measure of an angle between 0° and 90°, and if $\cos x° = \frac{4}{5}$, what is the value of $\tan x°$?

A) $\frac{5}{4}$

B) $\frac{3}{4}$

C) $\frac{4}{3}$

D) $\frac{3}{5}$

9

The height of Jenna's best pole vault last season was in the 85th percentile of all high school pole vaults in the country last season. Which choice is the best interpretation of this statistic?

A) Jenna's best pole vault was 15% lower than the best vault in the country last season.

B) Jenna's best pole vault was 85% higher than the nationwide average last season.

C) Jenna's vault was at least as high as the best vault of 85% of the pole vaulters in the country last season.

D) Jenna won the pole vault event in 85% of the meets in which she participated last season.

10

Frannie has budgeted $60 to buy new running shoes at Bay Island Running Company, which is having a storewide 20% off sale. If a 5% sales tax is also added to her purchase, what is the maximum price of running shoes, before discount and sales tax, that Frannie can buy?

A) $68.50

B) $70.64

C) $71.42

D) $78.26

11

If $a = \frac{1}{b}$ and $a+b=0$, what is the value of a?

A) $\sqrt{2}$

B) $2\sqrt{2}$

C) 4

D) The value of a cannot be a real number.

12

A new colony of black garden ants with an initial population of p_0 grows at a rate of 20% per week for the first 5 weeks after the initial colonization. Which of the following equations models the population, p, of the colony w weeks after the initial colonization, where $0 \le w \le 5$?

A) $p = p_0(1.2)^w$

B) $p = p_0(1.02)^w$

C) $p = (1.2p_0)^w$

D) $p = (1.02p_0)^w$

CONTINUE

4 4

13

Which of the following equivalent forms of the equation $x - 3y - 12 = 0$ shows the slope and the x-intercept of its graph in the xy-plane as constants or coefficients?

A) $x-3y=12$

B) $y=\frac{1}{3}x-4$

C) $y=\frac{1}{3}(x-12)$

D) $x=3(y+4)$

14

Which of the following functions corresponds to a graph in the xy-plane with x-intercepts at -2 and 3, and a y-intercept of 12?

A) $f(x)=(-2x-4)(2x-6)$

B) $f(x)=(-2x-4)(x-3)$

C) $f(x)=(4x+2)(3x-9)$

D) $f(x)=(2x+4)(2x+6)$

15

Which choice represents the values of x for which the function $y=\frac{x^2+2x-3}{x^2-5x-6}$ is undefined?

A) $x=-1, x=6$

B) $x=-2, x=3$

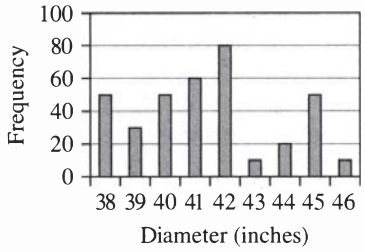
C) $x=1, x=-6$

D) $x=2, x=-3$

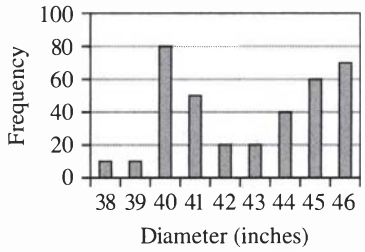
16

A botanist hypothesizes that, in a certain patch of forest, the median diameter of the 360 white oak trees is 42 inches. The histograms below show the data collected on the diameters of 360 white oak trees. Which graph is consistent with the botanists' hypothesis?

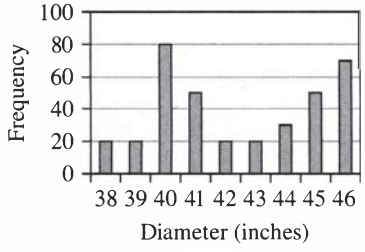
A)



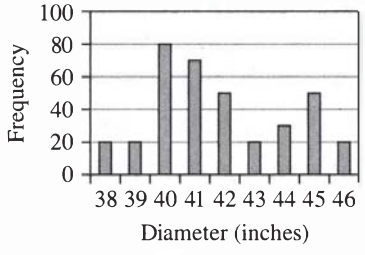
B)



C)



D)



4 4

17

In the xy-coordinate plane, line l has a slope of $\frac{1}{4}$ and passes through points $(26, -p)$ and $(-2p, 4)$. What is the value of p?

A) -12

B) -7

C) 12

D) 21

18

Which expression indicates both solutions of the equation $3x^2+4x-10=0$?

A) $\frac{-2\pm\sqrt{34}}{3}$

B) $\frac{-2\pm i\sqrt{104}}{3}$

C) $\frac{-2\pm 2\sqrt{34}}{3}$

D) $\frac{-4\pm i\sqrt{104}}{3}$

19

The mean value of all the cars on a used car lot is \$11,000 and the median value of these cars is \$7,000. Which of the following offers the best explanation for the discrepancy between the mean car value and the median car value?

A) Many of the cars are valued between \$7,000 and \$11,000.

B) There are a few cars that cost much less than the other cars do.

C) There are many cars priced at \$7,000.

D) There are a few cars that cost much more than the other cars do.

Questions 20 and 21 refer to the following information.

Week	Reported Cases
1	300
2	1,500
3	7,500
4	37,500

The table above gives the approximate number of reported cases of influenza in a country in the first four weeks of an outbreak.

20

Which of the following functions best models the number of reported cases, C, in the nth week of the outbreak?

A) $C=1{,}500(10^{n-1})$

B) $C=300(10^{n-1})$

C) $C=300(5^{n-1})$

D) $C=150(2^n)$

21

Which choice best characterizes the data shown in the table?

A) The number of reported cases increases exponentially with time, increasing by 400% each week.

B) The number of reported cases increases exponentially with time, increasing by 40% each week.

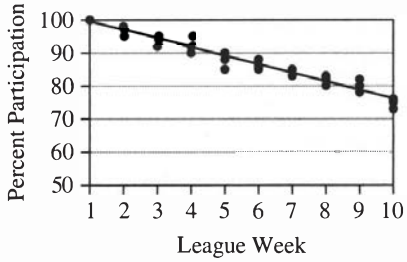
C) The number of reported cases increases linearly with time, increasing by approximately 12,000 cases per week.

D) The number of reported cases increases linearly with time, increasing by approximately 1,200 cases per week.

CONTINUE

4 4

22



The scatter plot above shows the percent participation for three kickball leagues for each week during a 10-week season. If there were a total of 108 players in all three leagues, then which choice best approximates the number of players who dropped out of the league each week?

A) 27

B) 9

C) 3

D) 2.5

23

If $m + n = 208$ and $\frac{m}{n} = \frac{4}{9}$, then what is the value of mn?

A) 36

B) 576

C) 3,328

D) 9,216

Questions 24 and 25 refer to the following information.

	Coral Heads/km^2	
Species	2005	2015
Acropora palmata	87	42
Acropora cervicornis	66	39
Monastrae annularis	26	24
Monastrae cavernosa	40	38

The table above shows the findings of marine biologists studying the decline of reef-building corals along the eastern Florida coast between 2005 and 2015.

24

Which coral species experienced the greatest percent decline between 2005 and 2015?

A) *Acropora palmata*

B) *Acropora cervicornis*

C) *Monastrae annularis*

D) *Monastrae cavernosa*

25

Marine biologists use species-specific models to predict future population decline. For *Acropora*, scientists assume a linear relationship between time and coral head decline. If p represents the average number of *Acropora palmata* coral heads per square kilometer that are lost each year and c represents the average number of *Acropora cervicornis* coral heads per kilometer that are lost each year, what is $\frac{p}{c}$?

A) $\frac{27}{3}$

B) $\frac{5}{3}$

C) $\frac{3}{5}$

D) $\frac{50}{9}$

CONTINUE

4 4

26

A bakery has a 52-pound supply of flour to make its swirled coffee cake fulfill holiday orders. The recipe calls for a 2:4:1 ratio, by weight, of sugar to flour to butter. How many pounds of sugar and butter combined does the bakery need to buy to complete the holiday orders for swirled coffee cake?

A) 91
B) 39
C) 26
D) 13

27

The dive tank managers at Seaside SCUBA Center try to keep their recreational SCUBA tanks filled with air at a pressure of approximately 3,000 pounds per square inch (psi). The maximum acceptable pressure for a tank is 3,300 psi, and the minimum acceptable pressure is 2,600 psi. Which inequality expresses the complete range of acceptable pressures, P, in psi, for the SCUBA tanks?

A) $|P-2{,}950| \leq 350$
B) $|P-2{,}950| \geq 350$
C) $|P-3{,}000| \leq 300$
D) $|P-3{,}000| \geq 400$

28

Derrick has completed a training run in which he ascended a mountain by the north trail and descended the same mountain by the east trail. He averaged a rate of 3 kilometers per hour during the ascent and 5 kilometers per hour during the descent. If the ascent took him $\frac{5}{8}$ of the total time of the run, then the north trail was what percent of the total distance of his run?

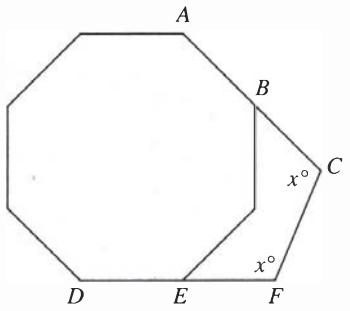
A) 37%
B) 48%
C) 50%
D) 63%

29

The sum of three integers, each less than 10, is 17. If the product of the two smallest integers is 50% less than the product of the two greatest integers, what is the value of the middle integer?

A) 4
B) 5
C) 6
D) 7

30



The figure above shows an octagon with eight congruent sides and eight congruent angles. Points A, B, and C are collinear and points D, E, and F are collinear. What is the value of x?

A) 112.5°
B) 132.5°
C) 135°
D) 150°

CONTINUE

4 4

31

Delaney attended a fundraiser that sold \$2 raffle tickets for regular prizes and \$5 raffle tickets for the grand prize. If Delaney spends a total of \$28 on both kinds of raffle tickets, how many \$2 raffle tickets could she have bought?

32

If 30% of a is equal to 10% of b, and $a + b = 80$, then what is the value of a ?

33

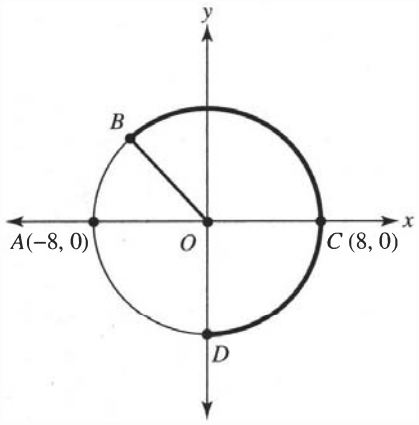
	Did not beat PR	Beat PR
Rest	76	114
Yoga	51	59

The table above shows the results of a study in which 300 sprinters competed in a 100-meter dash. The day before the race, each sprinter either took a day of rest or had a light yoga workout. It was then recorded whether or not each sprinter surpassed his or her personal record (PR) in the race. Of the sprinters who took a day of rest, what percentage did not beat their personal record? (Ignore the % symbol when gridding. For instance, enter 34.5% as 34.5.)

34

When a basketball player takes a free throw, the probability that she will make it is p, where $0 < p < 1$, and the probability that she will miss it is q, where $q = 1 - p$. When she takes two consecutive free throws, the probability that she will make both is p^2, and the probability that she will make one and miss one is $2pq$. Whenever Kia takes two free throws, she has a 0.64 probability of making both of them, what is the probability that she will miss both free throws?

35



The figure above shows a circle centered at the origin. If arc BCD has a length of $\frac{60\pi - 32}{5}$, what is the measure of angle AOB, in radians?

36

If $4(2^{y-2}) = 32^x$, what is the value of $\frac{y}{x}$?

CONTINUE

4 4

Questions 37 and 38 refer to the following information.

Dave, a head glassblower at a factory, has a steady weekly production load of 1,000 units. However, he has announced his retirement and will reduce his weekly production load by 4% each week for the next year. This phase-out process can be modeled by the equation $m=1{,}000k^n$, where m is the number of units that Dave will produce in the nth week after he first announced his retirement, and k is a constant.

37

According to the formula, what will Dave's weekly production load be, in units, in the 3rd week after he announces his retirement? (Round to the nearest integer.)

38

Throughout the year of Dave's phase-out, the three other glassblowers agree to absorb the balance of Dave's weekly production load. (The "balance" refers to the difference between Dave's weekly production and his weekly production before the phase-out began.) The most experienced glassblower will take 50% of this balance, and the least experienced glassblower will take 25% of this balance. In the 52nd and final week of Dave's phase-out, how many more units will the most experienced glassblower absorb from Dave's workload than the least experienced glassblower? (Round your answer to the nearest whole number.)

STOP

If you finish before time is called, you may check your work on this section only. Do not turn to any other section of the test.

SAT PRACTICE TEST 5 ANSWER KEY

Section 1: Reading

1. D
2. B
3. A
4. B
5. C
6. A
7. D
8. C
9. D
10. D
11. B
12. A
13. B
14. A
15. B
16. A
17. A
18. D
19. B
20. D
21. A
22. A
23. C
24. B
25. D
26. D
27. C
28. A
29. D
30. D
31. D
32. D
33. B
34. D
35. B
36. A
37. A
38. C
39. B
40. D
41. B
42. C
43. A
44. B
45. D
46. B
47. C
48. B
49. D
50. A
51. C
52. A

Total Reading Points (Section 1)

Section 2: Writing and Language

1. D
2. C
3. C
4. A
5. A
6. D
7. C
8. B
9. A
10. C
11. C
12. C
13. D
14. A
15. A
16. D
17. B
18. C
19. A
20. B
21. A
22. B
23. D
24. C
25. B
26. A
27. A
28. D
29. B
30. D
31. B
32. B
33. D
34. A
35. B
36. C
37. C
38. A
39. C
40. B
41. C
42. B
43. C
44. B

Total Writing and Language Points (Section 2)

Section 3: Math (No Calculator)

1. D
2. D
3. A
4. B
5. A
6. C
7. C
8. C
9. A
10. C
11. B
12. D
13. C
14. D
15. C
16. 8, 10, or 12
17. 16
18. 10
19. 3
20. 25/6 or 4.16 or 4.17

Section 4: Math (Calculator)

1. A
2. B
3. A
4. B
5. B
6. D
7. A
8. B
9. C
10. C
11. D
12. A
13. C
14. B
15. A
16. B
17. B
18. A
19. D
20. C
21. A
22. C
23. D
24. A
25. B
26. B
27. A
28. C
29. B
30. A
31. 4 or 9
32. 20
33. 40
34. .04 or 1/25
35. .8 or 4/5
36. 5
37. 885
38. 220

Total Math Points (Section 3 + Section 4)

illogical in choice C. Choice B is incorrect because the prepositional phrase *in reducing* is neither logical nor idiomatic.

34. **A** **Diction/Cohesiveness**

The passage as a whole explains the dramatic effects that Gutenberg's invention had on the entire continent of Europe. Therefore, it supports the contention that the choice was an *apt* (fitting) one.

35. **B** **Coordination/Pronoun Antecedents**

Choices A and D are incorrect because the pronoun *this* has no unambiguous antecedent: is it referring to the *printing press*, to *moveable type*, or to *the era of mass communication*? Choice C is incorrect because it forms an illogical prepositional phrase. The only choice that logically coordinates the clauses is choice B.

36. **C** **Coordination**

The second clause logically contrasts with the first, so the conjunction *but* is most logical.

37. **C** **Development/Cohesiveness**

This sentence should not be included, because it introduces an idea that is at odds with the discussion in the rest of the paragraph, which is about the effect that the printing press had on literacy and culture in 15th-century Europe.

38. **A** **Diction/Tone**

The original phrasing conveys a logical idea with a tone that is consistent with the rest of the passage. Choice B is incorrect because *busted* is inappropriate in both meaning and tone. Choice C is incorrect because *obliterated* is too violent a term and *what was in the way* is a vague reference. Choice D is incorrect because *provided a kick to* is too violent and informal in tone.

39. **C** **Pronoun Agreement/Logical Comparisons**

In the original phrasing, the pronoun *which* refers to the immediately preceding noun, *movable type*. Since it is illogical to say that *the Chinese were doing movable type*, or that *movable type was done by the Chinese*, choices A and B are incorrect. Choice C conveys a logical idea, that *movable type was first used by the Chinese*. Choice D is illogical because it implies a comparison rather than a contrast.

40. **B** **Diction**

The most effective choice for describing an unworkable task is choice B, *impractical*. Choice A is incorrect because this situation implies no drama or histrionics. Choice C is incorrect because *outrageous* means *worthy of outrage*, which is clearly not the case here. Choice D is incorrect because *extreme* is too vague for this context.

41. **C** **Coordination**

The second clause of this sentence indicates a logical consequence to the first clause, so the conjunction *so* is most logical.

42. **B** **Pronoun Agreement/Modifier Use**

The pronoun *they* in the original phrasing lacks a logical antecedent, as it does in choice C. Choice D is incorrect because the modifier *long* is misplaced, and because it uses the progressive aspect instead of the consequential aspect. Choice B is the only choice that avoids these problems.

43. **C** **Idiom/Clear Expression of Ideas**

The two clauses in this sentence are *He engineered* and *they would fit*. The original phrasing is incorrect because it inserts a superfluous verb. Choice B is incorrect because this clause is indicating a fact, not a hypothetical situation. Choice C uses the correct idiomatic form, *engineered to be the same size*, to convey intent. Choice D is incorrect because this clause is not making a comparison.

44. **B** **Diction/Clear Expression of Ideas**

Since the sentence is about the decline use of printers, the best choice is *falling into use*. Choice A, *fading*, implies that the printers are losing their color or intensity. Choice C, *withering*, implies that they are something like dying leaves. Choice D, *growing dull*, implies that they are tarnishing pieces of metal.

Section 3: Math (No Calculator)

1. **D** **Algebra (word problems) EASY**

Since there are 12 months in a year, there are 36 months in three years. If the store's bag usage is reduced to 0 by reducing the usage by 100 bags per month for 36 months, it must have begun with a usage rate of (36)(100) = 3,600 bags per month.

2. **D** **Advanced Mathematics (radicals) EASY**

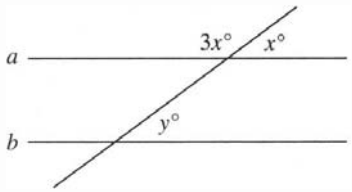
The square root of 81 is 9, and the square root of 36 is 6. Therefore the sum is 9 + 6 = 15.

3. **A** **Algebra (word problems) EASY**

The cost of renting a car with GPS navigation is $25 + $10 = $35 per day, and Carla rents the car for three days for a cost of (3)($35) = $105 without gas. If gas costs $0.10

per mile, then gas for x miles costs $\$0.10x$. Therefore, the total cost, in dollars, is $0.10x + 105$.

4. **B** **Additional Topics (parallel lines) EASY**



If lines a and b are parallel, then $x = y$ because corresponding angles must be congruent. The top two angles form a line, so $x + 3x = 4x = 180°$. Dividing both sides by 4 gives $x = 45°$, and therefore $x + y = 45 + 45 = 90$.

5. **A** **Algebra (linear equations) EASY**

Original equation to be solved for x: $\frac{1}{2}(2x-6) = -2(-2x-7.5)$

Distribute: $x - 3 = 4x + 15$
Subtract x from both sides: $-3 = 3x + 15$
Subtract 15 from both sides: $-18 = 3x$
Divide by 3 on both sides: $-6 = x$

6. **C** **Advanced Mathematics (exponents) MEDIUM**

Original expression: $\frac{x^{-\frac{1}{3}}y^3}{y^{-2}}$

Use the exponential rule $x^{-n} = \frac{1}{x^n}$: $\frac{y^3}{x^{\frac{1}{3}}y^{-2}}$

Use the exponential rule $\frac{x^a}{x^b} = x^{a-b}$: $\frac{y^5}{x^{\frac{1}{3}}}$

Use the exponential rule $x^{\frac{1}{n}} = \sqrt[n]{x}$: $\frac{y^5}{\sqrt[3]{x}}$

7. **C** **Advanced Mathematics (equivalent expressions) MEDIUM**

Original expression: $x(2x-4)(-x+3)$
Factor out a 2 from the middle term and –1 from the third term: $x(2)(x-2)(-1)(x-3)$
Use the Commutative Law of Multiplication to combine the numerical factors: $-2x(x-2)(x-3)$

8. **C** **Algebra (linear equations) MEDIUM**

It's important first to notice that the first equation is in "slope-intercept" form ($y = mx + b$), the second is not, because x is isolated rather than y. It's also important to remember that parallel lines have equal slopes and perpendicular lines have slopes that are the opposite reciprocals of each other.

First equation: $y = -\frac{1}{2}x - 4$
Second equation: $x = 2y + 2$
Subtract 2: $x - 2 = 2y$
Divide by 2 (and "swap" the sides of the equation): $y = \frac{1}{2}x - 1$

This shows that the first equation has a slope of $-1/2$ and the second has a slope of $1/2$. Since these slopes are neither equal nor opposite reciprocals, the two lines are neither parallel nor perpendicular. They are just intersecting lines.

9. **A** **Algebra (interpreting formulas) MEDIUM**

Consider what the equation $p = 7x - 125$ would give in the situation where no smoothies are sold, that is $x = 0$. In that case, $p = 7(0) - 125$, which means that the smoothie cart would lose \$125 on a day when no smoothies are sold. This indicates that 125 is the daily cost, in dollars, to run the smoothie cart.

10. **C** **Advanced Mathematics (dividing polynomials) MEDIUM-HARD**

There are three ways to answer this question. One way is simply to divide the polynomials just as you would divide two numbers by long division:

$$\begin{array}{r} x-1 \\ x-4\overline{)x^2-5x+6} \\ \underline{x^2-4x} \\ x+6 \\ \underline{x+4} \\ 2 \end{array}$$

This shows that the remainder is 2. If you're clever, you can accomplish the same task by using synthetic division (if you can remember it). The third way, for <u>really</u> clever folks, is to use the Remainder Theorem, which says that the remainder when the polynomial $P(x)$ is divided by $x - k$ is always equal to $P(k)$. In this case, that means that the remainder is $P(4) = (4)^2 - 5(4) + 6 = 16 - 20 + 6 = 2$.

11. **B** **Advanced Mathematics (functions and symmetry) MEDIUM-HARD**

If $g(-x) = -g(x)$ for all x in the domain of g, then g is an <u>odd</u> function, which means that its graph in the xy-plane is symmetric with respect to the origin (that is, the graph looks the same upside-down or right-side-up). If g is a polynomial, then it must only have odd-degree terms (this is why it is called <u>odd</u> symmetry). Only choices A and B are odd functions, and only choice B satisfies the second condition, because $g(-2) = (-2)^3 = -8$.

12. **D** **Advanced Mathematics (quadratics) MEDIUM-HARD**

One way to solve this equation is to use the quadratic equation, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, to find the two solutions, then add them together. For this quadratic, $a=3$, $b=-12$, and $c=7$, so: $r=\frac{12\pm\sqrt{(-12)^2-4(3)(7)}}{2(3)}$

Simplify: $r=\frac{12\pm\sqrt{60}}{6}$

Simplify: $r=2\pm\frac{\sqrt{15}}{3}$

Therefore the two solutions are $2+2\sqrt{10}$ and $2-2\sqrt{10}$, and their sum is 4.

But there is also a simpler way of answering this question, without having to find the individual solutions. You may recall the theorem that any quadratic in the form $x^2 - mx + n = 0$ (where m and n are constants) has two solutions whose sum is m and whose product is n. The quadratic $3r^2 - 12r + 7 = 0$ is not quite in this form, but we can easy get it in the right form by dividing both sides by 3, giving us $r^2-4r+\frac{7}{3}=0$, which tells us that the two solutions to this equation have a sum of 4 and a product of $\frac{7}{3}$.

13. **C** **Additional Topics (complex numbers) MEDIUM-HARD**

Original equation: $i^x i^y = 1$

Use the exponential identity $(x^m)(x^n) = x^{m+n}$: $i^{x+y} = 1$

Recall the "cycle" of the powers of i:

$i^0 = \mathbf{1}$, $i^1 = \boldsymbol{i}$, $i^2 = \mathbf{-1}$, $i^3 = \boldsymbol{-i}$, $i^4 = \mathbf{1}$, $i^5 = \boldsymbol{i}$ . . .

This shows that $i^n = 1$ if and only if n is a multiple of 4. Therefore, $x + y$ must be a multiple of 4. The only choice in which $x + y$ is a multiple of 4 is choice C.

14. **D** **Algebra (rates) HARD**

Ibrahim can collect and catalog flower specimens at a rate of 4 every hour, or 4 every 60 minutes, or 1/15 of a flower per minute. He can collect and catalog leaf specimens at a rate of 2 every 20 minutes, or 1/10 of a leaf per minute. Recall that, since *work* = *rate* × *time*, we can find the time with the formula *time* = *work* ÷ *rate*. Therefore, the time it takes Ibrahim to collect and catalog f flowers at a rate of 1/15 of a flower per minute is $f \div (1/15) = 15f$ minutes. The time it takes him to collect and catalog l leaves at 1/10 of a leaf per minute is $l \div (1/10) = 10l$ minutes. Therefore, the total time required is $15f + 10l$ minutes, which is equivalent to choice D.

15. **C** **Advanced Mathematics (solving quadratics) HARD**

Subsitute $y = -6x + 10$ into the second equation to solve for x: $-6x+10=x^2-7x+10$

Subtract 10: $-6x=x^2-7x$

Add $7x$: $x=x^2$

Subtract x: $0=x^2-x$

Factor: $0=(x)(x-1)$

By the Zero Product Property, the solutions are $x = 0$ and $x = 1$.

If $x = 0$: $y=-6(0)+10=10$

If $x = 1$: $y=-6(1)+10=4$

Therefore, $x + y$ can equal $0 + 10 = 10$ or $1 + 4 = 5$. Therefore, the correct answer is C.

16. **8, 10, or 12** **Algebra (systems of linear equations) EASY**

Let l represent the number of long tosses that Mike makes and s represent the number of short tosses he makes. The total points he scores can be represented by the equation: $15l+5s=100$

Divide by 5 on both sides: $3l+s=20$

Mike makes at least 4 long tosses, so let's begin with $l = 4$:

If $l = 4$, then $3(4) + s = 20$, so $s = 8$ and the total number of tosses is $4 + 8 = 12$.

If $l = 5$, then $3(5) + s = 20$, so $s = 5$ and the total number of tosses is $5 + 5 = 10$.

If $l = 6$, then $3(6) + s = 20$, so $s = 2$ and the total number of tosses is $6 + 2 = 8$.

17. **16** **Algebra (manipulating expressions) EASY**

The expression $a^2-2ab+b^2$ can be factored as $(a-b)^2$. Since $a = b + 4$, subtracting b from both sides gives us $a - b = 4$. Therefore $(a-b)^2 = 4^2 = 16$.

18. **10** **Advanced Mathematics (radicals) MEDIUM**

Plug $w=3\sqrt{5}$ into the equation $2w-\sqrt{20}=\sqrt{8y}$:

$2(3\sqrt{5})-\sqrt{20}=\sqrt{8y}$

Simplify: $6\sqrt{5}-\sqrt{20}=\sqrt{8y}$

Simplify $\sqrt{20}$ by factoring: $6\sqrt{5}-\sqrt{4}\sqrt{5}=\sqrt{8y}$

Simplify: $6\sqrt{5}-2\sqrt{5}=4\sqrt{5}=\sqrt{8y}$

Square both sides of the equation: $(4\sqrt{5})^2=8y$

Simplify: $80=8y$

Divide by 8: $10=y$

19. **3** **Algebra (systems of linear equations) MEDIUM**

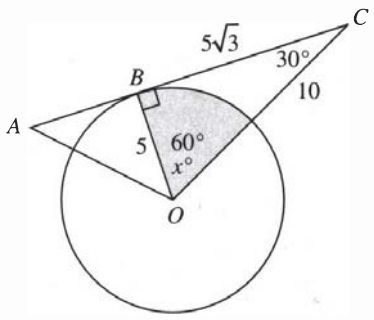
If a system has infinitely many solutions, then the two equations represent the same graph in the xy-plane.

First equation: $mx+ny=3$

Multiply both sides by 17 so that the equations "match" on the right side: $17mx+17ny=51$

This equation must be equivalent to $34x + 17y = 51$, so $17m = 34$ and $17n = 17$, and therefore $m = 2$ and $n = 1$. Therefore, $m + n = 3$.

20. **25/6 or 4.16 or 4.17** **Additional Topics (sectors, tangents, and triangles) HARD**



If AC is a tangent to the circle, then it is perpendicular to the radius to point B. Recall that the sine of an acute angle in a right triangle is the ratio of the opposite side to the hypotenuse. Therefore, if $\sin(x°) = \frac{\sqrt{3}}{2}$: $\frac{5\sqrt{3}}{OC} = \frac{\sqrt{3}}{2}$

Cross-multiply: $10\sqrt{3} = (OC)(\sqrt{3})$
Divide by $\sqrt{3}$: $10 = OC$
Now we can use the Pythagorean Theorem to find the length of OB: $(OB)^2 + (5\sqrt{3})^2 = 10^2$
Simplify: $(OB)^2 + 75 = 100$
Subtract 75 from both sides: $(OB)^2 = 25$
Take the square root of both sides: $OB = 5$
Therefore the circle has a radius of 5 and the circle has an area of $\pi(5)^2 = 25\pi$.

You should recognize this ratio of sides, $x : x\sqrt{3} : 2x$ as belonging to a 30°-60°-90° triangle, therefore $x = 60$ (you should also recall the basic trig fact that $\sin 60° = \frac{\sqrt{3}}{2}$), and therefore the sector is 60/360 = 1/6 of the entire circle. So the area of the sector is $25\pi/6$, and so $k = 25/6$.

Section 4: Math (Calculator)

1. **A** **Algebra (linear equations) EASY**

The y-intercept of a line is the value of y for which $x = 0$. Therefore, $3(0) - 2y = 12$, and so $-2y = 12$ and $y = -6$ is the y-intercept.

2. **B** **Algebra (inequalities) EASY**

If $4n \geq 9$, then the LEAST possible value for $4n$ is 9. Therefore, the least possible value of $4n + 1$ is 10.

3. **A** **Algebra (systems of linear equations) EASY**

Original system: $\begin{cases} 4c + 7d = 29 \\ 2c + 3d = 13 \end{cases}$
Multiply the second equation by 2: $\begin{cases} 4c + 7d = 29 \\ 4c + 6d = 26 \end{cases}$
Subtract the corresponding sides of the equations to eliminate c: $d = 3$
Substitute $d = 3$ into first equation: $4c + 7(3) = 29$
Simplify: $4c = 8$
Divide by 4 on both sides: $c = 2$

4. **B** **Problem Solving and Data Analysis (proportions) EASY**

Priscilla buys a 2 bags of soil for every 6 bags of fertilizer. If x represents the number of bags of fertilizer Priscilla bought, then we can set up a proportion: $\frac{2}{6} = \frac{x}{132}$
Cross-multiply: $6x = 264$
Divide by 6: $x = 44$

5. **B** **Algebra (word problems) EASY**

Each addition costs \$0.30, so n additions cost $\$0.30n$. A \$2.50 coffee with n additions therefore costs $\$(0.3n + 2.5)$.

6. **D** **Algebra (compositions of functions) EASY**

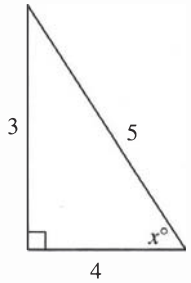
Substitute $g(x) = 2x - 5$ into $f(g(x))$: $f(g(x)) = f(2x - 5) = -(2x - 5) + 3 = -2x + 5 + 3 = -2x + 8$

7. **A** **Advanced Mathematics (subtracting polynomials) EASY**

Subtracting a number is equivalent to adding its opposite, so to simplify this expression we should distribute then combine like terms: $(3x^2 - 4x + 2) - (4x^2 - 3x - 1) = 3x^2 - 4x + 2 - 4x^2 + 3x + 1 = -x^2 - x + 3$

8. **B** **Additional Topics (trigonometry) EASY**

If $\cos x° = 4/5$ and $0 < x < 90$, then x can be considered as an acute angle in a triangle like the one below, in which the length of the adjacent side is 4 and the length of the hypotenuse is 5. You should recognize this as a 3-4-5 right triangle (or use the Pythagorean Theorem to find the missing side), and see that the tangent of x is therefore 3/4.



9. **C** **Problem Solving and Data Analysis (statistical interpretation) MEDIUM**

An 85th percentile score is a score that "beats or equals" at least 85% of the other scores. In this context, that means that Jenna's best pole vault last year was at least as high as the best vault of 85% of the pole vaulters in the country last year.

10. **C** **Problem Solving and Data Analysis (percents and inequalities) MEDIUM**

Frannie's buying constraints can be represented by the equation $60 \geq 1.05(0.8x)$, in which x is the original cost, in dollars, of the running shoes. In this inequality, $0.8x$ represents the cost after the 20% discount has been applied (the cost is 80% of the original price) and 1.05 is the factor that adds the 5% sales tax. $60 \geq 1.05(0.8x)$

Simplify: $60 \geq 0.84x$

Divide by 0.84 on both sides: $71.428 \geq x$

Therefore the maximum original cost for her shoes is \$71.42.

11. **D** **Advanced Mathematics (quadratic equations) MEDIUM**

Substitute $a = \frac{1}{b}$ into $a + b = 0$: $\frac{1}{b} + b = 0$

Multiply both sides by b: $1 + b^2 = 0$

Subtract 1 from both sides: $b^2 = -1$

Since the square root of –1 is not a real number (it is the imaginary number i), then the solutions to this system are not real: $a = i$ and $b = -i$.

12. **A** **Advanced Mathematics (exponential growth) MEDIUM**

If the population grows by 20% each week, then the population is multiplied by 1.20 each week. If this happens over the course of w consecutive weeks, the population is $p = p_0(1.2)^w$.

13. **C** **Algebra (linear equations) MEDIUM**

Original equation: $x - 3y - 12 = 0$

Add 12 to both sides: $x - 3y = 12$

This puts the equation into "standard form" ($ax + by = c$). The graph of a linear equation in this form has a slope of $-a/b$ and an x-intercept of c/a. (You should verify this yourself.) This means that this line has a slope of 1/3 and an x-intercept of 12/1 = 12. The only equation that includes these two values as constants or coefficients is C.

14. **B** **Advanced Mathematics (solving quadratics) MEDIUM**

The x-intercepts of the graph correspond to the zeros of the function. These can be found by setting each factor of the polynomial equal to 0. When we do this, choices A and B both yield x-intercepts of –2 and 3. However, only choice B yields a graph with a y-intercept of 12, because $f(0) = (-2(0) - 4)(0 - 3) = (-4)(-3) = 12$.

15. **A** **Advanced Mathematics (analyzing polynomial graphs) MEDIUM**

This function is undefined at those values of x that yield 0 in the denominator, since division by 0 is undefined. We can find these values by factoring the denominator and using the Zero Product Property:

$(x^2 - 5x - 6) = (x - 6)(x + 1)$; therefore, the denominator is 0 when $x = 6$ and $x = -1$.

16. **B** **Problem Solving and Data Analysis (central tendency) MEDIUM**

The median is the middle value of the data when it is arranged in ascending or descending order. For 360 data points, the median is the average of the 180th and 181st data point. For choices A, C, and D, 41 inches would represent the median as both the 180th and 181st terms are represented by that value. In choice B, the correct answer, the median is 42 inches.

17. **B** **Algebra (linear equations) MEDIUM-HARD**

To find p, we can use the equation, slope $= \frac{y_2 - y_1}{x_2 - x_1}$:

$$\frac{4 - (-p)}{-2p - 26} = \frac{1}{4}$$

Simplify: $\frac{4 + p}{-2p - 26} = \frac{1}{4}$

Cross-multiply: $16 + 4p = -2p - 26$

Add $2p$ to both sides: $16 + 6p = -26$

Subtract 16 from both sides: $6p = -42$

Divide by 6 on both sides: $p = -7$

18. **A** **Advanced Mathematics (solving quadratics) MEDIUM-HARD**

The solutions to the polynomial $3x^2 + 4x - 10 = 0$ can be found by using the quadratic equation, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, for which $a = 3$, $b = 4$, and $c = -10$:

$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-10)}}{2(3)}$$

Simplify: $x = \frac{-4 \pm \sqrt{136}}{6}$

Simplify $\sqrt{136}$: $x = \frac{-4 \pm 2\sqrt{34}}{6}$

Divide numerator and denominator by 2: $x = \frac{-2 \pm \sqrt{34}}{3}$

19. **D** **Problem Solving/Data Analysis (central tendency) MEDIUM-HARD**

When the mean of a set of numbers is significantly greater than the median, it is generally because of "upper outliers," that is, extreme values in the set that pull the average up without affecting the median. If a set

of numbers is "symmetrical about the median," that is, they are symmetrically spaced on either side of the middle value, then the median and the average are equal, as in {1, 2, 5, 8, 9} where both the median and the average are 5. However, if we change 9 to 90, making it an "upper outlier," we get {1, 2, 5, 8, 90}, in which the median is still 5, but the average is now 21.2.

20. **C** **Algebra (expressing functional relationships) MEDIUM-HARD**

We can plug in values from the table and work by process of elimination. We can begin by using the fact that $C = 1{,}500$ when $n = 2$.

A) $C(2) = 1{,}500(10^{2-1}) = 15{,}000$
B) $C(2) = 300(10^{2-1}) = 3{,}000$
C) $C(2) = 300(5^{2-1}) = 1{,}500$
D) $C(2) = 150(2^2) = 6{,}000$

Choice C is the only one that yields the correct value.

21. **A** **Problem Solving and Data Analysis (tables) MEDIUM-HARD**

Each week, the number of reported cases is 5 times the number of the previous week. This means that the number is increasing by 400% each week, because $(5x - x)/x = 4 = 400\%$.

22. **C** **Problem Solving and Data Analysis (graphical analysis) MEDIUM-HARD**

During week 1, all three leagues have 100% participation. According to the line of best fit, all three leagues have about 75% participation by week 10, so there was a drop of 25% participation over a 9-week span. Since (25%)(108) = 27, this means that 27 players dropped out over the course of 9 weeks, or at the rate of 27/9 = 3 players every week.

23. **D** **Advanced Mathematics (systems) MEDIUM**

Second given equation: $\frac{m}{n} = \frac{4}{9}$

Cross-multiply: $9m = 4n$

Divide by 4: $\frac{9}{4}m = n$

First given equation: $m + n = 208$

Substitute $n = \frac{9}{4}m$: $m + \frac{9}{4}m = 208$

Simplify: $\frac{13}{4}m = 208$

Multiply both sides by $\frac{4}{13}$: $m = 64$

Substitute into $n = \frac{9}{4}m$: $n = \left(\frac{9}{4}\right)(64) = 144$

Therefore, $mn = (64)(144) = 9{,}216$

24. **A** **Problem Solving/Data Analysis (percent change) MEDIUM-HARD**

The "percent change" formula is percent change = $\frac{\text{final quantity} - \text{original quantity}}{\text{original quantity}} \times 100\%$. For *Acropora palmata*, then, the percent change is $\frac{87-42}{87} \times 100 \approx 52\%$, which represents the greatest percent decline of any of the coral species in the table over the 10-year span.

25. **B** **Algebra (linear equations) MEDIUM-HARD**

For *Acropora palmata*, the number lost per year is $p = \frac{87-42}{2005-2015} = -\frac{45}{10}$. For *Acropora cervicornis*, the number lost per year is $c = \frac{66-39}{2005-2015} = -\frac{27}{10}$. Therefore, $\frac{p}{c} = \frac{-\frac{45}{10}}{-\frac{27}{10}} = -\frac{45}{10} \times -\frac{10}{27} = \frac{45}{27} = \frac{5}{3}$.

26. **B** **Problem Solving and Data Analysis (ratios) HARD**

The bakery's recipe calls for a 2:4:1 ratio of sugar to flour to butter. This means that the ratio, by weight, of flour to the other ingredients is 4:(2+1) = 4:3. If the weight of the flour is 52 pounds and x represents the combined weight, in pounds, of sugar and butter, then: $\frac{4}{3} = \frac{52}{x}$

Cross-multiply: $4x = 156$

Divide by 4: $x = 39$

27. **A** **Algebra (analyzing formulas) HARD**

The average of the upper limit and the lower limit of pressures is $\frac{3{,}300 + 2{,}600}{2} = 2{,}950$. This means that the acceptable values are centered on 2,950. Notice that the upper limit and the lower limit are both exactly 350 units away from this average so we can express the range of values as "all values that are within 350 units of 2,950," which can be expressed as P, where $|P - 2{,}950| \leq 350$.

28. **C** **Problem Solving/Data Analysis (rates) MEDIUM-HARD**

Recall the formula *distance* = *rate* × *time*. Let's define t as the total time of Derrick's run. Derrick ascended the mountain at a rate of 3 km/hour, which took him $\frac{5}{8}$ of the time, so his distance on the north trail was $3\left(\frac{5}{8}t\right) = \frac{15}{8}t$. He descended the mountain at a rate of 5 km/hour, which took him $\frac{3}{8}$ of the time, so his distance on the east trail was $5\left(\frac{3}{8}t\right) = \frac{15}{8}t$. Since these two distances are

obviously equal, the north trail must have been 50% of the total distance.

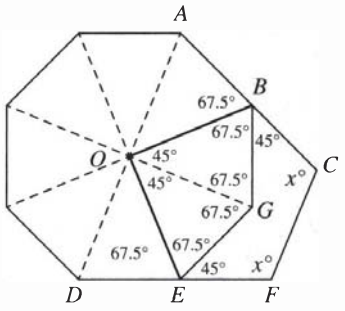
29. **B** **Problem Solving/Data Analysis (numerical reasoning) HARD**

Let's call the three integers, in ascending order, a, b, and c. Their sum is 17, so $a + b + c = 17$. If the product of the two smallest integers is 50% less than the product of the two greatest integers, then: $ab = 0.5bc$
Divide by b: $a = 0.5c$
Multiply by 2: $2a = c$

Since all of the numbers are integers, this indicates that c, the largest number, must be even. Since each integer is less than 10, the largest c could be is 8. Let's see if that works. If $c = 8$, then a must equal half of that, or 4. Since the sum of all three integers must be 17, b must equal $17 - 8 - 4 = 5$. Therefore, $a = 4$, $b = 5$, and $c = 8$.

30. **A** **Additional Topics (geometry/polygons) HARD**

As with many difficult geometry questions, it helps to draw some extra lines on the diagram. Label the center of the octagon O and consider the pentagon $OBCFE$, as show below.



Each of the eight central angles at O is congruent, so each has the measure $360° \div 8 = 45°$. Likewise, each external angle to the octagon, like angle GBC, has the measure $360° \div 8 = 45°$. Each of the triangles inside the octagon is isosceles, so their base angles are congruent, so each must measure $(180° - 45°)/2 = 67.5°$. Recall that the sum of the interior angles to any n-sided polygon is $180°(n - 2)$, so the sum of the interior angles to pentagon $OBCFE$ must be $180°(5 - 2) = 540°$: $90 + 112.5 + x + x + 112.5 = 540$
Simplify: $315 + 2x = 540$
Subtract 315: $2x = 225$
Divide by 2: $x = 112.5$

31. **4 or 9** **Problem Solving/Data Analysis (proportions) EASY**

If Delaney bought one \$5 grand prize raffle ticket, she would have \$23 left to buy regular prize raffle tickets. However, since \$23 is not evenly divisible by \$2, she would have to buy either 2 or 4 grand prize raffle tickets. If she bought 2 grand prize raffle tickets for \$10, she could buy 9 regular prize raffle tickets for \$18. If she bought 4 grand prize raffle tickets for \$20, she could buy 4 regular prize raffle tickets for \$8.

32. **20** **Algebra (systems) MEDIUM**

If 30% of a is equal to 10% of b: $0.3a = 0.1b$
Multiply both sides by 10: $3a = b$
Substitute $b = 3a$ into $a + b = 80$: $a + 3a = 80$
Simplify: $4a = 80$
Divide by 4: $a = 20$

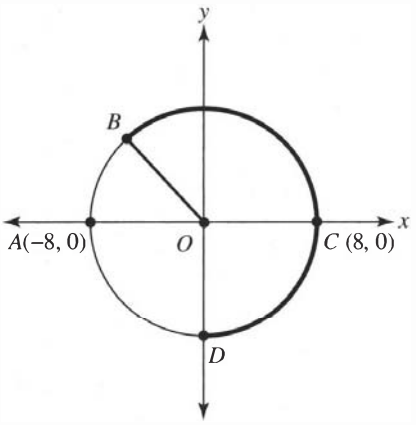
33. **40** **Problem Solving and Data Analysis (percents) MEDIUM**

A total of 76 + 114 = 190 sprinters had a day of rest the day before the race. Of those 190 people, 76 did not beat their PR. $\frac{76}{190} \times 100\% = 40\%$.

34. **.04 or 1/25** **Problem Solving and Data Analysis (probability) MEDIUM**

The probability that she makes both is $p^2 = 0.64$, and therefore $p = 0.8$. This means that the probability that she will miss a free throw is $1 - 0.8 = 0.2$. The probability that she will miss two consecutively, then, is $(0.2)(0.2) = 0.04$.

35. **.8 or 4/5** **Advanced Mathematics (radians) MEDIUM-HARD**



The circle has a radius of 8, and so its circumference is $2\pi(8) = 16\pi$. Arc $ABCD$ is three-fourths of the circumference, so it has a length of $(0.75)(16\pi) = 12\pi$. Since arc BCD has a length of $\frac{60\pi - 32}{5}$, arc AB has a length of $12\pi - \frac{60\pi - 32}{5} = \frac{60\pi}{5} - \frac{60\pi - 32}{5} = \frac{32}{5} = 6.4$. The radian measure of angle AOB is equal to the ratio of the length of its intercepted arc to the length of the radius: $6.4 \div 8 = 0.8 = 4/5$.

36. **5** **Advanced Mathematics (exponentials) MEDIUM**

Original equation:	$4(2^{y-2}) = 32^x$
Substitute $4 = 2^2$ and $32 = 2^5$:	$2^2(2^{y-2}) = (2^5)^x$
Simplify both sides:	$2^y = 2^{5x}$
If $\|x\| > 1$ and $x^a = x^b$, then $a = b$:	$y = 5x$
Divide both sides by x:	$\frac{y}{x} = 5$

37. **885** **Problem Solving (interpreting formulas) HARD**

In the formula $m = 1{,}000k^n$, the constant k represents the factor that multiplies Dave's production load each week. Since this workload is decreasing by 4% each week, he is doing 96%, or 0.96, of the previous week's work. Therefore, $k = 0.96$, and so the complete equation relating m and n is $m = 1{,}000(0.96)^n$. To find his production load in the third week, then we simply let $n = 3$: $1{,}000(0.96)^3 = 884.74$, which rounds to 885.

38. **220** **Problem Solving (percents/exponential expressions) HARD**

In the 52nd week of the phase out, Dave's production load is $1{,}000(0.96)^{52} = 119.70$ units, which rounds to 120 units. This is 1,000 − 120 = 880 fewer units than he had been producing before the phase-out. This means that the most experience glassblower must absorb (0.50)(880) = 440 units and the least experience glassblower must absorb (0.25)(880) = 220 units, so the difference is 440 − 220 = 220 units.

3 3

Math Test—No Calculator

25 MINUTES, 20 QUESTIONS

Turn to Section 3 of your answer sheet to answer the questions in this section.

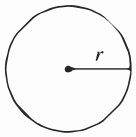
DIRECTIONS

For questions 1–15, solve each problem, choose the best answer from the choices provided, and fill in the corresponding circle on your answer sheet. **For questions 16–20**, solve the problem and enter your answer in the grid on the answer sheet. Please refer to the directions before question 16 on how to enter your answers in the grid. You may use any available space in your test booklet for scratch work.

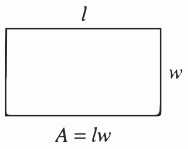
NOTES

1. The use of a calculator **is not permitted**.
2. All variables and expressions used represent real numbers unless otherwise indicated.
3. Figures provided in this test are drawn to scale unless otherwise indicated.
4. All figures lie in a plane unless otherwise indicated.
5. Unless otherwise indicated, the domain of a given function f is the set of all real numbers x for which $f(x)$ is a real number.

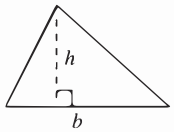
REFERENCE



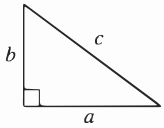
$A = \pi r^2$
$C = 2\pi r$



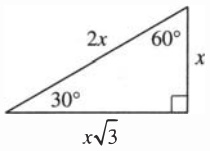
$A = lw$

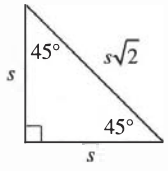


$A = \frac{1}{2}bh$

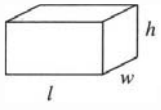


$c^2 = a^2 + b^2$

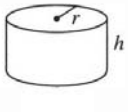




Special Right Triangles

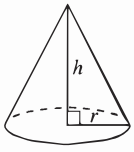


$V = lwh$

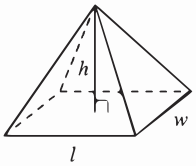


$V = \pi r^2 h$

$V = \frac{4}{3}\pi r^3$



$V = \frac{1}{3}\pi r^2 h$

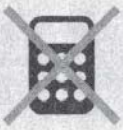


$V = \frac{1}{3}lwh$

The number of degrees of arc in a circle is 360.
The number of radians of arc in a circle is 2π.
The sum of the measures in degrees of the angles of a triangle is 180.

CONTINUE

3



3

1

If $3b = 2$ what is the value of $\frac{b}{2}$?

A) $\frac{1}{6}$

B) $\frac{1}{3}$

C) $\frac{2}{3}$

D) $\frac{3}{4}$

2

Which of the following is equivalent to $\frac{x^3}{x} + \frac{x^5}{x^3}$ for all positive values of x?

A) $2x^2$

B) $2x^4$

C) $2x^6$

D) $2x^8$

3

If $m < -3$ and $-3m \leq 2n$ what is the least possible integer value of n?

A) -5

B) -4

C) 5

D) 6

4

Janie runs a dog-sitting service in which she charges \$12 per hour for the first dog and \$5 per hour for each additional dog. She also charges an additional \$10 fee to take up to three dogs to the dog park. Which choice represents the total charge, in dollars, for Janie to sit 3 dogs for n hours, including one trip to the dog park?

A) $17n + 10$

B) $22n + 10$

C) $22n + 30$

D) $36n + 20$

5

An oceanographer is studying the growth of two artificial reefs. Osprey Reef has an area of 400 square meters and is growing at a rate of 2.5 square meters per month. Pelican Reef has an area of 360 square meters and is growing at a rate of 3 square meters per month. At these rates, in how many months will the two reefs have the same area?

A) 40

B) 42

C) 64

D) 80

6

If $x < -10$, which of the following has the least value?

A) $\frac{1}{x^2}$

B) $\frac{1}{x^3}$

C) $\frac{1}{x^4}$

D) $\frac{1}{x^5}$

CONTINUE

3 3

7

Which choice is equivalent to $\sqrt{-27}+\sqrt{-48}$?

A) $-5\sqrt{3}$

B) $\sqrt{-75}$

C) $5i\sqrt{3}$

D) $7i\sqrt{3}$

8

$$n=416-4T$$

The equation above models the relationship between n, the number of cars per week at a gas station requesting full service, and T, the average weekly temperature in degrees Fahrenheit, where $30 \le T \le 80$. Which of the following is the best interpretation of the number 4 in this equation?

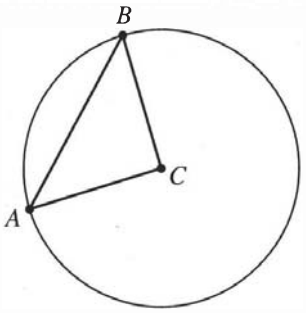
A) For every 4-degree increase in average weekly temperature, 1 fewer car requests full service per week.

B) For every 4-degree increase in average weekly temperature, 1 fewer car requests full service per day.

C) For every 1-degree increase in average weekly temperature, 4 fewer cars request full service per week.

D) For every 1-degree decrease in average weekly temperature, 4 fewer cars request full service per week.

9



In the figure above, C is the center of the circle and segment AC is perpendicular to segment BC. If the area of triangle ABC is 8 square units, what is the length of arc AB?

A) 2π

B) 4π

C) 8π

D) 16π

10

Which of the following expressions is equivalent to

$$\frac{a}{b}(a-b)(a+b)?$$

A) $\frac{a^3}{b}-ab$

B) $\frac{a^2}{b}-b$

C) $\frac{a^3}{b}+\frac{2a^2}{b}+ab$

D) $\frac{a^2}{b}-ab$

CONTINUE

3

3

11

$$f(x) = -\frac{1}{2}x + 1$$

$$g(x) = -4x - 1$$

When the line $y = g(f(x))$ is graphed in the xy-plane, what is its y-intercept?

A) -5

B) -3

C) 2

D) $\frac{3}{2}$

12

If $p(x) = 6x^2 + 2x$, which of the following is a factor of $p(x) - 4$?

A) $x + 4$

B) $x + 3$

C) $x + 2$

D) $x + 1$

13

If the equation $6x - xy = x^2 + 9$ is graphed in the xy-plane, at which point does the graph touch the x-axis?

A) $(-6, 0)$

B) $(-3, 0)$

C) $(3, 0)$

D) $(6, 0)$

14

For which of the following equations is $\frac{-3-\sqrt{3^2+4(2)(5)}}{4}$ a solution?

A) $2x^2 + 3x = -5$

B) $2x^2 + 3x = 5$

C) $2x^2 - 3x = -5$

D) $2x^2 - 3x = 5$

15

$$\frac{x^3}{x-2} = x^2 + 2x + 4 + \frac{b}{x-2}$$

If the equation above is true for all values of x where $x \neq 2$, what is the value of b?

A) -8

B) -4

C) 4

D) 8

CONTINUE

3 3

16

In the xy-plane, the graph of $y=\frac{1}{3}x-1$ passes through the point $(a, 1)$. What is the value of a?

17

$$2x^2 - bx + b = 0$$

If $x = 3$ is a solution to the equation, above, what is the value of b?

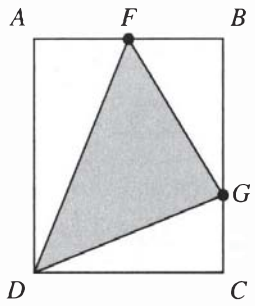
18

If $2 \le 3|3-x| \le 3$, what is one possible value of x?

19

If $\frac{2+i}{2-i}=a+bi$, where $i = \sqrt{-1}$, what is the value of $a+b$?

20



In the figure above, point F is the midpoint of side $\overline{AB}$ and G is a point on $\overline{BC}$ such that $BG = 2GC$. What fraction of the area of rectangle $ABCD$ is shaded?

STOP

If you finish before time is called, you may check your work on this section only. Do not turn to any other section of the test.

4 4

Math Test—Calculator

55 MINUTES, 38 QUESTIONS

Turn to Section 4 of your answer sheet to answer the questions in this section.

DIRECTIONS

For questions 1–30, solve each problem, choose the best answer from the choices provided, and fill in the corresponding circle on your answer sheet. **For questions 31–38**, solve the problem and enter your answer in the grid on the answer sheet. Please refer to the directions before question 31 on how to enter your answers in the grid. You may use any available space in your test booklet for scratch work.

NOTES

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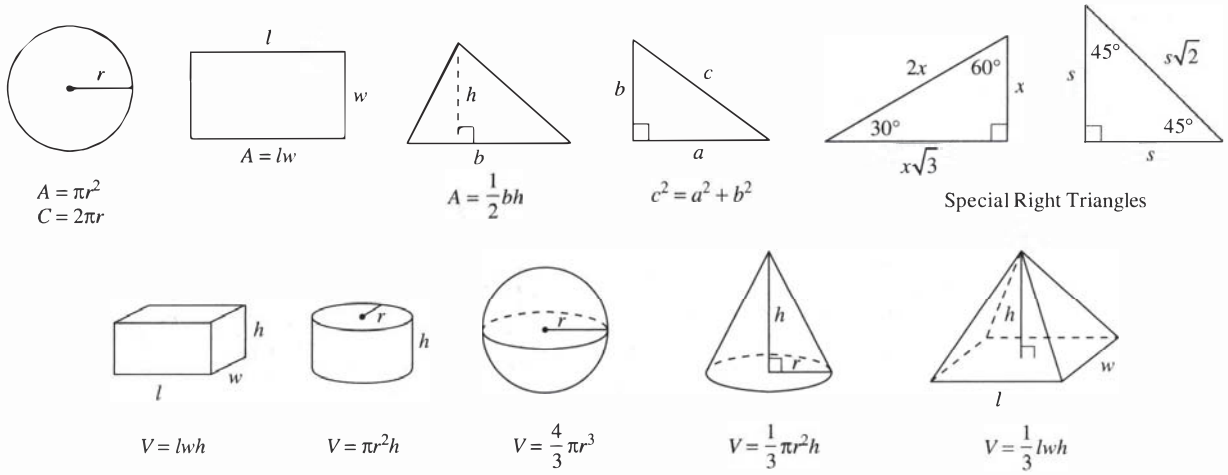
REFERENCE



$A = \pi r^2$
$C = 2\pi r$

$A = \frac{1}{2}bh$

$c^2 = a^2 + b^2$

Special Right Triangles

$V = lwh$ $V = \pi r^2 h$ $V = \frac{4}{3}\pi r^3$ $V = \frac{1}{3}\pi r^2 h$ $V = \frac{1}{3}lwh$

The number of degrees of arc in a circle is 360.
The number of radians of arc in a circle is 2π.
The sum of the measures in degrees of the angles of a triangle is 180.

CONTINUE

4 4

1

Emma can fill a 12-gallon tank of gasoline for a cost of $24.36. At this rate, how much should it cost her to fill a 16-gallon tank of gasoline?

A) $32.48

B) $34.88

C) $35.12

D) $36.54

2

If the average of a and b is 6 and the average of a, b, and c is 8, what is the value of c?

A) 8

B) 10

C) 12

D) 16

3

$$x = 3a + b$$

$$y = a - b$$

Given the two equations above, which choice is equivalent to xy?

A) $3a^2 - b^2$

B) $3a^2 - 4ab - b^2$

C) $3a^2 - 2ab - b^2$

D) $3a^2 - 2ab + b^2$

4

Which of the following systems of equations has no solution?

A) $\begin{cases} 2x+y=0 \\ 2x+y=1 \end{cases}$

B) $\begin{cases} 2x-y=0 \\ 2x+y=0 \end{cases}$

C) $\begin{cases} 2x+y=0 \\ -2x+y=0 \end{cases}$

D) $\begin{cases} 2x+y=1 \\ -2x+y=0 \end{cases}$

CONTINUE

4  4

5

Grade	Attack	Midfield	Defense	Goal	Total
12th	6	5	3	2	16
11th	4	6	4	1	15
10th	3	3	6	2	14
Total	13	14	13	5	45

The table above shows the distribution by grade and position for the 45 players on a boys' high school lacrosse team. If an attacker or midfielder is chosen at random from this team, what is the probability that he is in the 12th grade?

A) $\frac{10}{27}$

B) $\frac{11}{27}$

C) $\frac{16}{27}$

D) $\frac{11}{16}$

6

Arkady is trying to save \$5,000 to make a down payment on a new car. If he has \$300 already saved and plans to save \$40 every week, which inequality can he use to predict w, the number of weeks it will take him until he has enough for the down payment?

A) $5{,}300 \leq 40w + 300$

B) $4{,}700 \leq 40w$

C) $5{,}000 \leq 40w - 300$

D) $4{,}700 \leq 40w + 300$

Questions 7 and 8 refer to the following information.

A rubber ball is dropped vertically from a 10-meter tower to a hard surface below. Each time it strikes the ground, it bounces back to a height 80% of its previous height.

7

What maximum height does the ball reach immediately after its third bounce?

A) 6.4 meters

B) 5.12 meters

C) 4.096 meters

D) 3.2768 meters

8

What total vertical distance, both up and down, has the ball traveled when it strikes the ground for the fourth time?

A) 38.80 meters

B) 39.04 meters

C) 45.64 meters

D) 49.04 meters

CONTINUE

4 4

9

If $u = x + 2$ and $u \geq 0$, which of the following is equivalent to $(x-1)^2\sqrt{x+2}$?

A) $(u-3)^2\sqrt{u}$

B) $(u-2)^2\sqrt{u}$

C) $(u-1)^2\sqrt{u}$

D) $(u+1)^2\sqrt{u}$

10

If $g(x) = 5x - 2$, and $g(a) = 8$, what is the value of $g(a+2)$?

A) 2

B) 12

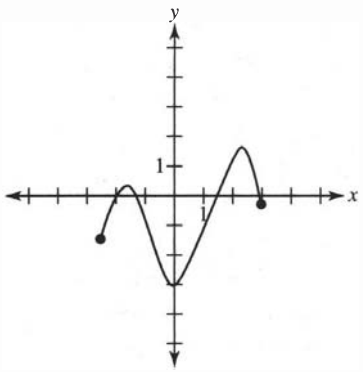
C) 18

D) 23

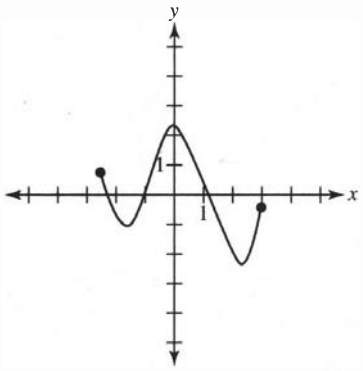
11

If the equation $f(x) = 2$ has four distinct real solutions, which of the following could be the complete graph of $y = f(x)$?

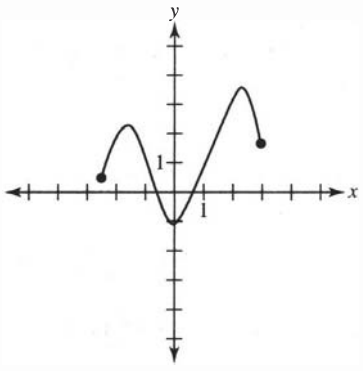
A)



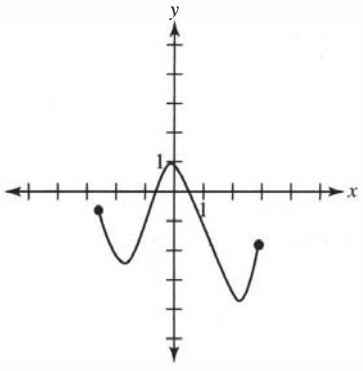
B)



C)



D)



CONTINUE

4 4

Questions 12 and 13 refer to the following information.

In an economic market, the supply and demand of any product depends on the price of that product. The supply, $S(p)$, of a product is the number of units of that product expected to be supplied at a price of p dollars per unit. The demand, $D(p)$, of a product is the number of units of that product expected to be sold at a price of p dollars per unit. The "equilibrium price" of one unit of the product is the value of p at which the supply and the demand are equal. The supply and demand functions for a particular product are $S(p) = 25p + 100$ and $D(p) = -35p + 1{,}300$.

12

For this particular product, which of the following is the best interpretation of the number 25 in this system?

A) For every 1 dollar increase in price, 25 more units are expected to be supplied.

B) For every 25 dollars increase in price, one more unit is expected to be supplied.

C) At least 25 units are expected to be supplied, even at a very low price.

D) For every 1 dollar increase in price, 25 more units are expected to be demanded.

13

What is the equilibrium price of one unit of this product?

A) $20.00

B) $23.33

C) $60.00

D) $120.00

14

$$y = 2x^2 + 2x + 1$$

$$y = b$$

When the equations above are graphed in the xy-plane, they form a parabola and a line. If the line intersects the parabola in exactly one point, what is the value of b?

A) $-\frac{1}{2}$

B) $-\frac{1}{4}$

C) $\frac{1}{2}$

D) 1

15

Joaquin is part of a crossword club. He earns 10 points for every easy puzzle he solves and 25 points for every hard puzzle he solves. If Joaquin solved 30 puzzles and earned a total of 375 points, how many more easy puzzles did he solve than hard puzzles?

A) 5

B) 10

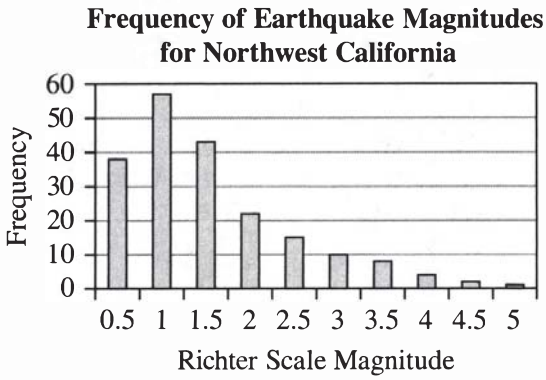
C) 20

D) 25

CONTINUE

4 4

16



The graph above shows the distribution of Richter scale magnitudes for 200 recent earthquakes in northwest California. What is the median magnitude of these 200 earthquakes?

A) 1.0

B) 1.5

C) 2.0

D) 2.5

17

What is the sum of the solutions to the equation

$$2(x^2 + 4x) = 20\ ?$$

A) −7

B) −4

C) 3

D) 4

18

A researcher is conducting a study to determine whether taking a zinc supplement can reduce the severity of cold symptoms. The treatment group for this study is 200 adult patients who will take a zinc supplement within 6 hours of first detecting cold symptoms. The researcher will then measure the length and severity of the cold symptoms for this treatment group. Which of the following comparison groups would best enable the researcher to reach a reliable conclusion about the ability of a zinc supplement to reduce the severity of cold symptoms?

A) A group of 200 adults without cold symptoms who also take the zinc supplement

B) A group of 200 children with cold symptoms who also take the zinc supplement

C) A group of 200 adults with cold symptoms who take sugar pills rather than the zinc supplements

D) A group of 100 children and 100 adults without cold symptoms who take sugar pills rather than the zinc supplements

19

Philip buys an annual membership to an online store for $80 in order to receive free shipping and handling on all of his orders for the year. Nonmembers pay a 2% shipping and handling surcharge on the cost of all orders after a 3.6% sales tax has been added. What is the approximate cost of merchandise Philip must purchase annually, before sales tax, in order for his savings on shipping and handling to cover the cost of his membership?

A) $2,940

B) $3,246

C) $3,861

D) $4,260

CONTINUE

4 4

20

Which of the following expressions is equivalent to $\frac{x-3}{x^2-1}$?

A) $\frac{2}{x+1}-\frac{1}{x-1}$

B) $\frac{1}{x+1}-\frac{2}{x-1}$

C) $\frac{2}{x+1}+\frac{1}{x-1}$

D) $\frac{1}{x+1}+\frac{2}{x-1}$

21

If $a=2^{3k}$ and $b=4^{-3k}$, which of the following is equivalent to $\frac{b}{a}$?

A) 2^{-9k}

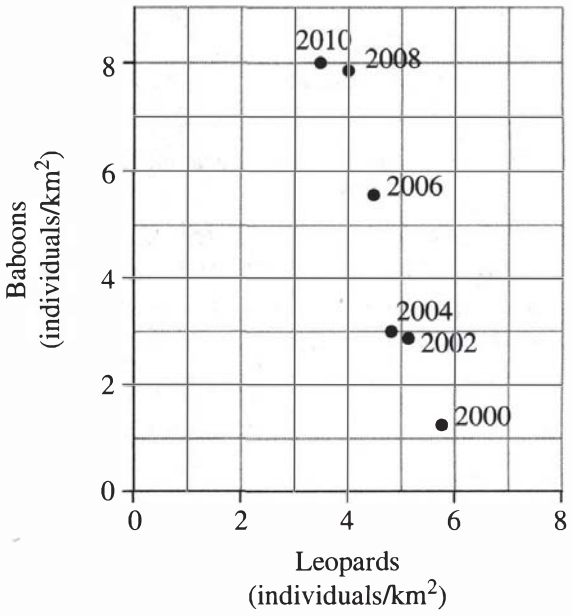
B) 2^{-6k}

C) 2^{-3k}

D) 2^{0}

Questions 22 and 23 refer to the following information.

Population Density of Baboons and Leopards in Western Cape of South Africa, 2000-2010



The scatterplot above shows the population densities of chacma baboons and leopards in the Western Cape of South Africa in 2-year intervals from 2000 to 2010.

22

What was the approximate percent change in the population density of chacma baboons from 2004 to 2006?

A) −6%

B) 30%

C) 55%

D) 90%

CONTINUE

4 4

23

Which of the following statements about the population densities of chacma baboons and leopards in the Western Cape of South Africa is most justified by this graph?

A) The population densities of both species decreased linearly with time from 2000 to 2010.

B) The population densities of both species increased linearly with time from 2000 to 2010.

C) The population density of chacma baboons varied linearly with the population density of leopards in the period from 2000 to 2010.

D) The population density of chacma baboons varied inversely with the population density of leopards in the period from 2000 to 2010.

24

Which of the following situations describes a population that is growing exponentially with time?

A) Every year, the population grows by 1% of the population of the previous year.

B) Every year, the population grows by 1,000 plus 2% of the original population.

C) Every year, the population grows by 2% of the original population.

D) Every year, the population grows by 10,000.

25

If $0 < a < 1$ and $-1 < b < 0$, which of the following must be true?

I. $a^2 > b^2$

II. $a^2 > (ab)^2$

III. $b^3 < \left(\frac{a}{b}\right)^3$

A) I only

B) II only

C) III only

D) I and III only

26

Rogerio drives 1 hour and 18 minutes to meet a friend. The first 30 minutes of his trip was mostly highway driving, so he averaged 64 miles per hour for this portion. For the last 48 minutes of the trip, he averaged 50 miles per hour. What was Rogerio's average speed, approximately, for the entire trip?

A) 60 miles per hour

B) 58 miles per hour

C) 55 miles per hour

D) 53 miles per hour

CONTINUE

4 4

27

Charlotte's club has several rectangular banners that it has been using around school to advertise a fundraiser. Charlotte wants a larger banner to use at the front entrance of the school, so she constructs a new rectangular banner that is 40% longer and k% taller than the smaller banners. If the new banners have an area that is 75% larger than the smaller banners, what is the value of k?

A) 20

B) 25

C) 30

D) 35

28

$$x + a = 4x - 8$$

$$y + b = 4y - 8$$

In the system of equations above, a and b are constants. If $a - b$ is equal to 1, which of the following must be true?

A) x is $\frac{1}{3}$ greater than y.

B) x is $\frac{1}{3}$ less than y.

C) x is 3 greater than y.

D) x is 3 less than y.

29

If y is 150% greater than x, where $x > 0$, then $x + y$ is what percent greater than y?

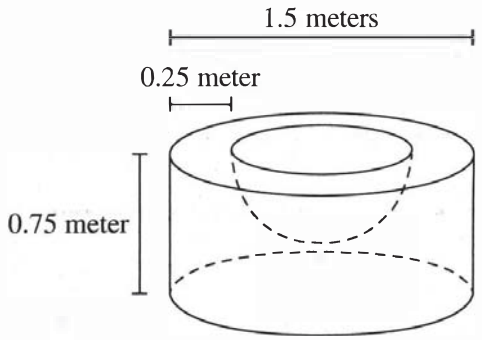
A) 40.0%

B) 66.7%

C) 71.4%

D) 150%

30



A raised cement fire pit is to be constructed according to the diagram above, consisting of a cylinder with a hemispherical cavity at the top. Which choice is closest to the volume, in cubic meters, of cement required to construct this fire pit?

A) 0.80 m^3

B) 0.97 m^3

C) 1.06 m^3

D) 1.59 m^3

CONTINUE

4  4

31

If the average of n numbers is 32, and the sum of these n numbers is 352, what is the value of n?

32

An auditorium with 120 seats begins to fill 20 minutes prior to the start of a lecture. If it fills at a rate of 12 students per minute, what fraction of the seats are empty 12 minutes prior to the start of the lecture?

33

$$\frac{(x+1)(x-2)}{2}+3x$$

If the expression above is rewritten in the form ax^2+bx+c, where a, b, and c are constants, what is the value of b?

34

$$y \le 1{,}000 - 12x$$

$$y \le 3x$$

In the xy-plane, if a point with coordinates (m, n) lies in the solution set of the system of inequalities above, what is the maximum possible value of n?

35

If x is the degree measure of an angle such that $\sin(x^\circ)=\sin\left(\frac{5\pi}{4}\right)$ and $\tan(x^\circ)=\tan\left(\frac{5\pi}{4}\right)$, and if $720 < x < 1{,}080$, what is the value of x?

36

$$\frac{4}{1-x}=\frac{2}{x+2}-\frac{1}{2}$$

What is the sum of the two solutions of the equation above?

CONTINUE

4 4

Questions 37 and 38 refer to the following information.

Projectile 1: $h_1(t) = -4.9t^2 + 28t + 10$

Projectile 2: $h_2(t) = -4.9t^2 + 7t + 19$

The functions above show the heights, in meters, t seconds after the simultaneous launch of two projectiles.

37

What is the height, in meters, of the two projectiles at the moment that their heights are equal?

38

What is the average vertical speed of Projectile 1, in meters per second, in the interval between its launch and the time it reaches its maximum height?

STOP

If you finish before time is called, you may check your work on this section only. Do not turn to any other section of the test.

SAT PRACTICE TEST 6 ANSWER KEY

Section 1: Reading

1. A
2. D
3. B
4. B
5. B
6. D
7. A
8. C
9. D
10. D
11. C
12. D
13. A
14. A
15. B
16. C
17. C
18. D
19. C
20. A
21. B
22. D
23. C
24. D
25. A
26. C
27. B
28. C
29. D
30. A
31. C
32. C
33. C
34. D
35. C
36. B
37. C
38. A
39. B
40. C
41. A
42. D
43. C
44. A
45. C
46. D
47. D
48. A
49. B
50. A
51. D
52. C

Total Reading Points (Section 1)

Section 2: Writing and Language

1. C
2. A
3. D
4. C
5. A
6. B
7. C
8. A
9. B
10. D
11. C
12. A
13. B
14. D
15. C
16. B
17. A
18. B
19. D
20. B
21. C
22. D
23. A
24. C
25. D
26. C
27. A
28. B
29. A
30. D
31. A
32. C
33. C
34. C
35. C
36. A
37. C
38. A
39. D
40. C
41. A
42. D
43. A
44. B

Total Writing and Language Points (Section 2)

Section 3: Math (No Calculator)

1. B
2. A
3. C
4. B
5. D
6. B
7. D
8. C
9. A
10. A
11. A
12. D
13. C
14. B
15. D
16. 6
17. 9
18. $2 \leq x \leq 2.33$ or $3.67 \leq x \leq 4$
19. 7/5 or 1.4
20. 5/12 or .417

Section 4: Math (Calculator)

1. A
2. C
3. C
4. A
5. B
6. B
7. B
8. D
9. A
10. C
11. C
12. A
13. A
14. C
15. C
16. B
17. B
18. C
19. C
20. A
21. A
22. D
23. C
24. A
25. B
26. C
27. B
28. A
29. A
30. C
31. 11
32. 1/5 or .2
33. 5/2 or 2.5
34. 200
35. 855
36. 11
37. 21.1
38. 14

Total Math Points (Section 3 + Section 4)

an amendment cannot be *designed.* Choice D is incorrect because the amendment was passed, and not merely *intended.*

36. **A** **Parallelism**

The original phrasing best maintains the parallel structure established in the preceding sentence: *Gone are the open slave markets . . . Gone also are the days when . . . .* The other choices disrupt this parallel structure.

37. **C** **Coordination/Comma Splices**

The original phrasing is incorrect because it forms a comma splice. Choice B is incorrect because a colon should never separate a modifier from the clause that it modifies. Choice D is incorrect because the phrase that follows the semicolon is not an independent clause. Choice C logically coordinates the main clause and the participial phrase.

38. **A** **Graphical Analysis**

The original phrasing provides the most accurate information from the graph. The six bars representing 2012 add up to approximately 6,000 cases. The other three statements are inaccurate, based on the graph.

39. **D** **Development**

This sentence *expands on the point made in the previous sentence* (specifically that the 6,000 cases processed by the IOM are a small fraction of the problem) by *providing specific details* (specifically about the number of victims both worldwide and in the United States).

40. **C** **Idiom**

The correct idiomatic phrasing here is to say that *human trafficking [has a status]* ***as*** *the [second] most profitable form of international crime.*

41. **A** **Cohesiveness/Redundancy**

Choice A provides the most concise and effective way to combine the sentences. Choices B and C are redundant, since being *afraid* is the same as having *fear.* Choice D is incorrect because the pronoun *it* has no clear and logical antecedent.

42. **D** **Idiom/Parallelism**

Choice D best maintains the parallelism in the list *to marshal . . . and to bring.*

43. **A** **Diction/Clear Expression of Ideas**

Choice A, *initiative,* best describes the effort of the Blue Campaign to raise public awareness about human trafficking. Choice B is incorrect because *achievement* focuses on giving credit for the program rather than on the program itself. Choice C is incorrect because *scheme* has too negative a connotation for this context. Choice D is incorrect because *enterprise* suggests a business venture rather than a social program.

44. **B** **Conclusion/Cohesiveness**

Choice B best links to the previous sentence, which describes the great *cost* of our previous efforts to end slavery. It also effectively concludes the passage by expressing a positive hope. Choices A, C, and D are incorrect because they do not link to the previous sentence.

Section 3: Math (No Calculator)

1. **B** **Algebra (linear equations) EASY**

Original equation: $3b = 2$

Divide both sides by 3: $b = \frac{2}{3}$

Divide both sides by 2: $\frac{b}{2} = \frac{1}{3}$

2. **A** **Algebra (simplifying expressions) EASY**

Given expression: $\frac{x^3}{x} + \frac{x^5}{x^3}$

Simplify using the identity $\frac{x^m}{x^n} = x^{m-n}$: $x^2 + x^2$

Combine like terms: $2x^2$

3. **C** **Algebra (inequalities) EASY**

Original inequality: $m < -3$

Multiply both sides by −3 and "flip" inequality: $-3m > 9$

Combine the two given inequalities into one statement: $9 < -3m \leq 2n$

By the Transitive Law of Inequality: $9 < 2n$

Divide by 2: $4.5 < n$

Since n must be greater than 4.5, the least integer value it can take is 5.

4. **B** **Algebra (expressing algebraic quantities) EASY**

When Janie sits 3 dogs, she charges \$12 per hour for the first dog and \$5 per hour each for the other two, for a total of \$22 per hour for all three dogs. Therefore, if she sits them for n hours her charge is $\$22n$. If she also takes all three dogs to the dog park, she charges an additional \$10 (remember, she charges a flat fee for a trip to the dog park with up to three dogs), and so she charges a total of $\$(22n + 10)$.

5. **D** **Algebra (linear relationships) EASY**

If Oprey Reef has an area of 400 square meters and is growing at a rate of 2.5 square meters per month, then it will have an area of $400 + 2.5m$ square meters after m months have passed. Similarly, if Pelican Reef has an area of 360 square meters and is growing at a rate of 3 square meters per month, it will have an area of $360 + 3m$ square meters after m months have passed. Their areas will be equal when $400 + 2.5m = 360 + 3m$

Subtract 360 and $2.5m$ from both sides: $40 = 0.5m$

Multiply both sides by 2: $80 = m$

6. **B** **Problem Solving (numerical reasoning) MEDIUM**

Since the only fact we are given is that x is less than -10, let's choose a convenient value for x, like -100. Now we can simply plug this value into the choices and choose the one with the least value:

A) $\frac{1}{x^2} = \frac{1}{(-100)^2} = \frac{1}{10{,}000} = 0.0001$

B) $\frac{1}{x^3} = \frac{1}{(-100)^3} = \frac{1}{-1{,}000{,}000} = -0.000001$

C) $\frac{1}{x^4} = \frac{1}{(-100)^4} = \frac{1}{100{,}000{,}000} = 0.00000001$

B) $\frac{1}{x^5} = \frac{1}{(-100)^5} = \frac{1}{-10{,}000{,}000{,}000} = -0.0000000001$

Remember that numerically "least" doesn't mean "closest to zero;" it means "farthest to the left on the number line." Among these four numbers, choice B, -0.000001, is farthest to the left.

7. **D** **Advanced Mathematics (complex numbers and radicals) MEDIUM**

Original expression: $\sqrt{-27} + \sqrt{-48}$

Factor out $\sqrt{-1}$ from both terms: $\sqrt{-1}\sqrt{27} + \sqrt{-1}\sqrt{48}$

Substitute $i = \sqrt{-1}$: $i\sqrt{27} + i\sqrt{48}$

Factor the perfect square from each radicand: $i\sqrt{9}\sqrt{3} + i\sqrt{16}\sqrt{3}$

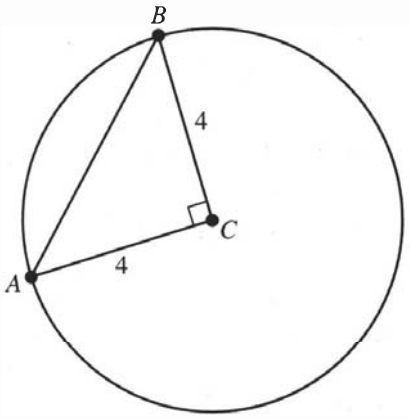
Simplify square roots of perfect squares: $3i\sqrt{3} + 4i\sqrt{3}$

Combine like terms: $7i\sqrt{3}$

8. **C** **Algebra (interpreting expressions) MEDIUM**

The equation $n = 416 - 4T$ or, equivalently, $n = -4T + 416$, is a linear equation in "slope-intercept" form. The "slope" of this linear relationship is -4, which represents the "change in n per unit change in T." Since n represents the number of cars requesting full service and T represents the temperature in degrees Fahrenheit, this slope represents the fact that 4 *fewer* (this is the meaning of the minus sign) cars request full service for every 1-degree increase in temperature.

9. **A** **Additional Topics (circles and arcs) MEDIUM**



When solving a geometry problem, it is usually very helpful to mark up the diagram, and even occasionally to re-draw it. In this case, the diagram is drawn to scale, so we can just mark it up. When we are asked to find an arc length, we should remember that an arc is just a part of the circumference, and we can always find the circumference of a circle with the formula $C = 2\pi r$. Therefore, our strategy should be to find the value of r, then use this to find the circumference, then use this to find the arc length. To find the value of r, we must use the fact that the right triangle has an area of 8 square units. Therefore: $\frac{1}{2}(r)(r) = 8$

Multiply both sides by 2: $r^2 = 16$

Take the square root of both sides: $r = 4$

This means that the circumference of the circle is $2\pi(4) = 8\pi$. Since the central angle of arc AB is 90°, it represents 1/4 of the circumference, or $(1/4)(8\pi) = 2\pi$.

10. **A** **Algebra (simplifying expressions) MEDIUM**

Original expression: $\frac{a}{b}(a-b)(a+b)$

FOIL the product of binomials: $\frac{a}{b}(a^2 - b^2)$

Distribute: $\frac{a^3}{b} - \frac{ab^2}{b}$

Simplify: $\frac{a^3}{b} - ab$

11. **A** **Advanced Mathematics (composition of functions) MEDIUM**

We simply need to find the equation that is equivalent to $y = g(f(x))$.

Substitute $f(x) = -\frac{1}{2}x + 1$: $y = g\left(-\frac{1}{2}x + 1\right)$

Substitute $-\frac{1}{2}x + 1$ as the input to $g(x)$: $y = -4\left(-\frac{1}{2}x + 1\right) - 1$

Distribute and simplify: $y = 2x - 4 - 1 = 2x - 5$

Since this equation is now in slope-intercept form, we can see that its y-intercept is -5.

12. **D** **Advanced Mathematics (factoring polynomials) MEDIUM**

If $p(x) = 6x^2 + 2x$, then $p(x) - 4 = 6x^2 + 2x - 4$, which can be factored as $2(3x - 2)(x + 1)$.

13. **C** **Advanced Mathematics (graphical analysis) MEDIUM**

The x-axis is equivalent to the line $y = 0$, so the points at which the graph of an equation touches the x-axis can be found simply by setting y equal to 0 and solving for x. If we take the equation $6x - xy = x^2 + 9$ and set y equal to 0, we get: $6x = x^2 + 9$
Subtract $6x$ from both sides: $0 = x^2 - 6x + 9$
Factor: $0 = (x - 3)^2$
Therefore, by the Zero Product Property, the x-intercept is (3, 0).

14. **B** **Advanced Mathematics (quadratic equations) MEDIUM-HARD**

First we should recognize that the solution is presented as a nonsimplified solution from the quadratic formula: if $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. However, the equations are not exactly in standard quadratic form, so we must first subtract the constant term from both sides so that each equation has 0 on the right side:

A) $2x^2 + 3x + 5 = 0$

B) $2x^2 + 3x - 5 = 0$

C) $2x^2 - 3x + 5 = 0$

D) $2x^2 - 3x - 5 = 0$

Now notice that the solution "fits" the quadratic formula only if $a = 2$, $b = 3$, and $c = -5$. This corresponds to the equation in choice B.

15. **D** **Advanced Mathematics (rational functions) MEDIUM-HARD**

If this equation is true for all values of x (except $x = 2$), as we are told, then we can just choose a value of x that lets us solve easily for b. A good option is $x = 0$:

$$\frac{x^3}{x-2} = x^2 + 2x + 4 + \frac{b}{x-2}$$

Substitute $x = 0$: $\frac{(0)^3}{(0)-2} = (0)^2 + 2(0) + 4 + \frac{b}{(0)-2}$

Simplify: $0 = 4 - \frac{b}{2}$

Multiply by 2 on both sides: $0 = 8 - b$
Add b to both sides: $b = 8$
Equivalently, we could have simply used long division (or synthetic division) to divide x^3 by $x - 2$ to show that the remainder is 8. Or, if you are *exceptionally* clever and recall the Remainder Theorem, which says that whenever the polynomial $P(x)$ is divided by $x - k$ the remainder is $P(k)$, you can see that b must be equal to $(2)^3 = 8$.

16. **6** **Algebra (linear equations) EASY**

If the point $(a, 1)$ is a solution, we can solve this equation by simply substituting $x = a$ and $y = 1$. When we substitute these values into $y = \frac{1}{3}x - 1$ we get $1 = \frac{1}{3}a - 1$

Add 1 to both sides: $2 = \frac{1}{3}a$

Multiply by 3: $6 = a$

17. **9** **Advanced Mathematics (solving quadratics) EASY**

Substituting $x = 3$ into the equation gives us: $2(3)^2 - b(3) + b = 0$
Simplify: $18 - 2b = 0$
Add $2b$ to both sides: $18 = 2b$
Divide by 2: $9 = b$

18. **$2 \le x \le 2.33$ or $3.67 \le x \le 4$** **Advanced Mathematics (inequalities) MEDIUM**

Original inequalities: $2 \le 3|3 - x| \le 3$
Since the problem asks us to find *one possible* value of x, we don't have to worry about finding the general solution. Therefore, we can find a particular solution by assuming that $3 - x$ (the expression inside the absolute value) is non-negative. This means that $3 - x \ge 0$ and therefore $x \le 3$. Now, recall that if a number is non-negative, then it is equal to its absolute value. Therefore, if $x \le 3$, we can rewrite the inequality and just ignore the absolute value signs: $2 \le 3(3 - x) \le 3$

Distribute: $2 \le 9 - 3x \le 3$
Subtract 9 from all three sides: $-7 \le -3x \le -6$
Divide by -3 and "flip" the inequalities: $\frac{7}{3} \ge x \ge 2$

Since 7/3 is 2.333 . . . , you may enter any value between 2 and 2.33.
To get the *complete* solution set, you must consider the possibility that $3 - x$ could be negative, and so $x > 3$. Remember that *if a quantity is negative, then its absolute value is its opposite.* Therefore, if $x > 3$, then the inequality can be rewritten as: $2 \le 3(x - 3) \le 3$

Distribute: $2 \le 3x - 9 \le 3$
Add 9 to all three sides: $11 \le 3x \le 12$
Divide by 3 on all three sides: $\frac{11}{3} \le x \le 4$

Since 11/3 = 3.666 . . . , you may enter any value between 3.67 and 4.

19. **7/5 or 1.4** **Additional Topics (complex numbers) MEDIUM-HARD**

Original expression: $\frac{2+i}{2-i}$

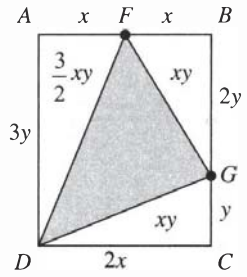
To simplify the quotient of complex numbers, multiply numerator and denominator by the *complex conjugate of the denominator*: $\left(\frac{2+i}{2-i}\right)\left(\frac{2+i}{2+i}\right)$

Multiply: $\frac{4+4i+i^2}{4-i^2}$

Substitute $i^2=-1$ and simplify: $\frac{4+4i+(-1)}{4-(-1)}=\frac{3+4i}{5}=\frac{3}{5}+\frac{4}{5}i$

Therefore, $a=\frac{3}{5}$ and $b=\frac{4}{5}$, so $a+b=\frac{7}{5}=1.4$

20. **5/12 or .417** **Additional Topics (areas) HARD**



As with every SAT geometry question, it's best to mark up the diagram. Since F is a midpoint of its side, AF and FB must have the same measure, so let's call them both x. Since BG is twice as long as GC, let's say that GC is y units long and BG is $2y$ units long. This means that the rectangle has a height of $3y$ and a width of $2x$, so its area is $(3y)(2x) = 6xy$. Now we can find the area of the three nonshaded triangles fairly easily, because their bases and heights are all labeled. Using the triangle area formula $A = (1/2)(b)(h)$, we can calculate the areas of the three nonshaded triangles: xy, xy, and $\frac{3}{2}xy$. The sum of these three areas is $\frac{7}{2}xy$, which is $\frac{\frac{7}{2}xy}{6xy}=\frac{7}{12}$ of the total area. Therefore, the *shaded* triangle is $1-\frac{7}{12}=\frac{5}{12}$ of the total area.

Section 4: Math (Calculator)

1. **A** **Problem Solving and Data Analysis (proportions) EASY**

The phrase *at this rate* tells us that we can set up a proportion based on equal rates: $\frac{12 \text{ gallons}}{\$24.36}=\frac{16 \text{ gallons}}{\$x}$

Cross-multiply: $12x = 389.76$

Divide by 12: $x = \$32.48$

2. **C** **Problem Solving and Data Analysis (averages) EASY**

Recall that the sum of any set of numbers is equal to their average times the number of numbers in the set. Therefore, if the average of a and b is 6, then

$$a+b=(6)(2)=12$$

If the average of a, b, and c is 8, then:

$$a+b+c=(8)(3)=24$$

Subtract the first equation from the second equation:

$$c=12$$

3. **C** **Algebra (multiplying binomials) EASY**

We can find the value of xy in terms of a and b by simply using the given equations to substitute:

$$xy=(3a+b)(a-b)=3a^2-3ab+ab-b^2=3a^2-2ab-b^2$$

4. **A** **Algebra (linear systems) EASY**

A system of two linear equations will have no solution if and only if the graphs of those two equations are parallel lines, which means they have the same slope but different y-intercepts. (If they have the same slope and the same y-intercept, then their graphs are identical and therefore the system has infinitely many solutions.) Remember that in standard $ax + by = c$ form, the slope of a line is always $-a/b$. Notice that choice A is the only system in which the lines are parallel. They both have a slope of –2, and the first line has a $y$intercept of 0, but the second line has a y-intercept of 1.

5. **B** **Problem Solving and Data Analysis (tables) EASY**

The table shows that the tem has a total of 13 attackers and 14 midfielders. There are 6 attackers who are in the 12th grade and 5 midfielders who are in the 12th grade, so $(6 + 5)/(13 + 14) = 11/27$ of this group are 12th graders.

6. **B** **Algebra (linear inequalities) EASY**

If he Arkady needs \$5,000 for the down payment, but already has \$300 saved, he only needs to save \$4,700 more. If he saves \$40 each week, he will have saved \$40w after w weeks. Since this amount must be at least 4,700 dollars, $4{,}700 \le 40w$.

7. **B** **Advanced Mathematics (geometric sequences) EASY**

After the first bounce, the ball rises to 80% of its previous height, or (0.8)(10) = 8 meters. After the second bounce, it rises to (0.8)(8) = 6.4 meters. After the third bounce, it rises to (0.8)(6.4) = 5.12 meters.

8. **D** **Additional Topics (geometric sequences) EASY-MEDIUM**

When the ball hits the ground for the first time, it has traveled 10 meters. It then goes up and down 8 meters

before it strikes the ground for the second time, then up and down 6.4 meters before striking the ground for the third time, then up and down 5.12 meters before striking the ground for the fourth time. Therefore, the total distance it travels is 10 + 2(8) + 2(6.4) + 2(5.12) = 49.04 meters.

9. **A** **Algebra (equivalent expressions) MEDIUM**

If $u = x + 2$, then $x = u - 2$. We can use this equation to substitute into $(x-1)^2\sqrt{x+2}$ to get $(u-2-1)^2\sqrt{u-2+2} = (u-3)^2\sqrt{u}$.

10. **C** **Advanced Mathematics (functional analysis) MEDIUM**

Use the definition of $g(x)$ to translate $g(a) = 8$:

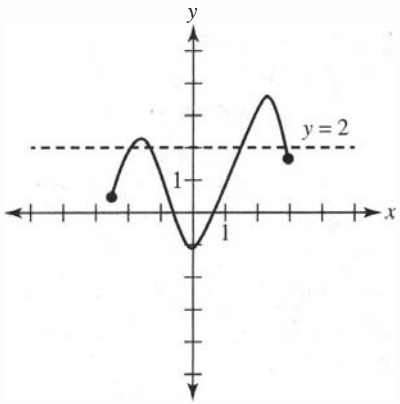
$$g(a) = 5a - 2 = 8$$

Add 2 to both sides: $5a = 10$

Divide both sides by 5: $a = 2$

Evaluate $g(a + 2)$: $g(a+2) = g(2+2) = g(4) = 5(4) - 2 = 20 - 2 = 18$

11. **C** **Advanced Mathematics (graphical analysis) MEDIUM**



The equation $f(x) = 2$ has real-valued solutions wherever the graphs of $y = f(x)$ and $y = 2$ intersect. Therefore, we should draw the line $y = 2$ on all of the graphs and choose the one that gives four distinct intersection points. As the figure above shows, the only choice that works is C.

12. **A** **Algebra (interpretation of expressions) MEDIUM**

The number 25 is the "slope" of the supply function $S(p) = 25p + 100$, where S represents the number of units supplied to the market and p is the price, in dollars. Like every "slope," it represents the change in the function value (usually y, but in this case S) for every unit increase in the input (usually x, but in this case p). Therefore, 25 represents the change in the number of units supplied for every dollar increase in price.

13. **A** **Advanced Mathematics (functional analysis) MEDIUM**

The equilibrium price, as the question explains, is the price at which the supply and the demand are equal. Therefore, we simply set the supply and demand expressions equal and solve for p:

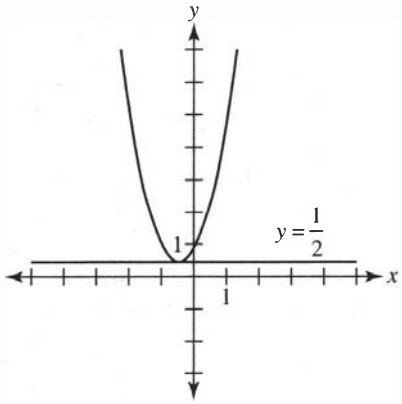
$$25p + 100 = -35p + 1{,}300$$

Add $35p$ to both sides and subtract 100 from both sides:

$$60p = 1{,}200$$

Divide by 60: $p = 20$

14. **C** **Advanced Mathematics (nonlinear systems) MEDIUM-HARD**



The graph of the equation $y = 2x^2 + 2x + 1$ is an "open-up" parabola, and the graph of the equation $y = b$ is a horizontal line. The only way that a horizontal line can intersect an "open-up" parabola in exactly one point is if it passes through the vertex of the parabola. Therefore, to solve this problem we must find the coordinates of the vertex of the parabola. The line of symmetry of the parabola described by $y = ax^2 + bx + c$ is the vertical line $x = -\frac{b}{2a}$. This line of symmetry must pass through the vertex, so the vertex must have an x-coordinate of $-\frac{b}{2a} = -\frac{2}{2(2)} = -\frac{1}{2}$. We can now find the y-coordinate by plugging $x = -\frac{1}{2}$ into the quadratic equation: $y = 2\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right) + 1 = 2\left(\frac{1}{4}\right) - 1 + 1 = \frac{1}{2}$. Therefore, the vertex of the parabola is $\left(-\frac{1}{2}, \frac{1}{2}\right)$, and so the horizontal line that passes through this vertex is the line $y = \frac{1}{2}$.

15. **C** **Algebra (word problems) MEDIUM**

Let's let a = the number of easy puzzles that Joaquin solves, and b = the number of hard puzzles that he solves. If he solved 30 puzzles, then $a + b = 30$. If he earned a total of 375 points, then $10a + 25b = 375$. Now let's solve the system:

$$a + b = 30$$
$$10a + 25b = 375$$

Multiply both sides of the first equation by 10: $10a + 10b = 300$
Subtract this from the second equation: $15b = 75$
Divide by 15: $b = 5$
Substitute $b = 5$ into the first equation: $a + 5 = 30$
Subtract 5 from both sides: $a = 25$
Therefore, Joaquin solved 25 – 5 = 20 more easy puzzles than hard puzzles.

16. **B** **Problem Solving and Data Analysis (medians) MEDIUM**

The median of 200 numbers is the average of the 100th and 101st of these numbers when listed in order. The sum of the two bars on the left is just under 100, and the sum of the first three bars is clearly greater than 130. Therefore, the 100th and 101st numbers are in the third bar, which represents a Richter scale number of 1.5.

17. **B** **Advanced Mathematics (solving quadratics) MEDIUM**

Although this question can be answered by finding the two solutions and then taking their sum, it turns out that these solutions aren't very pretty: $-2+\sqrt{14}$ and $-2-\sqrt{14}$. So it is helpful to remember that, for any quadratic equation of the form $ax^2 + bx + c = 0$, the sum of the two (perhaps identical) solutions will always be $-b/a$, and the product of those solutions will always be c/a. Therefore, we simply need to put the equation into standard quadratic form and find $-b/a$. The equation $2(x^2 + 4x) = 20$ can be rewritten as $2x^2 + 8x - 20 = 0$, and so $-b/a = -8/2 = -4$, which is obviously the sum of $-2+\sqrt{14}$ and $-2-\sqrt{14}$.

18. **C** **Problem Solving and Data Analysis (study design) MEDIUM-HARD**

A proper study requires a *control group*, which is a group that is identical in every relevant way to the experimental group *except* for the treatment variable. Without a control group, it can be difficult if not impossible to tell if any effect is due to the treatment or to some other variable. The treatment variable in this case is the taking of the zinc supplements. Therefore, the best comparison group is one that is identical to the treatment group (100 adults with cold symptoms who are taking some kind of pill), but differs in the treatment: the subjects take a harmless sugar pill (a "placebo") instead of the actual zinc supplement.

19. **C** **Problem Solving and Data Analysis (percent change) MEDIUM-HARD**

Let's define x as the minimum cost, in dollars, of purchases that Philip must make to save enough on shipping and handling costs to cover his annual fee of $80. First, we must calculate the amount he would spend in total on those purchases if he did *not* have the annual membership. With the 3.6% tax and 2% shipping and handling on every purchase, the total would be $(1.036)(1.02)(x) = 1.05672x$. If, however, he did not have to pay shipping and handling, the total would be only $1.036x$. The difference between these two is $1.05672x - 1.036x = 0.02072x$.

This difference must be at least $80 in order for Philip to cover his annual fee, so $0.02072x \geq 80$. Dividing both sides by 0.02072 gives $x \geq 3{,}861$.

20. **A** **Advanced Mathematics (rational expressions) HARD**

Notice that this question is not asking us to "simplify" the expression. In fact, each answer choice is written as a sum, which is an "expanded" version of the original. Fortunately, the answer choices have some elements in common that help us simplify this problem. Notice that each sum contains fractions with the same denominators: $(x+1)$ and $(x-1)$. The common denominator, then, is their product: $(x+1)(x-1) = x^2 - 1$, which is the denominator in the original expression. This shows us that one way we can solve this problem is by simplifying each choice until we find one that is equivalent to the original expression:

A) $\frac{2}{x+1} - \frac{1}{x-1} = \frac{2(x-1)}{(x+1)(x-1)} - \frac{1(x+1)}{(x+1)(x-1)} = \frac{2x-2}{x^2-1} - \frac{x+1}{x^2-1}$

$= \frac{x-3}{x^2-1}$

B) $\frac{1}{x+1} - \frac{2}{x-1} = \frac{1(x-1)}{(x+1)(x-1)} - \frac{2(x+1)}{(x+1)(x-1)} = \frac{x-1}{x^2-1} - \frac{2x+2}{x^2-1}$

$= \frac{-x-3}{x^2-1}$

C) $\frac{2}{x+1} + \frac{1}{x-1} = \frac{2(x-1)}{(x+1)(x-1)} + \frac{1(x+1)}{(x+1)(x-1)} = \frac{2x-2}{x^2-1} + \frac{x+1}{x^2-1}$

$= \frac{3x-1}{x^2-1}$

D) $\frac{1}{x+1} + \frac{2}{x-1} = \frac{1(x-1)}{(x+1)(x-1)} + \frac{2(x+1)}{(x+1)(x-1)} = \frac{x-1}{x^2-1} + \frac{2x+2}{x^2-1}$

$= \frac{3x+1}{x^2-1}$

21. **A** **Advanced Mathematics (exponentials) MEDIUM-HARD**

We can use the given equations to substitute into the expression $\frac{b}{a}$: $\frac{b}{a} = \frac{4^{-3k}}{2^{3k}}$

One way to simplify this expression is to express both exponentials in terms of the same base. Since $4 = 2^2$, we can rewrite the expression as

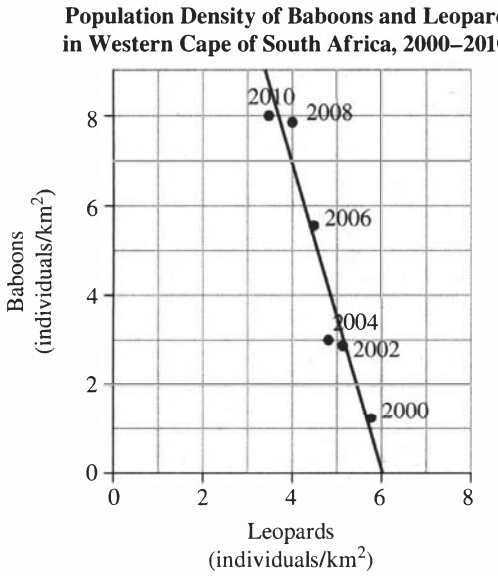
$$\frac{b}{a} = \frac{4^{-3k}}{2^{3k}} = \frac{(2^2)^{-3k}}{2^{3k}} = \frac{2^{-6k}}{2^{3k}} = 2^{-6k-3k} = 2^{-9k}$$

22. **D** **Problem Solving and Data Analysis (scatter plots) MEDIUM-HARD**

The point corresponding to the year 2004 has coordinates of approximately (4.8, 3), which means that in 2004, the population density of leopards was 4.8 individuals per square kilometer and the population density of baboons was 3 individuals per square kilometer. The point corresponding to 2006 has coordinates of about (4.5, 5.6). This means that the population density of baboons went from 3 to 5.6, which is a change of approximately $(5.6 - 3)/3 = 2.6/3 = 0.87$ or about 87%, which is closest to choice D.

23. **C** **Data Analysis and Problem Solving (scatter plots) MEDIUM**

Choice A is incorrect because the population density of baboons *increased* between 2000 and 2010, from about 1.2 to about 8 individuals per square kilometer. Choice B is incorrect because the population density of leopards *decreased* between 2000 and 2010, from about 5.8 to about 3.5 individuals per square kilometer. Choice D is incorrect because if two variables vary inversely, then their products are (approximately) constant. However, if we take the product of each pair of coordinates, we get (5.8)(1.2) = **6.96** for 2000, (5.1)(2.9) = **14.79** for 2002, (4.8)(3) = **14.4** for 2004, (4.5)(5.6) = **25.2** for 2006, (4)(7.9) = **31.6** for 2008, and (3.5)(8) = **28** for 2010. Since these products are not even approximately equal, we cannot say that the two variables vary inversely with one another. Choice C, however, is correct, because the 6 points fall very close to a "line of best fit" relating the two population densities.



24. **A** **Advanced Mathematics (exponentials) MEDIUM-HARD**

The correct answer is A, because in order to increase a number by x%, we must multiply it by $(1 + x/100)$. Therefore, increasing a number by 1% is equivalent to multiplying it by 1.01. So if an initial population P_0 grows at a rate of 1% *every* year, the population after t years is $P = P_0(1.01)^t$, which is an exponential function. Choice C looks very similar to choice A, but it is *not* an exponential relationship, but rather a *linear* relationship, because each year the population is growing by a *constant* amount, which is 2% of the *original* population. For instance, in choice C, if the initial population were 100, then each year the population would grow by (0.02)(100) = 2, so after t years the population would be $P = 100 + 2t$, which is a linear relationship, not an exponential one. Choices B and D also indicate linear relationships.

25. **B** **Problem Solving (numerical reasoning) MEDIUM-HARD**

It's very important to remember the difference between proving that something *can be true* and proving that something *must be true*. When solving *must be true* questions like this, it's often best to try first to find counterexamples for the statements, that is, examples that satisfy the conditions of the problem but do *not* satisfy the statements. A statement with a counterexample obviously does *not* have to be true.

Let's start by choosing values for a and b. Since a must be between 0 and 1, let's choose $a = 0.5$. Since b must be between −1 and 0, let's choose $b = -0.5$. Now we can check statement I. Since $(0.5)^2 = 0.25$ and $(-0.5)^2 = 0.25$, it is *not* necessarily true that $a^2 > b^2$. Since statement I is not necessarily true, we can eliminate choices A and D. When we plug these values in for statement II, we get $(0.5)^2 > (0.25)^2$, which is true. However, one successful example does not prove that statement II *must* be true, so let's check statement III. Since $(-0.5)^3 < \left(\frac{0.5}{-0.5}\right)^3$ simplifies to $-0.125 < -1$, which is *not* true, we can eliminate choice C, and so the correct answer must be choice B.

26. **C** **Algebra (inequalities) MEDIUM**

Recall the formula that *average speed = total distance traveled ÷ total time*. The total time of the trip is 1 hour and 18 minutes, which is 1.3 hours. We can find the total distance traveled by using the formula *distance = average speed × time*. If he drove for 30 minutes (or 0.5 hour) at 64 mph, he traveled (64)(0.5) = 32 miles. If he then drove for 48 minutes (or 0.8 hour) at 50 mph, he traveled (50)(0.8) = 40 miles. Therefore, he traveled a total of 32 + 40 = 72 miles in 1.3 hours, which means his average speed was 72 ÷ 1.3 = 55.38 mph, which is closest to choice C.

27. **B** **Advanced Mathematics (graphing parabolas) MEDIUM-HARD**

Let's start by assuming that the smaller banners have a height of h and a length of l, and therefore have an area of hl. The larger banners have a height of $(1 + k/100)h$ and a

length of $(1.40)l$. Since the larger banner has an area that is 75% larger than the smaller banner:

$(1 + k/100)h(1.40)l = 1.75hl$

Divide both sides by hl: $(1 + k/100)(1.40) = 1.75$

Divide both sides by 1.40: $1 + k/100 = 1.25$

Subtract 1 from both sides: $k/100 = 0.25$

Multiply by 100: $k = 25$

28. **A** **Algebra (interpreting systems) MEDIUM-HARD**

First, let's simplify our given equations to make them easier to work with.

First equation: $x + a = 4x - 8$

Subtract x from both sides: $a = 3x - 8$

Second equation: $y + b = 4y - 8$

Subtract y from both sides: $b = 3y - 8$

Since $a - b$ is equal to 1: $a - b = (3x - 8) - (3y - 8) = 1$

Simplify: $3x - 3y = 1$

Divide both sides by 3: $x - y = \frac{1}{3}$

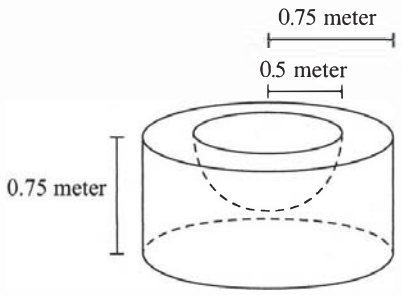
Add y to both sides: $x = y + \frac{1}{3}$

Which means "x is 1/3 greater than y."

29. **A** **Problem Solving and Data Analysis (percentages) MEDIUM-HARD**

One way to approach this question is to pick values for x and y. Since y must be 150% greater than x, let's choose $x = 100$ and so $y = 250$. (Be careful here—notice that y is not 150! If it is 150% *greater* than 100, then it is (1 + 150/100)(100) = (1 + 1.5)(100) = (2.5)(100) = 250.) Therefore, $x + y = 100 + 250 = 350$. To see what percentage 350 is greater than 250, we use the formula (350 – 250)/250 = 100/250 = 0.4 or 40%.

30. **C** **Additional Topics (volumes) MEDIUM-HARD**



To find the volume of the fire pit, we must subtract the volume of the hemisphere from the volume of the cylinder. Recall (from the reference information at the beginning of the test) that the volume of a cylinder is $\pi r^2 h$ and the volume of a sphere is $\frac{4}{3}\pi r^3$. Since the diameter of the cylinder is 1.5 meters, its radius is 0.75 meter, and its height is also 0.75 meter. Therefore, its volume is $\pi(0.75)^2(0.75) = 0.421875\pi$. The hemisphere has a radius of 0.75 – 0.25 = 0.5 meter, so it has a volume of $\frac{2}{3}\pi(0.5)^3 = .083333\pi$. Therefore, the fire pit has a volume of $0.421875\pi - 0.083333\pi = 0.338542\pi = 1.063561$ cubic meters, which is closest to choice C.

31. **11** **Problem Solving and Data Analysis (averages) EASY**

Recall the formula *average* = *sum* ÷ n: $32 = \frac{352}{n}$

Multiply both sides by n: $32n = 352$

Divide both sides by 32: $n = 11$

32. **1/5 or .2** **Problem Solving and Data Analysis (rates) EASY-MEDIUM**

Since the auditorium began to fill 20 minutes prior to the lecture, by the time it was 12 minutes prior to the lecture the auditorium had been filling up for 8 minutes at a rate of 12 students per minute. This means that (8)(12) = 96 seats were filled, which is 96/120 = 4/5 of the seats, and so 1 – 4/5 = 1/5 of the seats are empty.

33. **5/2 or 2.5** **Algebra (equivalent expressions) MEDIUM**

Original expression: $\frac{(x+1)(x-2)}{2} + 3x$

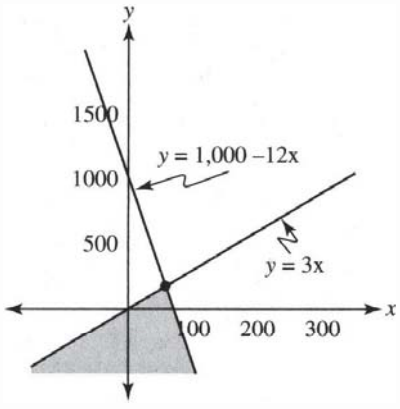
Distribute the 0multiplication in the numerator:

$\frac{x^2 - x - 2}{2} + 3x$

Distribute the division: $\frac{1}{2}x^2 - \frac{1}{2}x - 1 + 3x$

Combine line terms: $\frac{1}{2}x^2 + \frac{5}{2}x - 1$

34. **200** **Algebra (systems of inequalities) MEDIUM**



The graph above shows the solution set to the given system of inequalities. The solution set to $y \leq 1{,}000 - 12x$ is the set of all points *on or under* the line $y = 1{,}000 - 12x$, and the solution set to $y \leq 3x$ is the set of all points *on or under* the line $y = 3x$. The solution to the system, then, is the region where these two sets overlap. The point in this region with the maximum y value is the intersection point of the two lines. We can find the x-value of

this point by substituting $y = 3x$ into $y = 1{,}000 - 12x$:

$$3x = 1{,}000 - 12x$$

Add $12x$ to both sides: $15x = 1{,}000$

Divide by 15: $x = \frac{1{,}000}{15} = \frac{200}{3}$

Now plug in to $y = 3x$ to find the y-coordinate:

$$y = 3\left(\frac{200}{3}\right) = 200$$

35. **855** **Additional Topics (trigonometry) HARD**

When you see an expression like $\sin\left(\frac{5\pi}{4}\right)$, it's important to remember that the "input" to any trigonometric function is always an angle measure, and if that angle measure does not have a degree symbol next to it, it is a *radian measure*. For instance, $\sin(1°)$ means the sine of a 1° angle, but $\sin(1)$ means the sine of a 1 *radian* (or approximately 57.3°) angle. An angle measure of $\frac{5\pi}{4}$ radians is equivalent to a measure of $\frac{5\pi}{4}\left(\frac{180°}{\pi}\right) = 135°$. If the sine of $x°$ is equal to the sine of 135° and the tangent of $x°$ is equal to the tangent of 135°, then the two angles are "co-terminal," which means that their terminal rays coincide when they are graphed in standard position on the xy-plane. This means that $x = 135 + 360n$, where n is any integer. If x must be between 720 and 1,080, then the only possibility is $n = 2$, which yields $x = 135 + 360(2) = 855$.

36. **11** **Advanced Mathematics (quadratics) MEDIUM-HARD**

Original equation: $\frac{4}{1-x} = \frac{2}{x+2} - \frac{1}{2}$

Multiply by the common denominator:

$$(2)(1-x)(x+2)\left(\frac{4}{1-x}\right) = \left(\frac{2}{x+2} - \frac{1}{2}\right)(2)(1-x)(x+2)$$

Simplify: $(8)(x+2) = (2)(2)(1-x) - (1-x)(x+2)$

Distribute: $8x + 16 = 4 - 4x - (x + 2 - x^2 - 2x)$

Distribute and combine like terms: $8x - 16 = x^2 - 3x + 2$

Subtract $8x$ and add 16 to both sides: $x^2 - 11x + 18 = 0$

Any quadratic of the form $ax^2 + bx + c = 0$ has two solutions that have a sum of $-b/a$; therefore, the two solutions of this quadratic are $-(-11)/1 = 11$.

37. **21.1** **Problem Solving (nonlinear systems) HARD**

If $h_1 = h_2$, then: $-4.9t^2 + 28t + 10 = -4.9t^2 + 7t + 19$

Add $4.9t^2$, subtract $7t$, and subtract 10: $21t = 9$

Divide by 21: $t = \frac{9}{21} = \frac{3}{7}$

Substitute $t = \frac{3}{7}$ into either equation to get the height:

$$-4.9\left(\frac{3}{7}\right)^2 + 28\left(\frac{3}{7}\right) + 10 = 21.1$$

38. **14** **Advanced Mathematics (interpreting quadratic models) HARD**

Average speed is can be found with the formula *average speed = total distance ÷ total time*. We are interested in the time between launch and when Projectile 1 reaches its maximum height. At the launch, $t = 0$ seconds, and so the height is $-4.9(0)^2 + 28(0) + 10 = 10$ meters. We can find the time at which it reaches its maximum height by noticing that the height function is quadratic in t, and so its graph is a parabola that reaches its greatest height at its vertex. Recall that the axis of symmetry of any quadratic function in the form $y = ax^2 + bx + c$ is is $x = -\frac{b}{2a}$. Therefore, the value of t when Projectile 1 reaches its maximum height is $t = -\frac{b}{2a} = -\frac{28}{2(-4.9)} = \frac{20}{7}$. The height at that time is therefore $-4.9\left(\frac{20}{7}\right)^2 + 28\left(\frac{20}{7}\right) + 10 = 50$. So the total distance it traveled is 50 − 10 = 40 meters in $\frac{20}{7}$ seconds; therefore, its average speed is 40 meters ÷ $\frac{20}{7}$ seconds = 14 meters per second.

(It's not a coincidence that this number is half of 28, which is the middle coefficient in the quadratic. In this function, 28 represents the vertical speed, in meters per second, of Projectile 1 when it is launched. When it reaches its maximum height, its vertical speed is 0. The average of 28 and 0 is, of course, 14.)