The Tutorverse

MAKING THE UNIVERSE BRIGHTER, ONE STUDENT AT A TIME

THE NEW SAT®

BONUS FULL-LENGTH PRACTICE TEST

1,500+ PRACTICE QUESTIONS

LOGICALLY grouped by section and question type

DETAILED EXPLANATIONS for every question

TIPS, TRICKS, & STRATEGIES for each section

MORE QUESTIONS FOR THE REDESIGNED SAT THAN 10 FULL-LENGTH EXAMS

Part Three: Math

Overview

The third and fourth parts of the SAT are the Math Tests, one of which does not allow the use of calculators, the other of which does allow the use of calculators. Some questions are multiple choice, while others are student-produced responses. These tests assess the student's ability to solve both standard and context-based math questions as they relate to the following units:

- Algebra: linear equations; systems of equations; graphing linear equations
- Data Analysis: ratios; proportions; percentages; probabilities; working with quantitative data
- Advanced Math: creating algebraic expressions; solving and graphing quadratic and nonlinear equations
- Additional Math Topics: area/volume; trigonometry; right triangles; unit circle; various theorems.

On the Actual Test

On the third part of the SAT (Math – No Calculators), students will have 25 minutes to answer 20 questions. 15 of these 20 questions will be multiple choice, and the remaining 5 questions will be student-produced responses. Of the 20 questions, 9 will relate to Advanced Math, 8 to Algebra, and 3 to Additional Math Topics.

On the fourth part of the SAT (Math – Calculators), students will have 55 minutes to answer 38 question. 30 of these 38 questions will be multiple choice, and the remaining 8 questions will be student-produced responses. Of the 38 questions, 17 will relate to Data Analysis, 11 to Algebra, 7 to Advanced Math, and 3 to Additional Math Topics.

It won't be possible to know in advance which math topic will be covered by multiple choice or student-produced responses.

How to Use This Section

Each of the 4 math units are further divided by topic. Many of these topics are related, but

- Unit: Algebra
  - Solving Linear Equations & Inequalities
  - Solving Systems of Equations
  - Linear Equations, Inequalities, & Systems in Word Problems
  - Linear Equations, Inequalities, & Systems on the Coordinate Grid
  - Absolute Value
- Unit: Data Analysis
  - Ratios & Proportions
  - Linear & Exponential Growth
  - Interpreting Graphs & Tables
  - Additional Data Analysis & Statistics
- Unit: Advanced Math
  - Working with Polynomials
  - Working with Polynomial Factors in Expressions & Equations
  - Quadratic Functions & Equations
  - Exponents & Radicals
  - Systems of Equations
  - Function Notation
  - Graphs of Functions
  - More Word Problems

- Unit: Additional Math Topics
  - Geometry
  - Equations of Circles
  - Trigonometry
  - Radians
  - Imaginary & Complex Numbers

There is a *Guided Practice* section for each of the above-listed math topics. These sections are designed to help build students' fluency in each topic by progressing from the more basic to the more advanced. The workbook will clearly indicate which questions should or should not be attempted with the use of a calculator. Students should take as much time as they need working through each topic's *Guided Practice* section.

There is a *Mixed Practice* section for each of the above-listed units. These sections contain a sampling of questions from each topic comprising the given unit. The workbook will clearly indicate which questions should or should not be attempted with the use of a calculator. We recommend that students give themselves approximately 1 minute and 15 seconds per question in each unit's *Mixed Practice* section. This emulates the average amount of time allotted per math question on the actual exam.

We recommend that students practice at least 2 *Guided Practice* topics per week in preparing for the exam (though this number should, of course, be tailored to fit a student's individual study plan).

Tutorverse Tips & Strategies

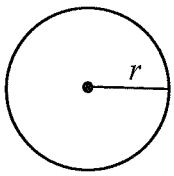
In addition to tips and strategies outlined in the *Test Overview* section of this workbook, consider employing the following Math-specific suggestions:

- *Don't assume you have to use a calculator, even on the calculator section.*
  Sometimes, students spend too much time plugging away at their calculators and not enough time thinking about the question itself. Oftentimes, questions do not require a calculator to solve. Instead, think about the question itself and how it might be possible to use any multiple choice answer choices to help solve the question.
- *Commit important formulas, theorems, and identities to memory.*
  The exam provides a list of basic formulas, relationships, and rules at the beginning of the math section, but it's best not to spend time flipping back and forth trying to apply a formula to a particular question. Instead, students should attempt to memorize as many important rules and theorems as possible – especially ones that may help save time on the actual exam.
- *Work out your answers on paper if you need to.*
  Many students attempt to solve math questions mentally. We strongly recommend students layout their work on paper. This helps not only ensure that calculations are performed correctly, but also facilitates a final review (if time allows) of answer choices on the actual exam.

Reference Equations & Formulas

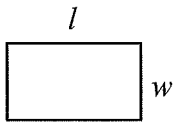
At the beginning of each math section (sections 3 and 4 of the actual exam) will be a sheet of reference formulas and equations. Some of these formulas and equations are reproduced below, which you may find on the test. These formulas and equations will be helpful to know for some, but not all problems on the test. As we mentioned previously, it is prudent to commit as many of these formulas to memory as possible.

A circle has 360 degrees of arc.
A circle has 2π radians of arc.
The sum of the measures of all angles in a triangle is 180.

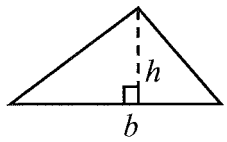


$A = \pi r^2$

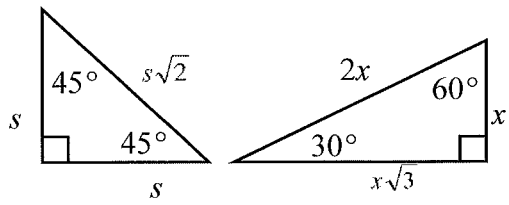
$C = 2\pi r$



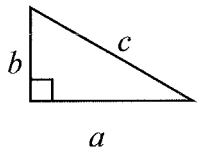
$A = lw$



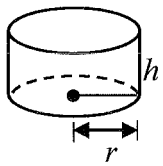
$A = \frac{1}{2}bh$



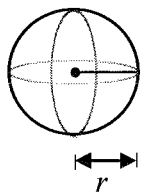
Special Right Triangles



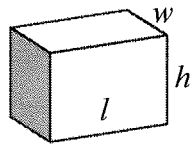
$c^2 = a^2 + b^2$



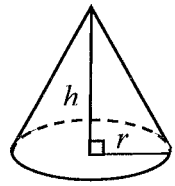
$V = \pi r^2 h$



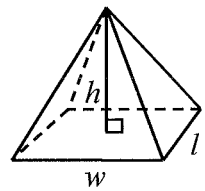
$V = \frac{4}{3}\pi r^3$



$V = lwh$



$V = \frac{1}{3}\pi r^2 h$



$V = \frac{1}{3}lwh$

Algebra

Guided Practice – Solving Linear Equations & Inequalities

(Answers & explanations begin on page 389).

Multiple Choice – No Calculator

1 If $\frac{x-1}{2} = k$ and $k = 4$, what is the value of x?

A) 2
B) 3
C) 8
D) 9

2 If $2x + 4 = 20$, what is the value of $4x + 5$?

A) 8
B) 13
C) 32
D) 37

3 If $\frac{5}{6}x - \frac{1}{6}x = \frac{2}{3} + \frac{1}{2} + \frac{5}{6}$, what is the value of x?

A) 2
B) 3
C) 5
D) 6

4 If $\frac{x+3}{4} = g$ and $g = 5$, what is the value of x?

A) 20
B) 18
C) 17
D) 6

5 $8(x - 1) = x(a + 3) - 8$

In the equation shown above, a is a constant. For what value of a does the equation have infinitely many solutions?

A) 0
B) 1
C) 4
D) 5

Student Produced Response – No Calculator

1 $-3(2x - 2.5) = -5(3x - 1.5)$

What is the solution to the equation above?

2 If $5c - 12 \geq 8$, what is the least possible value of c?

3 If $3x + 7 = 22$, what is the value of $6x - 8$?

4 If $\frac{5}{8}x - \frac{3}{8}x = \frac{3}{4} + \frac{1}{2}$, what is the value of x?

5 $2(\frac{1}{2} - p) = \frac{3}{4} + 8p$

What is the solution to the equation above?

6 $-3(4x - 2.8) = -5(4x - 2.8)$

What is the solution to the equation above?

Multiple Choice – Calculator

1 If $2b - 4 > 6$, which of the following best describes all possible values of b?

A) any value less than $\frac{1}{2}$
B) any value greater than 1
C) any value less than $\frac{3}{2}$
D) any value greater than 5

2 If $12 + 3x$ is 10 more than 17, what is the value of $4x$?

A) 3
B) 5
C) 20
D) 40

3 If $\frac{1}{2}g = \frac{3}{5}$, what is the value of g?

A) $\frac{6}{5}$

B) $\frac{4}{3}$

C) $\frac{3}{5}$

D) $\frac{5}{6}$

4 If $\frac{1}{4}a > -\frac{5}{8}$, which of the following best describes the possible range of values of a?

A) $a > -\frac{5}{2}$

B) $a > -\frac{5}{32}$

C) $a < -\frac{5}{32}$

D) $a < -\frac{5}{2}$

5 If $6 + 9x$ is 20 more than 4, what is the value of $5x$?

A) 2
B) 10
C) 18
D) 22

6 If $7x + 3 = 38$, what is the value of $4.5x$?

A) 5
B) 17.5
C) 22.5
D) 28

7 If $\frac{x+2}{x+1} = 5$, what is the value of x?

A) -3

B) $-\frac{3}{4}$

C) $\frac{3}{4}$

D) 3

8 $x + 2x + 3 = 3(-5x + 4) - 6$

What is the value of x?

A) $-\frac{1}{6}$

B) $\frac{1}{6}$

C) 1
D) 6

9 Which of the following numbers is NOT a solution of the inequality $2x + 3 \geq 3x - 2$?

A) -6
B) -4
C) 4
D) 6

10 $2 + 8x - 3 = (a + 1) \times -1$

In the equation above, a is a constant. For what value of a does the equation have infinitely many solutions?

A) 2
B) 5
C) 7
D) 12

11 If $2a - 3 \geq 1$, what is the least possible value of $2a + 3$?

A) 2
B) 3
C) 7
D) 9

12 Let x and y be numbers such that $-y < x < y$. Which of the following must be true?

I. $x > 0$
II. $x > y$
III. $|x| < y$

A) I only
B) II only
C) III only
D) I and III only

13 If $4 > -2x - 5$, which inequality represents the possible range of values of $-10x - 25$?

A) $-10x - 25 > -0.5$
B) $-10x - 25 < -0.5$
C) $-10x - 25 < 20$
D) $-10x - 25 > 20$

14 $2(3x + 2) < 3(2x - 1)$
Which of the following best describes the solutions to the above inequality?

A) All real numbers

B) $x < -\frac{7}{6}$

C) $x > \frac{7}{12}$

D) No solution

15 $-x + 5 + 12x > -8 + 11x + 2$
In the equation shown above, x is a constant. For what value of x does the inequality have infinite solutions?

A) All real numbers

B) $x > -\frac{1}{2}$

C) $x > 5$

D) No solution

16 The sum of three numbers is 670. One of the numbers, x, is 50% more than the sum of the other two numbers. What is the value of x?

A) 335

B) 402

C) 432

D) 455

17 If x is the average (arithmetic mean) of m and 8, y is the average of $2m$ and 12, and z is the average of $3m$ and 16, what is the average of x, y, and z in terms of m?

A) $m + 6$

B) $m + 12$

C) $2m + 9$

D) $3m + 18$

Guided Practice – Solving Systems of Equations

(Answers & explanations begin on page 390).

Multiple Choice – No Calculator

1 $3x+2y=2$
$2y-x=-6$
What is the solution (x,y) to the system of equations above?
A) (2,–2)
B) (2, 2)
C) (2,–4)
D) (2, 4)

2 $\frac{x}{y}=4$
$3(y+1)=x$
If (x,y) is the solution to the system of equations above, what is the value of y?
A) 1
B) 3
C) 9
D) 12

3 $x+y=3$
$3x+2y=12$
Which of the following ordered pairs (x,y) satisfies the system of equations above?
A) (–2,–2)
B) (3, –3)
C) (6, –3)
D) (3, 6)

4 $2x+3y=-6$
$3x-2y=17$
If (x,y) is a solution to the system of equations above, what is the value of $x-y$?
A) 11
B) 9
C) 7
D) 5

5 $3x-y=y+8$
$x+2y=8$
Based on the system of equations above, what is the value of the product of xy?
A) 2
B) 6
C) 8
D) 16

Multiple Choice – Calculator

1 $x=2y$
$y=2x$
How many solutions does the system of equations have?
A) 0
B) 1
C) 2
D) infinitely many

2 $3x+2y=-2$
$6x+ay=-6$
In the system of equations above, for which value of a does the system have no solutions?
A) 2
B) 3
C) 4
D) 6

3 $2x + b = y$
$2y + b = x$
In the equations above, b is a constant. How many solutions does this system have?
A) 0
B) 1
C) 2
D) infinitely many

4 $2x + b = 3x - 5$
$2y + c = 3y - 5$
In the equations above, b and c are constants. If b is equal to c plus $\frac{1}{2}$, which of the following is true?
A) x is y plus $\frac{1}{6}$.
B) x is y plus $\frac{1}{2}$.
C) x is y plus 1.
D) x is y minus $\frac{1}{2}$.

5 $-3x = 6y + 12$
$2(2y + 3) = 3x - 2$
What is the solution (x,y) to the system of equations above?
A) (0,–2)
B) (–2,0)
C) (0, 2)
D) (2,–2)

6 $3x + 5y = -2$
$6x - ay = -4$
In the system of equations above, a is a constant. For what value of a does the equation have infinite solutions?
A) –10
B) –5
C) 5
D) 10

7 $-10x = -5y + 15$
$4y + 3 = 12x - 5$
How many solutions does the system of equations have?
A) 0
B) 1
C) 2
D) infinitely many

8 $-2x = 6y + 8$
$3(ay + 5) = 4x - 3$
For which value of a does the system have no solutions?
A) –4
B) –3
C) $-\frac{1}{3}$
D) 4

Student Produced Response – Calculator

1 $ax + by = 10$
$3x + 4y = 20$
In the system of equations above, a and b are constants. If the system has infinitely many solutions, what is the value of $\frac{a}{b}$?

2 $y \le -15x + 54$
$y \le 3x$
On the xy-plane, if a point with coordinates (a,b) lies in the solution set of the system of inequalities above, what is the maximum possible value of b?

3 $3x + 2y = -4$
$2(x - 4) = 3y$
How many solutions does the above system of equations have?

Guided Practice – Linear Equations, Inequalities, & Systems in Word Problems

(Answers & explanations begin on page 392).

Multiple Choice – No Calculator

1 Jack and Amy work at an electronics store, and one of their duties is to email customers when the items they've ordered are ready to be picked up in the store. One afternoon, Jack sent m email messages each hour for 3 hours. Amy sent a email messages each hour over 4 hours. Which of the following represents the total number of emails sent by Jack and Amy that afternoon?

A) $7ma$
B) $12ma$
C) $3m + 4a$
D) $4m + 3a$

2 $w = 4.5a + 2$

A veterinarian uses the model above to estimate the weight, w, of a kitten, in ounces, in terms of the kitten's age, a, in weeks, between when the kitten is 8 and 16 weeks old. Based on the model, what is the estimated increase, in ounces, in the kitten's weight each week?

A) 38
B) 36
C) 6.5
D) 4.5

3 A restaurant has 15 tables that can sit a total of 86 people. Some tables seat 4 people and others seat 6 people. How many tables sit 6 people?

A) 2
B) 7
C) 8
D) 13

4 $a = 1.0 + 0.2x$

$s = 0.4 + 0.4x$

In the equations above, a and s represent the price per pound, in dollars, of aluminum and steel, respectively, at a scrap yard, plus an additional surcharge, x weeks after delivery to the yard. What was the price per pound of aluminum when it was the same as the price per pound of steel?

A) \$1.20
B) \$1.60
C) \$2.00
D) \$5.00

5 While training for a swim meet, Toby created a training schedule in which the distance he swims each week is increased by a constant amount. If Toby's training schedule requires that he swim 400 meters in week 4 and 1,000 meters in week 16, which of the following best describes how the distance Toby swims every week changes between week 4 and week 16 of his swimming schedule?

A) Toby increases his swim distance by 5 meters every week.
B) Toby increases his swim distance by 100 meters every week.
C) Toby increases his swim distance by 100 meters every 2 weeks.
D) Toby increases his swim distance by 50 meters every 2 weeks.

6 A charity organization is recruiting volunteers for a fundraiser. Each volunteer can sign up to set up tables before the event or hand out flyers during the event. A volunteer can set up 6 tables per hour or hand out 20 flyers per hour. There are 180 tables and 300 flyers. If the volunteers each work a 3-hour shift, how many volunteers does the organization need?

A) 5
B) 10
C) 15
D) 20

7 $d = 145 + 0.1x$

$p = 75 + 0.2x$

In the equations above, d and p represent the price, in dollars, of down coats and pea coats, respectively, x days after Labor Day last year. What was the price of a down coat when it was equal to the price of a pea coat?

A) \$145
B) \$152
C) \$205
D) \$215

8 Jane bought 4 paperback and 2 hardcover books at a used book store and spent a total of $36. Her friend, Sarah, bought 3 paperbacks and 5 hardcover books and spent a total of $69. What would be the total cost, in dollars, of 1 paperback and 1 hardcover?

A) 2
B) 5
C) 9
D) 15

9 Dara works in a clothing store and earns a percent of the price of each item sold as a commission, in addition to her regular hourly salary. Her manager calculates her weekly paycheck by using the expression $14h + 0.10p$, where h is equal to the number of hours she works and p is equal to the total price of the items sold. Which of the following is the best interpretation of 0.10 in the expression?

A) Dara earns $0.10 for every item sold.
B) Dara's commission goes up $0.10 for each item she sells.
C) Dara's commission is equal to 10% of the total price of the items she sells.
D) Dara works 10 hours every week.

10 The twelfth grade class is holding a bake sale for student activities. Cookies cost $2 each and brownies cost $3 each. The class is hoping to earn at least $500. Students have provided a total of 400 cookies and brownies. Solving which of the following systems of equations yields the number of cookies c and brownies b the twelfth grade class needs to sell to make its goal?

A) $b + c < 400$
$3c + 2b \geq 500$

B) $b + c < 400$
$3c + 2b \leq 500$

C) $b + c \leq 400$
$2c + 3b \geq 500$

D) $b + c < 400$
$\frac{c}{2} + \frac{b}{3} \geq 500$

11 $b = 6.5 + 0.5x$
$f = 4.5 + 0.7x$

A collector created a formula to represent roughly how much trading cards appreciate in value. In the equations above, b and f represent the price, in dollars, of baseball and football cards, respectively, x years after the cards were released. After how many years are baseball and football cards equal in value?

A) 5
B) 10
C) 50
D) 100

12 At a sporting goods store, baseballs cost $3 each and tennis balls cost $2 each. A physical education teacher has a $150 budget and wants to buy at least 50 baseballs and tennis balls in total. Solving which system of inequalities will help him determine how many baseballs, b, and tennis balls, t, to buy?

A) $b + t \leq 50$
$3b + 2t \geq 150$

B) $b + t \geq 50$
$3b + 2t \leq 150$

C) $b + t \geq 50$
$3b + 2t \geq 150$

D) $b + t \leq 50$
$3b + 2t \leq 150$

13 A bakery sells muffins in boxes of 4 and cupcakes in boxes of 6. At the end of the day, the baker does an inventory and sees that they have sold 30 boxes and a total of 140 cupcakes and muffins. How many boxes of muffins did the bakery sell?

A) 10
B) 15
C) 20
D) 25

14 Gina sells custom-made tee-shirts at a craft fair. She sold twice as many shirts on Saturday as she did Friday. If she sold 26 shirts on Friday and x is equal to the number of shirts she sold Saturday, which of the following equations is true?

A) $26x = 2$

B) $2x = 26$

C) $\frac{x}{2} = 26$

D) $x + 26 = 2$

Student Produced Response – No Calculator

1 $a = 10 + 0.1m$

$b = 5.5 + 0.25m$

In the equations above, a and b represent different Wi-Fi plan options at a hotel, with a given base price, in dollars, and an additional surcharge per minute. After how many minutes will both plans have the same price?

2 A butcher shop sells chicken for $3 per pound and pork for $4 per pound. If the shop sold a total of 15 pounds of meat one day and made $50, how many pounds of chicken did it sell?

3 The temperature of the ocean decreases at a constant rate as the depth increases. In one part of the ocean, the temperature at the surface is 22°C and the temperature at 500 meters below the surface is 14°C. For every additional 100 meters, the temperature decreases by k°C, where k is a constant. What is the value of k?

4 $t = 2.6 - 0.5x$

$l = 1.8 - 0.3x$

In the equations above, t and l represent the price per square foot of tile and linoleum flooring, respectively, at a home improvement store, x weeks after delivery to the store. What was the price per square foot of tile, in dollars, when it was the same as the price per square foot of linoleum?

5 Tamara owns a gym and wants to start offering yoga classes. Each yoga class is 1.5 hours long. Instructors are paid $40 per hour, but Tamara doesn't want to spend more than $1,000 on instructors. What is the maximum number of yoga classes Tamara can offer per week?

Multiple Choice – Calculator

1 A magazine has both print and online subscriptions available. A print subscription costs $12.50 per month and an online subscription costs $9.99 per month. Which of the following expressions represents the amount, in dollars, the magazine receives for subscriptions if there are x print subscriptions and y online subscriptions?

A) $12.50x + 9.99y$

B) $12.50x - 9.99y$

C) $9.99x + 12.5y$

D) $9.99x - 12.5y$

2 A premium cable channel has a monthly subscription fee of $9.98. Viewing programming on the channel is included in the membership fee, but there is an additional $2.50 fee to watch on-demand programming. For one month, Sam's bill was $17.48. How many on-demand programs did Sam watch that month?

A) 1

B) 2

C) 3

D) 4

3 Last week, Joanna worked 12 more hours than Rafael. If they worked a combined total of 64 hours, how many hours did Joanna work?

A) 52
B) 38
C) 26
D) 12

4 Admission at a theme park is \$12.50 for adults and \$6.75 for students. A twelfth grade class went to the theme park. There were 209 adults and students on the trip in all, and the total admission fee was \$2,118. Solving which of the following systems of equations yields the number of adults, x, and the number of students, y, who went to the theme park?

A) $x + y = 2{,}118$
$12.5x + 6.75y = 209$

B) $x + y = 209$
$6.75x + 12.5y = 2{,}118$

C) $x + y = 209$
$12.5x + 6.75y = 2{,}118$

D) $x + y = 209$
$12.5x + 6.75y = 2{,}118 \times 2$

5 A small library has a maximum capacity of 15,000 books. When the shelves are full, librarians remove old books to make space for the new ones. On January 1st, the library had 13,500 books. Each month, they receive an additional 95 books. If y represents the time, in months, after January, which of the following inequalities describes the set of months where the library is at or above capacity?

A) $15{,}000 - 95 \le y$
B) $15{,}000 \le 95y$
C) $15{,}000 - 13{,}500 \ge 95y$
D) $13{,}500 + 95y \ge 15{,}000$

6 A TV network sells ads in 30-second and 60-second time slots. A 30-minute TV program has 6 minutes of ads. The network airs programming for 18 hours a day. The network sold a total of 325 slots one day and filled every available ad minute. Solving which of the following systems of equations yields the number of 30-second ads, x, and the number of 60 second ads, y, that aired in one 18-hour period?

A) $x + y = 325$
$0.5x + y = 216$

B) $x + y = 325$
$0.5x + y = 1{,}080$

C) $x + y = 216$
$0.5x + y = 325$

D) $x + y = 325$
$0.5x + 6 = 108$

7 A cereal factory ships boxes of cereal that are 8 ounces or 14 ounces. Let x be the number of 8-ounce boxes and y be the number of 14-ounce boxes. The cereal ships in cases that can carry at most 12 boxes and the total weight cannot exceed 96 ounces. Which of the following systems of inequalities represents this relationship?

A) $8x + 14y \le 96$
$x + y \le 12$

B) $\frac{x}{8} + \frac{y}{14} \le 96$
$x + y \le 12$

C) $8x + 14y \le 12$
$x + y \le 96$

D) $x + y \le 96$
$8x + 14 \le 96$

8 At a sandwich shop, a foot-long sandwich, f, costs \$2.50 more than a six-inch sandwich, s. One family orders 2 foot-long sandwiches and 4 six-inch sandwiches for a total of \$37.70. Which system of equations could you use to find the cost of a foot-long sandwich?

A) $f = 2.5s$
$2f + 4s = 37.7$

B) $f = s + 2.5$
$2f + 4s = 37.7$

C) $f = s + 2.5$
$2f - 4s = 37.7$

D) $f = 2.5s$
$4f - 2s = 37.7$

9 A plane has a maximum fuel capacity of 5,300 gallons. The airline requires that the pilot fly no more than 400 minutes before refueling. A one-way flight to a certain destination requires 2,000 gallons of fuel and 190 minutes of flight time. How many round trips can the pilot make before having to refuel?

A) 1
B) 2
C) 3
D) 4

10 Certain shipping crates can hold either 50 pounds or 80 pounds each. Let x be the number of 50-pound crates and y be the number of 80-pound crates. A truck can hold either 30 boxes or a weight of 2,000 pounds. Which of the following systems of inequalities represents this relationship?

A) $x + y \le 2{,}000$
$50x + 80y \le 2{,}000$

B) $\frac{x}{50} + \frac{y}{80} \le 2{,}000$
$x + y \le 30$

C) $50x + 80y \le 30$
$x + y \le 2{,}000$

D) $50x + 80y \le 2{,}000$
$x + y \le 30$

11 At a pottery store, each cup costs \$6.50 less than each plate. If Maggie spends a total of \$55.50 for 5 cups and 3 plates, what is the price of each cup?

A) \$3.50
B) \$4.50
C) \$6.50
D) \$11.00

12 $d = \frac{1}{2}p + 65$
$s = 200 - p$

In the equations above, d is equal to the quantity of a product displayed for sale at a store at p, the price in dollars and s is equal to the quantity of the product that the store sold. At what price will the quantity of the product displayed be equal to the quantity of the product sold?

A) \$90
B) \$100
C) \$115
D) \$135

13 In a trivia game, each player starts with k points and loses 5 points each time a question is answered incorrectly. If a player gains no additional points but answers 30 questions incorrectly has a score of 50 points, what is the value if k?

A) 0
B) 100
C) 200
D) 300

14 Every morning, Sharon bikes at a constant speed of 9 miles per hour and then walks at 3 miles per hour to get to her job. Her goal is to travel the 6 miles to work in less than 1 hour. If Sharon bikes b miles and walks w miles, which of the following systems represents Sharon's goal?

A) $\frac{b}{9}+\frac{w}{3}<1$
$b+w=6$

B) $\frac{b}{9}+\frac{w}{3}\geq 1$
$b+w<6$

C) $9b+3w\geq 6$
$b+w=1$

D) $9b+3w<1$
$b+w\geq 6$

15 It takes Michaela 90 minutes and 3 sheets of paper to complete a writing assignment. It takes her 20 minutes and 1 sheet of paper to complete a math assignment. Michaela is required to spend more than 300 minutes to complete assignments and she can use as many as 20 sheets of paper. If w represents the number of writing assignments and m is the number of math assignments, which system of inequalities represents Michaela's homework?

A) $90w+20m\leq 300$
$3w+m\geq 20$

B) $90w+20m>300$
$3w+m\leq 20$

C) $\frac{90}{w}+\frac{20}{m}>300$
$3w+m<20$

D) $90w+20m>20$
$3w+m\leq 300$

16 $s=5.5h+\frac{t}{6}$
$t=0.15d$

The first equation above shows the earnings of a server at a certain restaurant for a night's work, where s is equal to the total earnings, in dollars, h is equal to the number of hours worked, and t is equal to the total amount of tips all servers earned, in dollars. Tips are, on average, 15% of the total dinner sales in dollars, d. If the restaurant made \$785 in dinner sales one night, how much did one server make during an 8 hour shift, rounded to the nearest cent?

A) \$19.73
B) \$44.00
C) \$47.72
D) \$63.63

17 A school is building a rectangular playground. Due to space constraints, the length, x, of the playground must be at least 20 feet longer than the width, y, and the perimeter will not be greater than 2,000 feet. Solving which of the following systems of inequalities yields the length and width of the playground?

A) $x-y\geq 20$
$2x+2y\leq 2{,}000$

B) $x-y\leq 20$
$2x+2y\geq 2{,}000$

C) $x-y\geq 20$
$2x+2y\geq 2{,}000$

D) $x-y\leq 20$
$2x+2y\leq 2{,}000$

18 A movie theater sells tickets for adults and children. One customer buys 4 adult tickets and 8 child tickets; the cost is \$116.00. Another customer buys 6 adult tickets and 2 child tickets and spends \$89.00. How much does an adult ticket cost?

A) \$8.50
B) \$10.00
C) \$12.00
D) \$14.50

19 Concert tickets cost \$12 for students and \$15 for adults. If Jason spends at least \$90 but no more than \$105 on x student tickets and 2 adult tickets, which is a possible value of x?

A) 3
B) 5
C) 7
D) 8

20 A comic book store sells new superhero comics for \$3.50 each and new graphic novels for \$12.50 each. If one day, the store sold 40 more superhero comics than graphic novels and made \$700 in total sales, how many superhero comics did the store sell?

A) 35
B) 40
C) 75
D) 100

21 A copy shop charges \$1.00 per visit to use its copy machine and an additional fee of \$0.10 per page. Which of the following represents the total charge, t, in dollars, to use this copy machine to make n copies in one visit?

A) $t = 1.10n$
B) $t = 1.00 + 10n$
C) $t = 1.00 + 0.10n$
D) $t = 1.10 + n$

22 Jake is driving cross country. Gas costs \$2.19 per gallon at the gas station chain that he prefers. On average, he can drive 23 miles per gallon of gas. He plans to drive 400 miles each day. Which of the following expressions represents the total cost, in dollars, of the gas needed for traveling x days?

A) $2.19x \div (400 \div 23)$
B) $2.19(x \cdot 400 \cdot 23)$
C) $2.19[(400 \div 23)x]$
D) $2.19(400 \div 23) \div x$

23 Courtney is measuring the height of geologic features from sea level, which has an altitude of 0 feet. The lowest elevation Courtney measures has an altitude of x feet. One feature is 4 times x plus 10 feet high, which Courtney calculates as being 6 feet above sea level. What is the height of another feature that is 2 times x plus 5 feet?

A) –4
B) –1
C) 3
D) 4

24 Mr. Lopez is redoing the front walkway at his house and deciding where to buy materials and rent tools for the project. The table below shows the materials' cost and daily rental costs for three different stores.

Store	Cost of Materials M (\$)	Rental cost of wheelbarrow W (\$ per day)	Paver saw (\$ per day)
A	500	25	70
B	450	20	80
C	550	15	85

The total cost, y, for buying materials and renting the tools in terms of the number of days, x, is given by $y = M + (W + K)x$. For what number of days, x, will the total cost of buying the materials and renting the tools from Store B be less than or equal to the total cost of buying the materials and renting the tools from Store A?

A) $x \leq 5$
B) $x \geq 5$
C) $x \leq 10$
D) $x \geq 10$

25 A machine packs 18 boxes of cereal into a carton. It fills c cartons in 1 hour. Which equation shows how many cartons the machine fills in 5 minutes?

A) $\frac{18c \cdot 5}{60}$

B) $5c \cdot 60$

C) $\frac{5c}{60}$

D) $\frac{5c \cdot 60}{18}$

26 John brings a water cooler holding n fluid ounces of water to distribute to all of the athletes on the football team. If he gives each player 8 fluid ounces of water, he will have 76 fluid ounces of water left over. In order to give each player 10 fluid ounces of water, he will need an additional 30 ounces. How many players are there?

A) 10
B) 41
C) 53
D) 106

Student Produced Response – Calculator

1 A geologist in Texas has calculated that the average erosion rate for the Gulf Coast is 2.4 feet per year. According to the geologist's estimate, how long will it take, in years, for the Texas Gulf Coast to erode by 36 feet?

2 A florist sells a bouquet of roses for $25 and a bouquet of daisies for $15. If one day, the florist sold 30 more bouquets of roses than daisies and made a total of $1,350, how many bouquets of roses did the store sell?

3 A landscaping company placed two orders with a nursery. The first order was for 13 bushes and 4 trees and totaled $485. The second order was for 6 bushes and 2 trees and totaled $230. The bills do not list a per-item price. What would be the total cost of 1 bush and 1 tree?

4 Geoff and Damon took turns driving their car across the country. The trip took a total of 55 hours. Geoff drove 15 hours more than Damon did. If Geoff's average speed was 55 miles per hour, what was the total distance he drove?

5 Pete bought a pair of pants and a backpack at a department store. The sum of the prices before sales tax was $78.00. There was no sales tax on the pants and a 6% sales tax on the backpack. The total Pete paid, including the sales tax, was $80.40. what was the price, in dollars, of the pants?

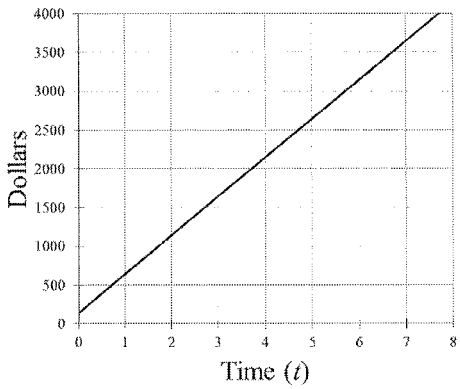
Guided Practice – Linear Equations, Inequalities, & Systems on the Coordinate Grid

(Answers & explanations begin on page 396).

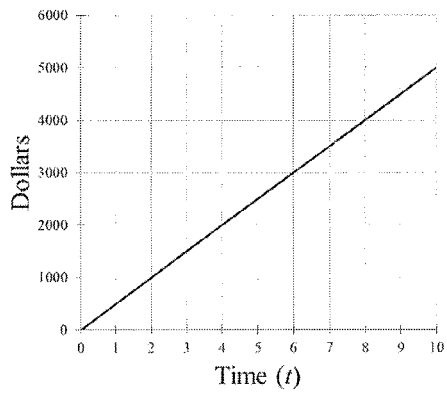
Multiple Choice – No Calculator

1 Jane opened a savings account with \$500. Each month, she deposits \$150. Which graph represents v, the value of the savings account, as a function of t, the number of months since the account was opened?

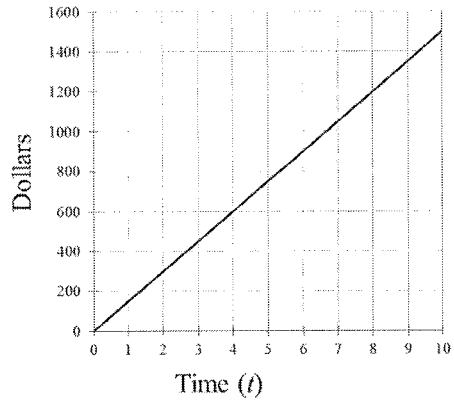
A)



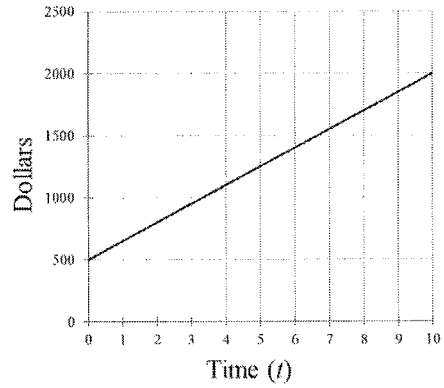
B)



C)



D)



2 Jake works as a computer specialist at an electronics store. Each week, he receives a certain number of computers that need to be repaired. The number of computers he has left to fix at the end of each day can be estimate with the equation $C = 95 - 12d$, where C is the number of computers left and d is the number of days he has worked that week. What is the meaning of the value 95 in the equation?

A) Jake repairs computers at a rate of 95 per day.
B) Jake repairs computers at a rate of 95 per hour.
C) Jake starts each week with 95 computers to fix.
D) Jake will complete the repairs within 95 days.

3 $2x + 5y = -2$

$3y - x = 12$

What is the solution (x,y) to the system of equations above?

A) (6,2)
B) (2,6)
C) (–6,2)
D) (6,–2)

4 Which of the following equations represents a line that is parallel to the line with equation $y = 4x + 3$?

A) $-4x + y = 12$
B) $4x + 2y = 8$
C) $2x + y = 6$
D) $-2x + 2y = 3$

5 $y = ax + 1$

$ax - y = 6$

In the system of equations above, a is a constant. The system has x-intercepts at (1,0) and (–6,0), respectively. If the system has no solutions, what is the value of a?

A) –6
B) –1
C) 0
D) 1

6 A line on the xy-plane passes through the origin and has a slope of $\frac{1}{5}$. Which of the following points lies on the line?

A) (1,5)
B) (5,1)
C) (0,5)
D) (5,5)

7 A sports equipment store rents out pairs of roller skates. They calculate the cost of the rental using the function $c = 7h + b$, where h is the number of hours a pair of roller skates can be rented for, b is a flat charge that varies based on the type of roller skate model, and c is the total cost of the rental. If Sue paid $26 to rent a pair of roller skates for 3 hours, what was the surcharge for the pair?

A) 2
B) 3
C) 5
D) 7

8 Which of the following equations represents a line that is perpendicular to the line with equation $y = -6x + 1$?

A) $-6x + y = 3$
B) $12x + 2y = 4$
C) $x + 6y = 3$
D) $x - 6y = 18$

Student Produced Response – No Calculator

1 A hotel charges guests for the use of its Wi-Fi. To calculate the cost for each guest, the hotel uses the function $c = ah + 10$, where the cost c is a function of a, the hourly rate (in dollars per hour), and h, the number of hours, plus a $10 flat fee. A guest pays $26 for 8 hours of Wi-Fi use. What was the hourly rate, in dollars per hour?

2 The graph of a line on the xy-plane has slope 3 and contains the point (1,9). The graph of a second line passes through the points (2,3) and (4,1). If the two lines intersect at the point (a,b), what is the value of $a + b$?

Multiple Choice – Calculator

1 If the relationship between the total cost, y, of renting ski equipment from Store A and the number of days, x, for which the equipment is rented is graphed on the xy-plane, what does the slope of the line represent?

A) The total cost of the rental.
B) The number of items rented.
C) The daily rental fee.
D) The total cost of each item rented.

2 $y = ax - 4$

$bx - y = 4$

In the system of equations above, a and b are both constants, and the system has infinite solutions. If the value of b is 4, what is the value of a?

A) –4
B) 4
C) 6
D) 8

3 $y = 2x - 3$

$x - y = 3$

A system of two equations is shown above. How many solutions does the system have?

A) Zero
B) One
C) Two
D) Infinite

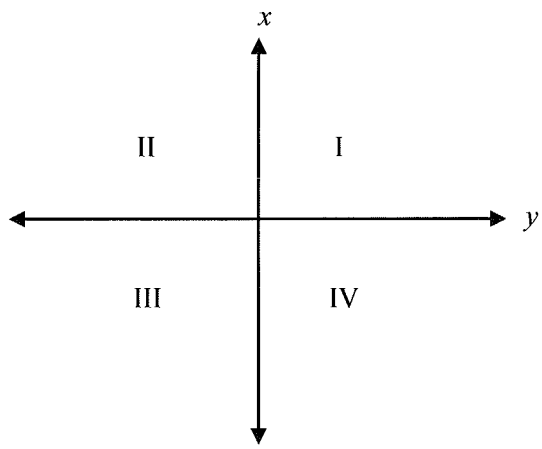
4 $y = 3x - 2$

$3x - y = 5$

A system of two equations is shown above. How many solutions does the system have?

A) Zero
B) One
C) Two
D) Infinite

5



If the system of inequalities $y \geq -3x + 1$ and $y > -\frac{1}{3}x + 1$ is graphed on the xy-plane above, which quadrant contains no solutions to the system?

A) Quadrant II
B) Quadrant III
C) Quadrant IV
D) There are solutions in all four quadrants.

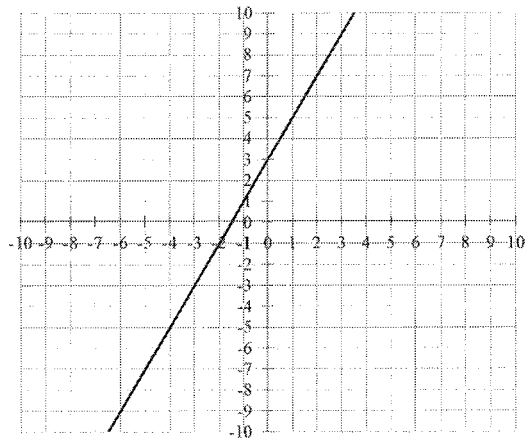
6 Line l on the xy-plane contains points from each of Quadrants I, II, and III, but no points from Quadrant IV. Which of the following must be true?

A) The slope of the line l is undefined.
B) The slope of the line l is zero.
C) The slope of the line l is positive.
D) The slope of the like l is negative.

7 Joanne drew a map to show Maple Street and Elm Street in a coordinate plane. She graphed Maple Street using the equation $y = 2x - 4$ and Elm Street using the equation $y = -3x + 6$. At what point on the xy-plane will the two streets meet?

A) (0,–2)
B) (–2,0)
C) (0,2)
D) (2,0)

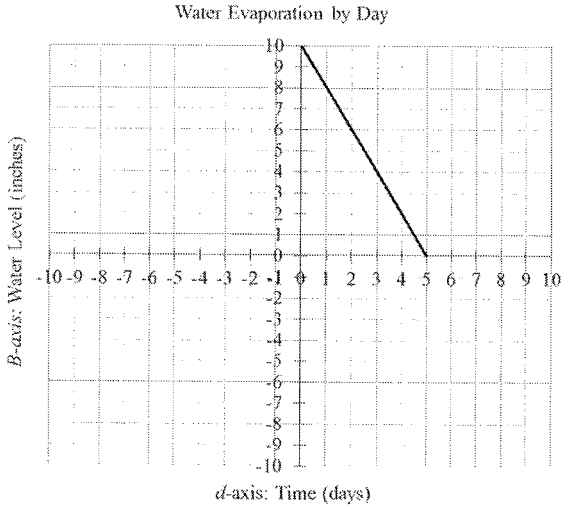
8



The graph above shows a line on the xy-plane. Which of these is the equation of a perpendicular line?

A) $2x - y = 3$
B) $y = -2x + 3$
C) $2y + x = 6$
D) $-2y + x = 4$

The next two questions refer to the below information:



The graph above displays the amount of water left in a bucket, B, after the water has been left to evaporate in the sun for d days.

9 What does the B-intercept represent in the graph?

A) The amount of water in the bucket on the first day.
B) The total amount of water that evaporated.
C) The total number of days it took for all the water to evaporate.
D) The amount of water that evaporated on the first day.

10 Which of the following represents the relationship between d and B?

A) $B = 2d$
B) $B = 2d + 10$
C) $B = -2d + 10$
D) $B = -2d$

Student Produced Response – Calculator

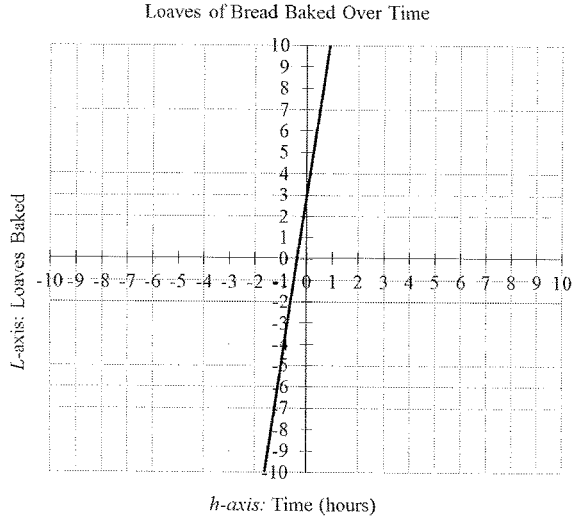
1 $y = -bx + 5$

$bx - y = 3$

In the system of equations above, b is a constant. On a coordinate grid, the graphs of the equations intersect at point (1,1). What is the value of b?

The next two questions refer to the below information:

A bakery tracked how many loaves of bread they baked per hour over h hours and recorded the data in the following graph:



11 What does the L-intercept represent in the graph?

A) The total number of loaves baked.
B) The total number of hours when bread is baked.
C) The increase in the number of loaves baked per hour.
D) The number of loaves the bakery had when they started tracking.

12 Which of the following represents the relationship between h and L?

A) $L = 8h$
B) $L = 8h + 3$
C) $L = \frac{3}{8}h + 3$
D) $L = -8h + 3$

2 Johnson's Paint Company estimates the price of painting a house in dollars, using the expression $30 + 12nh$, in which there is a flat fee of \$30, an hourly rate of \$12, n is equal to the number of painters, and h is equal to the number of hours. Clark's Paint Company estimates a similar job using the expression $42 + 10nh$. Dawn wants to hire one of the companies to paint her house using 3 painters. How many hours will it take for the jobs to be the same price from both companies?

Guided Practice – Absolute Value

(Answers & explanations begin on page 398).

Multiple Choice – No Calculator

1 If $|x-4|+3=7$, which of the following is a possible value of x?

A) 0
B) 3
C) 4
D) 7

2 Which of the following expressions is equal to 0 for some value of x?

A) $|x-2|-2$
B) $|2-x|+2$
C) $|x+2|+2$
D) $|x-2|+2$

3 Solve $|5x+4|+10=2$.

A) –8
B) 4/5
C) 2
D) There is no solution.

4 If $|2-4x|>10$, which of the following is a possible value of x?

A) –4
B) –1
C) 2
D) 3

5 The requirements to join a dance group state that dancers must be within 4 inches of 70 inches tall. Which of the following inequalities can be used to assess which dancers are the correct heights?

A) $|h-4|\le 70$
B) $|h-4|\ge 70$
C) $|h-70|\le 4$
D) $|h-70|\ge 4$

6 If $f(x)=|5-4x|$, then $f(2)$ =

A) $f(-2)$
B) $f(-1)$
C) $f(4)$
D) $f(\frac{1}{2})$

7 If a is a negative number and b is a positive number, what is the absolute value of a plus the absolute value of b?

A) $-a+b$
B) $-(a+b)$
C) $-a-b$
D) $a+b$

Student Produced Response – No Calculator

1 $|x-3|-7=-5$

What is the least value of x that satisfies the equation above?

2 $|8-k|=5$

$|k-2|=11$

What is the value of k that satisfies both equations?

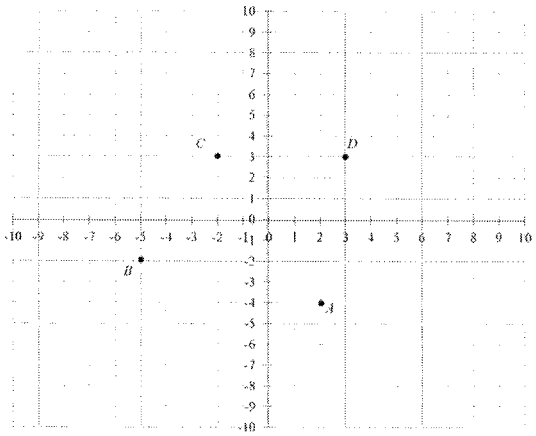
3 A band releases a new single. Weekly sales s (in thousands) increase steadily for a while and then decrease as given by the function $s=-2|t-20|+40$ where t is the time in weeks. What was the maximum number of singles sold in week number one?

Multiple Choice – Calculator

1 For what value of n is $|n-2|+2$ equal to 0?

A) 0
B) 2
C) 4
D) There is no such value of n.

2 Which of the lettered points in the figure below has coordinates (x,y) such that $|x|-|y|=3$?



A) A
B) B
C) C
D) D

3 A carpenter is buying a metal rod for a project. The ideal diameter is 2.5 inches with an allowable error of at most 0.05 inches. He measures all of the rods available at the store. Which size, in inches, should he buy?

A) 2.4418
B) 2.4671
C) 2.5512
D) 2.6518

4 Let x and y be numbers such that $-y < x < y$. Which of the following must be true?

I. $x > 0$
II. $y > 0$
III. $|x| < y$

A) I only
B) III only
C) II and III only
D) I, II, and III

5 The ideal diameter of a gear for a certain type of clock is 12.25 mm. An actual diameter can vary by as much as 0.06 mm. Which of the following shows the range of possible diameters for the clock gear?

A) $|x-12.25| \le 0.06$
B) $|x+12.25| \le 0.06$
C) $|0.06+x| \le 12.25$
D) $|x-0.06| \ge 12.25$

6 Simplify $-|3^2-10|+|-(3)^2-2|$

A) 9
B) 10
C) 11
D) 12

7 The temperature on Mars roughly satisfies the inequality $|t-75| \le 145$ where t is the temperature in Fahrenheit. Which of the following shows the range of temperatures on Mars?

A) $-220 \le t \le 220$
B) $-70 \le t \le 145$
C) $-70 \le t \le 220$
D) $-220 \le t \le 70$

8 The average temperature in Minneapolis last week was 34°F. The temperature fluctuated between 7 degrees cooler or warmer. Which equation represents the high and low temperature, t, in Minneapolis last week?

A) $|t+7|=34$
B) $|t+34|=7$
C) $|t-7|=34$
D) $|34-t|=7$

9 If $|26x|=13$, the possible values for x are

A) only $\frac{1}{2}$ and $-\frac{1}{2}$.
B) all real numbers for which $-\frac{1}{2} \le x \le \frac{1}{2}$.
C) all real numbers except $\frac{1}{2}$ and $-\frac{1}{2}$.
D) all real numbers for which $-2 \le x \le 2$.

10 At a factory, a machine fills bags with coffee beans. After the bags are filled, another machine weighs them. If the bag's weight differs from 16 ounces by more than 0.5 ounces, the bag is rejected. Which equation could be used to find the heaviest and lightest acceptable weights?

A) $|x-0.5|=16$

B) $|x-16|=0.5$

C) $|x+0.5|=16$

D) $|x+16|=0.5$

11 Dominique is baking pastries. The oven is set to 180°C but vacillates 5°C during baking. Which of the following represents all the temperatures that the oven reaches during baking?

A) $|x-5|\leq 180$

B) $|x-180|\geq 5$

C) $|x-180|\leq 5$

D) $|x-5|\geq 180$

12 It usually takes Jane between 30 and 40 minutes to drive to visit her friend's house. The house is 25 miles away. Which of the following shows the range of speed, in miles per hour, that Jane likely travels to get to her friend's house?

A) $|x-30|\leq 10$

B) $|x-43.75|\leq 6.25$

C) $|x-50|\leq 12.5$

D) $|x+37.5|\leq 12.5$

13 If $x<0$, $|-x|=$

A) $-x$

B) x

C) 0

D) $-|x|$

Student Produced Response – Calculator

1 The average score on Mr. Rodriguez's chemistry final was a 72. Mr. Rodriguez calculated that the rest of the scores varied by ±15% of the average, which can be represented by the inequality $|x-72|\leq 10.8$. What score did the lowest-scoring student receive on the test, rounded to the nearest whole number?

2 A café owner hangs art on the walls that is also for sale. She sets the price for each painting at $110.00, but she has given instructions to her employees that the price can vary by as much as 15%. Employees use the inequality $|x-110|\leq 16.5$ to calculate the possible prices. What is the lowest possible price an employee could offer for a painting, in dollars?

3 Tom earned $100 last week. He has budgeted $30 to spend at the movies, and he wants to keep his spending within $5 of his budget. A ticket costs $7.50. A medium popcorn costs $3.75. A soft drink costs $2.25. His spending at the movies can be represented by the inequality $|s-30|\leq 5$, where s equals Tom's total spending. If he buys a medium popcorn and soft drink just for himself, and also buys movie tickets for himself and his friends, what is the maximum number of friends he can bring with him while staying within his preferred budget?

Mixed Practice – Algebra

(Answers & explanations begin on page 399).

Multiple Choice – No Calculator

1 If $3x = 27$, what is the value of $6x + 5$?

A) 9
B) 30
C) 54
D) 59

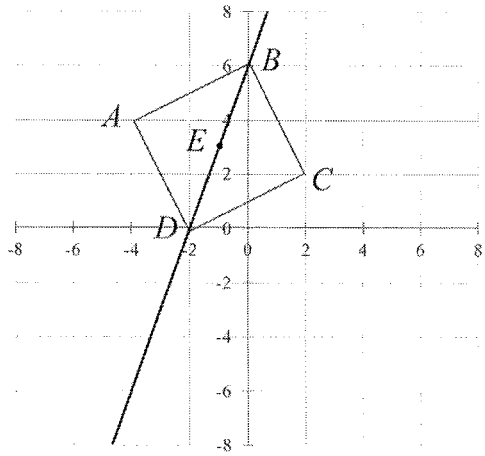
2 For a certain brand of snack bars, the peanut butter bars have 40 more calories than the dried fruit bars. If 3 peanut butter bars and 4 dried fruit bars have a total of 750 calories, how many calories does 1 peanut butter bar have?

A) 84
B) 90
C) 130
D) 360

3 Julio needs to score at least an 80 on his next math test in order to maintain his A average, which is an average of 90 or higher. His next test has a maximum point score of 100. Which of the following inequalities describes all the possible scores that Julio can get on his test and maintain his A average?

A) $|x - 80| \leq 100$
B) $|x - 100| \leq 80$
C) $|x - 90| \leq 10$
D) $|x - 10| \leq 90$

4



On the xy-plane above, $ABCD$ is a square and point E is the center of the square. The coordinates of points B and E are (0,6) and (–1,3) respectively. Which of the following is an equation of the line that passes through points A and C?

A) $y = 3x + 6$
B) $y = -3x - 6$
C) $y = \frac{1}{3}x + 2\frac{2}{3}$
D) $y = -\frac{1}{3}x + 2\frac{2}{3}$

5 $3x + b = 4x - 7$
$3y + c = 4y - 7$

In the equations above, b and c are constants. If b is c plus $\frac{1}{2}$, which of the following is true?

A) x is y plus $\frac{1}{4}$.
B) x is y plus $\frac{1}{2}$.
C) x is y plus 2.
D) x is y plus 4.

Student Produced Response – No Calculator

1 $ax - y = -3$
$y = bx - 5$
In the system of equations above, a and b are constants and $a \neq b$. The system has one solution at (1,–1). What is the value of b?

2 If $3a - 5 \geq 1$, what is the least possible value of $3a + 5$?

Multiple Choice – Calculator

1 When 3 times the number x is added to 9, the result is 3. What number results when 2 times x is added to 15?
A) –2
B) 5
C) 11
D) 14

2 Lisa and Stacey ate dinner out together. The price of Lisa's dinner was x dollars, and the price of Stacey's dinner was $3.00 more than the price of Lisa's dinner. If Lisa and Stacey split the cost of the meal evenly and each paid a 18% tip, which of the following expressions represents the amount, in dollars, each of them paid? (Assume there is no sales tax.)
A) $0.18x + 0.2$
B) $1.5x + 0.06$
C) $1.18x + 0.27$
D) $1.36x + 1.5$

3 Which of the following numbers is NOT a solution of the inequality $3b - 3 \geq 5b + 5$?
A) –6
B) –5
C) –4
D) –3

4 Sean wants to download songs and podcasts to his smart phone. Every song requires 3.5 megabytes of data, and every podcast requires 20 megabytes of data. Sean wants to download 25 songs and podcasts, but he only has 300 megabytes available on his phone. What is a possible number of podcasts Sean can download?
A) 12
B) 15
C) 17
D) 20

5 $|m - 4| = 5$
$|k + 5| = 10$
In the equations above $m < 0$ and $k < 0$. What is the value of $m - k$?
A) –16
B) –5
C) 4
D) 14

Student Produced Response – Calculator

1 $a = 26 + 0.3x$
$b = 18 + 0.5x$
In the equations above, a and b represent the price per square foot of Alex's and Berenice's houses, respectively, x months after they each purchased them. What was the price per square foot of Alex's house, in dollars, when it was the same as the price per square foot of Berenice's house?

Data Analysis

Guided Practice – Ratios & Proportions

(Answers & explanations begin on page 401).

Multiple Choice – Calculator

1. If $y = kx$, where k is a constant, and $y = 42$ when $x = 7$, what is y when $x = 9$?
   A) 49
   B) 54
   C) 63
   D) 72

2. A bakery sells cupcakes for $2.75 each, or a dozen for $21. How much cheaper is the unit price of the cupcakes sold by the dozen than one sold individually?
   A) $0.75
   B) $1.00
   C) $1.75
   D) $4.89

3. Comparing the growth of two dogs, Dora notices that Rover has gained 30% more weight than Spot. When Dora first observed both dogs, they both weighed 20 kg and Rover now weighs 46 kg. How much does Spot weigh now?
   A) 35.4 kg
   B) 40.0 kg
   C) 53.8 kg
   D) 59.8 kg

4. A rectangle is 20 meters long and 20 meters wide. If the length is increased by 10% and the width is decreased by 10%, what is the new area?
   A) 320
   B) 396
   C) 400
   D) 484

5. A rectangle is 50 feet long and 40 feet wide. The length is decreased by 20% and the width is increased by 20%. What is the percent decrease in the area, rounded to the nearest percent?
   A) 0%
   B) 1%
   C) 4%
   D) 10%

6. A bike messenger is paid $5 per delivery and can on average deliver one package every 15 minutes. If she worked five hours on Thursday and four hours on Friday, how much money did she earn in those two days if her rate of delivery was equal to the average?
   A) $36
   B) $300
   C) $180
   D) $144

The next two questions refer to the following information.

In a sales position, the salesperson's earnings are in direct proportion to the number of items sold. If the salesperson sells 40 items, he earns $1,500.

7. How much will the salesperson earn if he sells 180 items?
   A) $5,200
   B) $5,850
   C) $6,275
   D) $6,750

8. The salesperson uses 67% of his earnings for living expenses. The remainder is placed in a savings account. How much will the salesperson place in the savings account after selling 40 items?
   A) $495
   B) $1,005
   C) $1,320
   D) $2,680

9. On Wednesday of Spirit Week, a survey was done of the juniors wearing the school colors. Out of 650 students, 48% were wearing the school colors. How many juniors were *not* wearing the school colors that day?
   A) 312
   B) 338
   C) 360
   D) 384

10 The spacecraft New Horizons took 9.5 years to travel from Earth to Pluto, a distance of 4.67×10^9 miles. What was the approximate average speed for the trip in miles per hour?
A) 3.23×10^7
B) 1.35×10^6
C) 5.61×10^4
D) 2.24×10^4

11 The atomic weight of the element mercury, in atomic mass units (amu), is approximately 200 amu. The atomic weight of cerium is approximately 30% less than that of mercury. Which of the following best approximates the atomic weight, in amu, of cerium?
A) 60 amu
B) 140 amu
C) 194 amu
D) 158 amu

12 A high school senior reviewed 25% more pages in her science textbook on Thursday than on Wednesday. If the total number of pages she reviewed on Thursday was 40, how many pages did she review on Wednesday?
A) 15
B) 32
C) 50
D) 65

13 A smartphone that regularly costs $600 is on sale for 40% off. A customer used a coupon for an additional discount on the marked sale price at checkout. If the final price of the smartphone is $252, what additional discount was applied to the sale price?
A) 5%
B) 18%
C) 30%
D) 43%

14 The density of an object is equal to the mass of the object divided by the volume of the object. Aluminum has a density of 2.7 grams per cubic centimeter. What is the mass, in grams, of a sample of aluminum with a volume of 8 cubic centimeters?
A) 0.3375
B) 2.96
C) 10.7
D) 21.6

15 A quality assurance analyst chooses 7 of every 200 widgets manufactured to inspect for quality of the manufacturing process. To ensure the same percentage of widgets checked, how many widgets should the analyst inspect from 35,000 produced?
A) 775
B) 1,225
C) 1,550
D) 1,725

16 1 furlong = 660 feet
1 mile = 5,280 feet
A wheat farmer drives his tractor 3 miles in one morning. How many furlongs did he drive?
A) 27
B) 24
C) 18
D) 15

17 1 acre = 43,560 square feet
1 square yard = 9 square feet
A real estate developer bought 3.5 acres of land. For development purposes, he divides the land into square yards on a topographical map. How many square yards did he buy?
A) 1,372,140
B) 112,011
C) 16,940
D) 1,383

18 14 pounds = 1 stone
1 pound = 16 ounces
A British doctor calculates the average weight of an adult male as 11.9 stone. What is the average weight of an adult male, in ounces?

A) 2,665.6
B) 2,332.4
C) 41.9
D) 13.6

19 When Ms. Martinez negotiated her annual salary 10 years ago, it was agreed that she would receive a raise every year of 5% of her starting salary. Her annual salary this year is $72,000. What was her first year's salary?

A) $22,000
B) $43,100
C) $44,200
D) $48,000

20 1 ounce = 28.35 grams
1 pound = 16 ounces
A jeweler is buying gold at 20% below the current market value of $40 per gram. Approximately how much will he pay for a gold necklace that weighs 0.1 pounds?

A) $54.4
B) $363
C) $1,451
D) $1,814

21 Last year, a driving instructor taught 300 students, 240 of which passed their driving test on the first try. This year, the instructor taught 240 students, 144 of which passed their driving test on the first try. Which of the following best approximates the percentage of all of the instructor's students in the past two years who passed their driving test on the first try?

A) 60%
B) 70%
C) 71%
D) 80%

22 A certain family kept track of their annual spending last year. The following table shows their results.

Expenses	Percentage of Total Income
Rent	26
Food	10
Car Expenses	14
Clothing	3
Insurance	9
Entertainment	4
Taxes	34

If the family spent $1,920 on clothing last year, approximately how much was spent on rent, in thousands of dollars?

A) 6
B) 17
C) 58
D) 64

23 Light can travel approximately 300 million meters in one second. What is that speed, in kilometers per minute?

A) 5×10^{3}
B) 1.8×10^{7}
C) 5×10^{9}
D) 1.8×10^{13}

24 A car is moving at a speed of 30 miles per hour. Which expression has a value equal to the car's speed, in feet per minute? (*Note: There are 5,280 feet in 1 mile.*)

A) $\dfrac{30\times5{,}280}{60}$
B) $\dfrac{60\times5{,}280}{30}$
C) $\dfrac{30\times60}{5{,}280}$
D) $\dfrac{60}{30\times5{,}280}$

25 In a certain high school, the ratio of boys to the entire student population is $\frac{3}{7}$. If there are 1,200 girls in the school, how many boys are there?

A) 300
B) 400
C) 900
D) 1,100

26 The average car contains approximately 330 pounds of aluminum and 2,400 pounds of steel. If a car manufacturer uses 200 tons of aluminum in one day, approximately how many pounds of steel does it use?

A) 57,000
B) 850,000
C) 3,000,000
D) 1,000,000,000

27 A car factory manufactures 10 cars every 12 minutes. Assuming the factory never closes, how many cars can it make in 12 hours?

A) 10
B) 100
C) 600
D) 7,200

28 An architect is designing a gallery space for an artist using the Golden Ratio. He wants the ratio of the length to the height of the feature wall to equal 1.618. If the wall is 13 feet high, approximately how long will the wall be?

A) 8
B) 11
C) 15
D) 21

29 Madden filled her car's gas tank to the top and then drove her car at a constant spend along a highway for one hour, using 16% of the gas in the tank. If her gas tank is depleted at a constant rate, approximately how many more hours will she be able to drive?

A) 5.25
B) 6.25
C) 15
D) 16

30 Between 1970 and 2006, carbon monoxide emissions in the United States fell from 197 million tons to 89 million tons, while sulfur dioxide emissions fell from 31 million tons to 15 million tons. Which had the greater percent decrease?

A) carbon monoxide emissions, because its 54.8% decrease is greater than a 51.6% decrease
B) carbon monoxide emissions, because its 48.4% decrease is greater than a 45.2% decrease
C) sulfur dioxide emissions, because its 54.8% decrease is greater than a 51.6% decrease
D) sulfur dioxide emissions, because its 48.4% decrease is greater than a 45.2% decrease

31 In 2011, the total world strawberry production in tons was 4,594,539, of which 1,312,960 tons were produced in the United States. What percent of the world's strawberries in 2011 were produced in places outside the United States?

A) 3.5
B) 28.6
C) 49.9
D) 71.4

Student Produced Response – Calculator

1 A woman is jogging at a rate of 2 meters per second. What is the woman's speed, rounded to the nearest integer, in feet per minute? (*Note: There are 0.305 meters in one foot.*)

2 Joni Construction is planning to pave over a park to build a parking lot. On the blueprint, the park takes up 120 in^2. The scale on the map shows that 1 in = 10 meters. They are planning on using one ton of concrete to build every 200 m^2 of the parking lot floor. Approximately how many tons of concrete will be needed to cover the entire park?

3 Susan took 8 math tests over the course of one semester. Her average score on the first 3 tests was 84. Her average score on her last 5 tests was 92. What was the overall average of all 8 tests?

4 The highest grossing film released in 2015 earned $935.8 million in ticket sales, which was 8.4% of the total amount earned in ticket sales by all films released that year. What was the total amount of money earned by all films, in billions of dollars, rounded to the nearest tenth of a billion?

Guided Practice – Linear & Exponential Growth

(Answers & explanations begin on page 404).

Multiple Choice – Calculator

1 Which of the following functions would describe the total amount, A, a bank customer would receive, in dollars, on a deposit of \$10,000 at an annual percentage rate of 15% compounded annually for t years?

A) $A=10{,}000(1.015)^t$

B) $A=10{,}000(11.5^t)$

C) $A=10{,}000(1+0.15)^t$

D) $A=10{,}000(0.15)^t$

2 Which of the following statements is *not* an example of exponential growth?

A) Every year, the amount of interest earned at a fixed rate during that year is added to the principle.

B) Every year, the amount of amount of money which earns interest increases by the amount of interest earned the previous year.

C) Every year, the principle increases based on the amount of interest accrued from the previous year.

D) Every year, the amount of money added to the account is constant.

3 Which one of the following sequences would be considered an example of exponential growth?

A) $\frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2},\frac{9}{2}$

B) $\frac{2}{3},\frac{4}{9},\frac{8}{27},\frac{16}{81},\frac{32}{243}$

C) $\frac{3}{2},\frac{9}{4},\frac{27}{8},\frac{81}{16},\frac{243}{32}$

D) 4,8,12,16,20

4 The population of lizards in a park is observed over the course of eighteen weeks, as shown in the table below.

Lizard Population

Week of Observation	Number of Lizards Observed
Initial Population	250
Week 3	220
Week 6	190
Week 9	160
Week 12	130
Week 15	100
Week 18	70

Which of the following best describes the relationship between time and the observable population of lizards during the eighteen weeks?

A) Increasing linear

B) Decreasing linear

C) Exponential growth

D) Exponential decay

5 The rule of 72 is a method of approximation based on the compound interest formula to determine how long an amount will take to double, given a fixed annual rate of interest. By dividing 72 by the annual rate of return, investors can roughly estimate the number of years it will take for the initial amount to double. Approximately how many years will it take for an initial amount of \$240 to accrue enough interest to become \$480 at a fixed annual interest rate of 6%?

A) 6

B) 8.33

C) 12

D) 72

6 The chart below displays the number of bacteria observed in a sample.

Expected Number of Bacteria

Time of Observation	Number of Bacteria
2:00 PM	100,000
2:30 PM	150,000
3:00 PM	225,000
3:30 PM	

The bacteria's method of reproduction leads to exponential growth. What is the expected number of bacteria at 4:00PM?

A) 300,000
B) 337,500
C) 375,000
D) 506,250

The next two questions refer to the information below:

The formula below is used for estimating the growth of a certain colony of bacteria based on the number of bacteria initially in the sample:

$$N_{NOW+20} = N_{NOW} + 0.7N_{NOW}\left(1 - \frac{N_{NOW}}{K}\right)$$

7 If $N_{NOW} = 10{,}000$ and $K = 50{,}000$, what is N_{NOW+20}, the number of bacteria twenty minutes from now?

A) 13,500
B) 15,600
C) 18,400
D) 19,700

8 What will be the value of K if the desired number of bacteria at the end of 20 minutes (N_{NOW+20}) is 14,200 and the initial number is 10,000?

A) 25,000
B) 30,000
C) 35,000
D) 40

9 An investor earned 100% annually on her investment in a strong performing business and reinvested her earnings into that business. Which of the following choices describes the growth of the amount she originally invested?

A) the amount will double every year
B) the amount will be 100 times larger each year
C) the amount will be unchanged each year
D) the amount will be $100 greater each year

10 Which of the following equations below describes the amount of interest that would be lost as a result of the interest rate on an annually compounded account decreasing from 9% to 7%?

A) $A = P(1-0.2)^t$

B) $A = P\left(\left(1+\frac{9}{100}\right)^t - \left(1+\frac{7}{100}\right)^t\right)$

C) $A = P(1.2)^t$

D) $A = P(0.8)^t$

11 In a wildlife refuge, the number of prairie dogs is observed over a four year period. The initial population, observed the first year as 16,000, and each succeeding year is recorded in the table below.

Prairie Dog Observation

Year of Observation	Number of Prairie Dogs Observed
1	16,000
2	20,000
3	25,000
4	31,250

What is the percent increase added to the population each year?

A) 50%
B) 37.5%
C) 33%
D) 25%

12 Two banks offer the same interest rate on deposits. One bank compounds annually and the other compounds semiannually. Which formula below would be used to calculate the difference, A_D, between the total value of an account compounded semiannually versus annual compounding on an initial amount of P dollars at rate r percent for t years?

A) $A_D = P\left(\left(1+\frac{2}{r}\right)^{\frac{t}{2}}\right) - (1+r)^t$

B) $A_D = P \cdot r^t - P \cdot r \cdot t$

C) $A_D = P\left(\left(1+\frac{2r}{2}\right)^{2t} - (1+r)^t\right)$

D) $A_D = P\left(\left(1+\frac{r}{2}\right)^{2t} - (1+r)^t\right)$

13 One bank in town has a promotion to attract new savings accounts. The bank offers three different levels of compounding interest, depending on how much is deposited. All three levels set the savings rate of interest to the same percent as the number of times per year the account is compounded. The highest rate is compounded monthly. If an annual compounding would result in the rate of 1% and a semiannual compounding would set the savings rate equal to 2%, which general formula below would describe the amounts in each of three accounts if n is the number of times the interest is compounded per year?

A) $A = 2^{nt}P$ $A = 2^{nt}P$

B) $A = 1.2^{nt}P$

C) $A = P(1+n)^{2nt}$

D) $A = P(1.01)^{nt}$

Student Produced Response – Calculator

1 $\frac{25}{9}, \frac{125}{27}, \frac{625}{81}, \frac{3{,}125}{243}$

In the above sequence of exponentially increasing numbers, what is the common ratio that is being raised to increasing integer powers?

2 The chart below displays the observed number of perch in an underwater wildlife preserve over the course of three years.

Perch Population Over Time

First Observation	4,290
1 year after first observation	4,590
2 years after first observation	4,912
3 years after first observation	5,255

What is the annual percentage growth of the number of perch in the underwater preserve during the three recorded years, to the nearest whole percent? (*Ignore the percent symbol when entering your answer.*)

3 During the first year of life, a certain species of panda increases its weight by 40% every six months. By what percent will the panda have increased its weight after 1 year?

4 Exponential growth has been observed in a certain species of rabbit at a wildlife preserve as summarized in the chart below.

Wildlife Preserve Rabbit Population

Year of Observation	Number of Prairie Dogs Observed
1	1,600
2	2,000
3	2,500
4	

At the rate deduced from the chart, what would be the expected number of rabbits in the fourth year?

5 Mark was looking at the chart shown below of the annually reported amounts for an account into which he had invested $40,000.

Account Balance Over Time

Year	Amount
0 (Initial Amount)	$ 40,000
1	$ 43,200
2	$ 46,656

What is the annual percentage rate of growth, based on annual compounding?

6 A scientist discovers a new element. When he exposes 48 grams of the element to oxygen, a chemical reaction happens and the amount of the element remaining decreases by 0.6g per second. When he exposes 48 grams of the element to hydrogen, a chemical reaction happens and the amount of the element remaining decreases by half of itself every 12 seconds. How many more grams are left of the oxygen-exposed element than the hydrogen-exposed element after one minute?

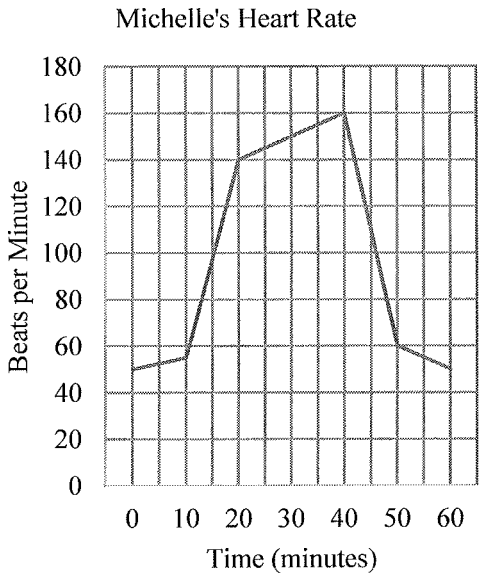
7 Two competing astrophysicists have each created a computer model that can discover new stars in the universe beyond the Milky Way. The first astrophysicist's model found 10 new stars on the first day and, every day after, finds twice as many new stars as the previous day. The second astrophysicist's model found 200 new stars on the first day and, every day after, finds 125 more new stars than the previous day. How many more stars does the first astrophysicist find than the second astrophysicist on Day 10?

Guided Practice – Interpreting Graphs & Tables

(Answers & explanations begin on page 406).

Multiple Choice – Calculator

1 Michelle does aerobic exercise for one hour every day. The graph below shows her target heart rate at different times during her workout.

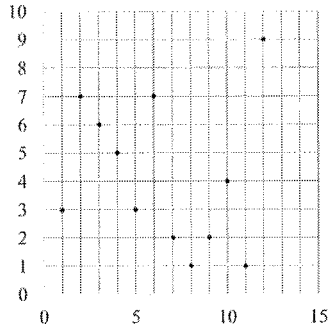


On which interval is the target heart rate strictly increasing then strictly decreasing?

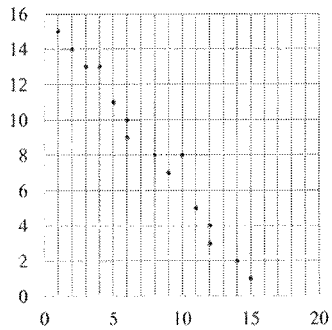
A) Between 0 and 20 minutes
B) Between 10 and 30 minutes
C) Between 30 and 50 minutes
D) Between 40 and 60 minutes

2 Which of the following graphs best shows a strong negative association between x and y?

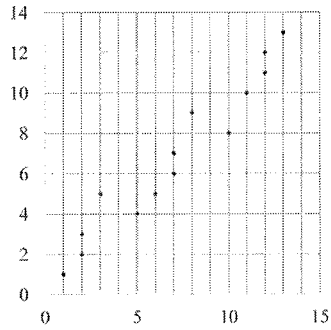
A)



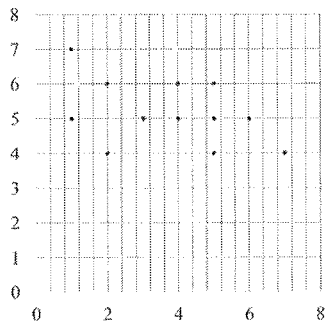
B)



C)



D)



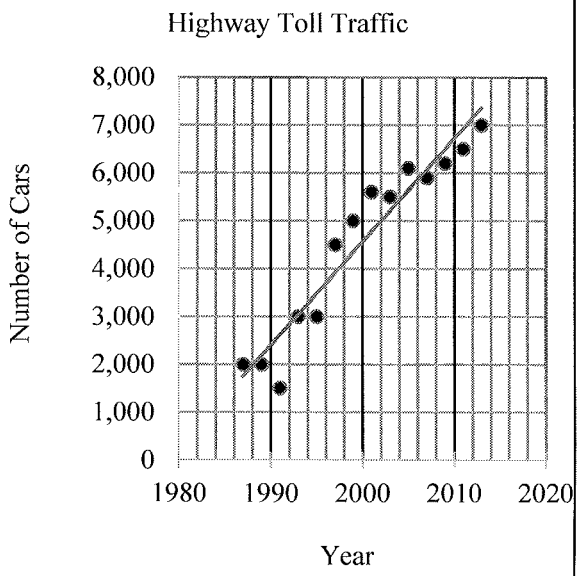
3 Jennifer is planning a school assignment. The table below shows information about the assignment.

Category	Measurement
# of hours Jennifer plans to spend on assignment per day	2
# of hours for research	3
# of pages in the assignment	15
# of words per page	500
# of words Jennifer writes per minute	70
# of words in the assignment	7,500

If Jennifer completes the assignment at the rate given in the table, which of the following is the closest to the number of days Jennifer will need to complete the assignment?

A) 2
B) 3
C) 5
D) 6

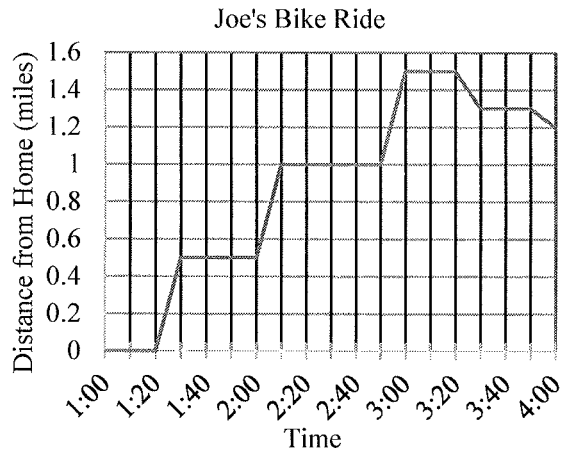
4



According to the line of best fit in the scatterplot above, which of the following best approximates the year in which the number of cars that passed through the toll was estimated to be 6,500?

A) 2005
B) 2007
C) 2010
D) 2013

5



The graph above shows Joe's distance from his house during a 3-hour bike ride. He stopped for 30 minutes to take photos in a park. Based on the graph, which of the following is closest to the time he finished taking photos and continued his bike ride?

A) 1:20
B) 2:00
C) 2:50
D) 3:10

6 The table below represents ticket sales for 35 productions put on at the Chestertown Theater, categorized by genre and intended audience.

Chestertown Theater Productions

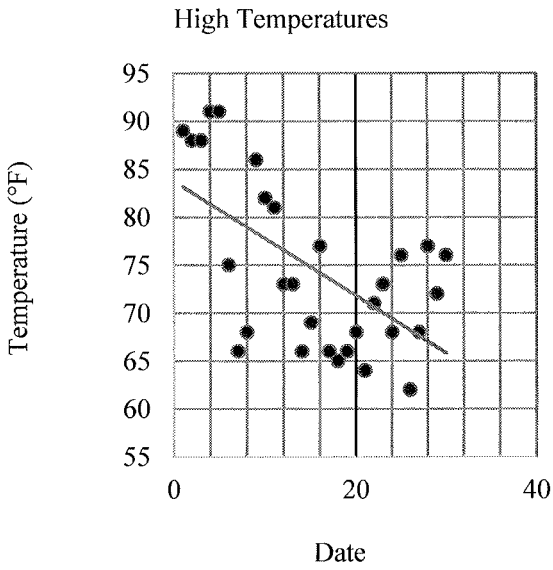
	Comedy	Drama	Musical	Total
Child	4	2	6	12
Any	5	3	2	10
Adult	3	5	5	13
Total	12	10	13	35

What fraction of the movies is dramas intended for adults?

A) $\frac{1}{7}$
B) $\frac{2}{7}$
C) $\frac{1}{3}$
D) $\frac{5}{13}$

The next two questions refer to the below information:

The scatter plot below shows the recorded high temperatures in September for a town in Minnesota.



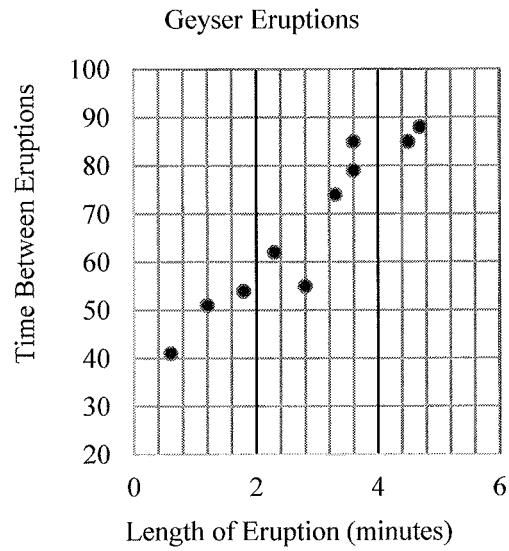
7 Based on the line of best fit to the data shown, which of the following values is closest to the average daily decrease in temperature?

A) 1°F
B) 3°F
C) 5°F
D) 20°F

8 Based on the graph, which of the following show the general trend in temperatures?

A) The temperature generally increased throughout the month.
B) The temperature generally decreased throughout the month.
C) The temperature increased until September 15 and then generally decreased.
D) The temperature generally remained steady throughout the month.

9 The scatter plot below shows how the interval between eruptions and the length of each eruption has changed over time.



If a line of best fit were drawn, which would best approximate the length of the eruption after an 80-minute interval?

A) 2 minutes
B) 3 minutes
C) 4 minutes
D) 5 minutes

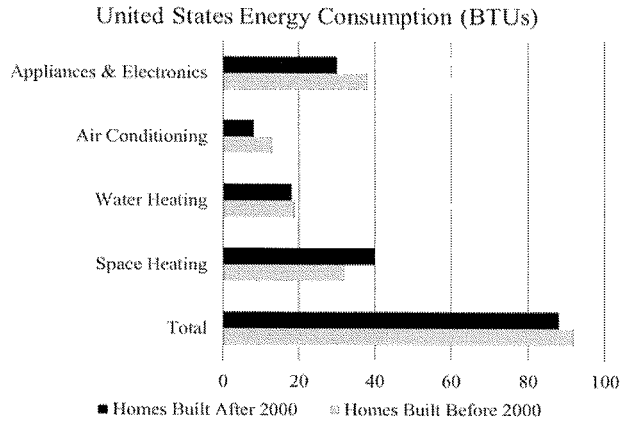
10 A survey was conducted of people who voted in a recent Senate election in one district. The table below displays a summary of the survey results.

Age	Candidate A	Candidate B	Total
18–29	255	383	638
30–44	656	684	1,340
45–64	1,801	1,200	3,001
65 or over	604	737	1,341

According to the table, for which age group did the greatest percentage of people within that age group vote for Candidate B?

A) 18–29
B) 30–44
C) 45–64
D) 65 or over

The next two questions refer to the below information:



The bar graph above shows energy consumption in millions of British thermal units (Btu) in the United States, by type of heating or cooling, for homes built before 2000 and homes built after 2000.

11 In a scatterplot of this data, where energy consumption in homes built before 2000 is plotted along the x-axis and energy consumption in homes built after 2000 is plotted along the y-axis for each of the given energy uses, how many data points would be above the line $y = x$?

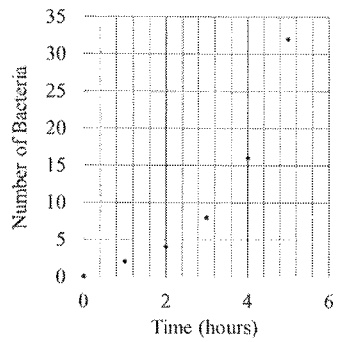
A) 1
B) 2
C) 4
D) 5

12 A researcher noticed that the number of household appliances in the average house built since 2000 has gone up by 15%. Which of the following conclusions is most valid?

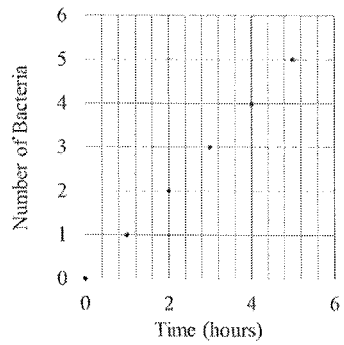
A) A decrease in the amount of energy consumed by an appliance leads to a decrease in the number of appliances installed in a house.
B) An increase in the amount of energy consumed by an appliance leads to an increase in the number of appliances installed in a house.
C) A decrease in the amount of energy consumed by an appliance leads to an increase in the number of appliances installed in a house.
D) Both energy consumption and the number of appliances per house has increased since 2000.

13 If the number of bacteria in a petri dish doubles each hour, which of the following graphs could model the number of bacteria?

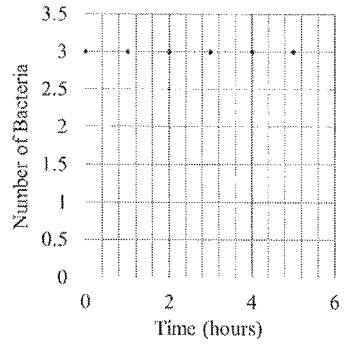
A)



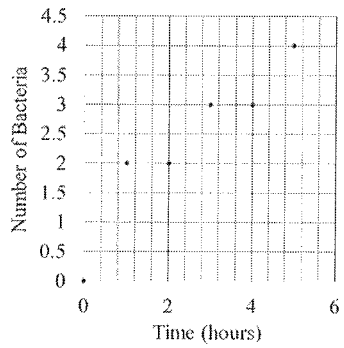
B)



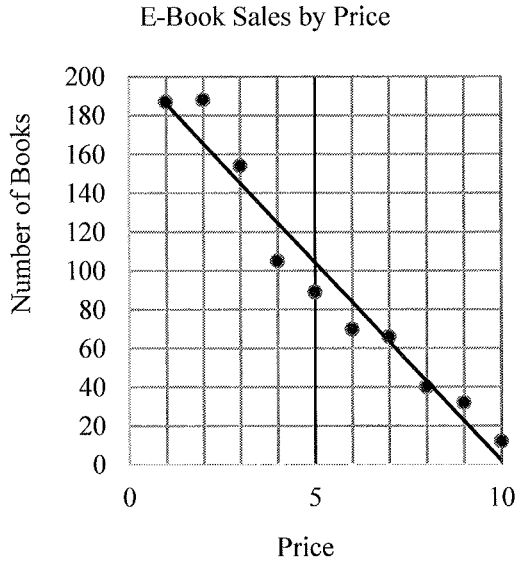
C)



D)



The next two questions refer to the below information:



A bookstore sells e-books through its website and tracks the number of sales of books at different price points. The equation for the line of best fit is $y = 196 - 18x$.

14 How many of the different price points sold more than was expected based on the line of best fit?

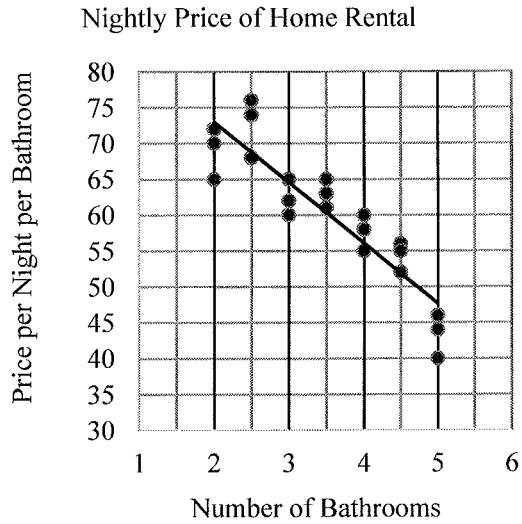
A) 2
B) 3
C) 5
D) 6

15 What is the best interpretation of the meaning of the x-intercept (not pictured) of the line of best fit?

A) The store would be giving away books for free.
B) There would be at least 10 books sold.
C) There would be 196 books sold.
D) Virtually no books would be sold at a price point of more than $10.

The next two questions refer to the below information:

A vacation home rental company tracked the average nightly price of 21 homes. The scatterplot below shows the number of bathrooms in each home and the price of the home per bathroom, along with the line of best fit and the equation for the line. The equation of the line of best fit is $y = 90 - 9x$.



16 According to the line of best fit, approximately how much would it cost to rent a home with three-and-a-half bathrooms for one night? (Round your answer to the nearest dollar.)

A) $55
B) $60
C) $65
D) $205

17 What is the best interpretation of the meaning of the slope of the line of best fit?

A) For each additional full bathroom, the rental price of the home decreases by $9.
B) For each additional full bathroom, the average price per bathroom decreases by $9.
C) For each additional full bathroom, the rental price of the home decreases by $90.
D) For each additional full bathroom, the average price per bathroom decreases by $4.50.

18 Tanisha opened a bank account that earns 1.5% interest compounded annually. Her initial deposit was $200. If Tanisha does not deposit any additional money, which of the following functions gives the total amount A, in dollars, that will be in Tanisha's account after t years?

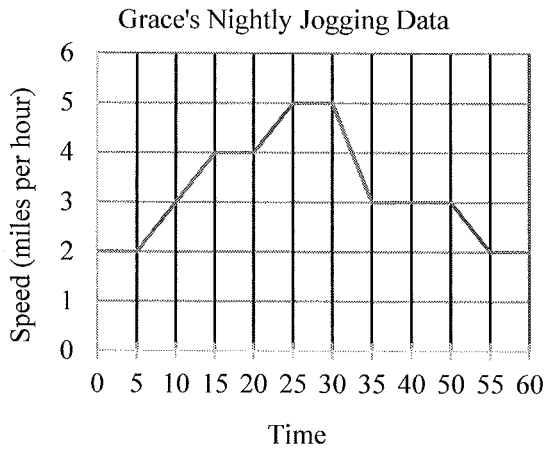
A) $A = 200(1.015t)$

B) $A = 200(1.15t)$

C) $A = 200(1.015)^t$

D) $A = 200(1.15^t)$

19



Each evening, Grace jogs for 60 minutes. The graph above shows Grace's speed during a particular night's jog. Which segment of the graph represents the times when Maria's speed is the greatest?

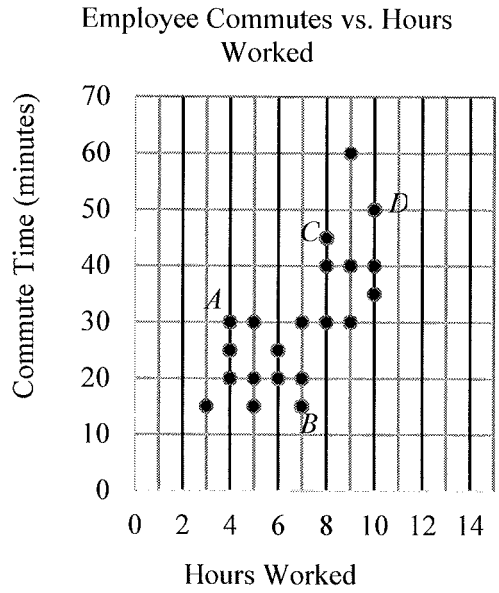
A) The segment from (5,2) to (15,4).

B) The segment from (25,5) to (30,5).

C) The segment from (35,3) to (50,3).

D) The segment from (55,2) to (60,2).

The next two questions refer to the below information:



The human resources department at a large company created the above scatterplot to examine the relationship between the length of time employees typically work in the office and the time they take to commute to the office.

20 How many hours does the person who lives farthest from the office typically work?

A) 8

B) 9

C) 10

D) Cannot be answered from the given information

21 The labeled points represent different employees. Which labeled employee spends the most time commuting per hour worked?

A) A (4,30)

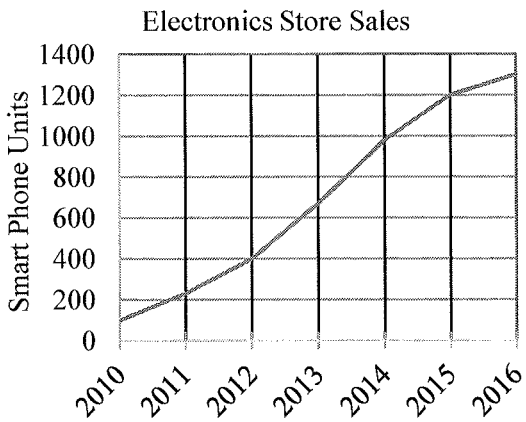
B) B (6,25)

C) C (8,45)

D) D (10,50)

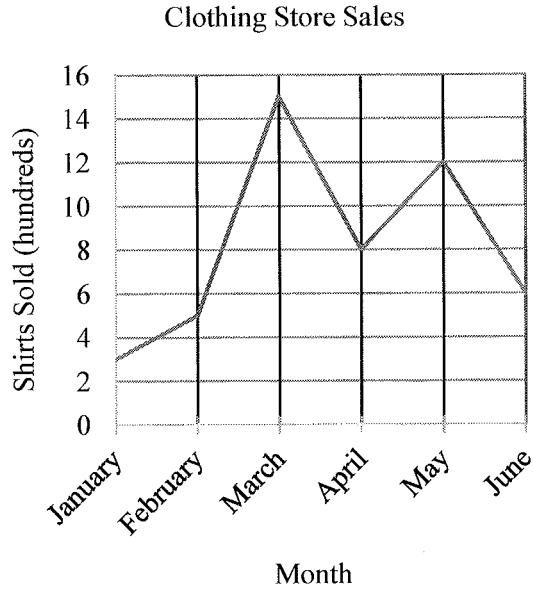
Student Produced Response – Calculator

1 The graph below shows the annual smart phone sales at an electronics store. According to the graph, the number of smart phone sales in 2012 is what fraction of those sold in 2015?



The next two questions refer to the below information:

A clothes store tracked sales of t-shirts for six months in the graph below.



2 According to the graph, the number of shirts sold in January is what fraction of those sold in April?

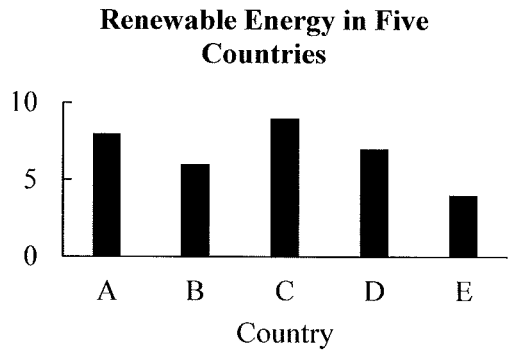
3 According to the graph, how many more shirts were sold in the month with the most sales than the month with the least sales?

Guided Practice – Additional Data Analysis & Statistics

(Answers & explanations begin on page 408).

Multiple Choice – Calculator

1 The number of households who have switched to using renewable energy in their homes instead of fossil fuels in 5 countries is shown in the chart below.



If the total number of households using renewable energy is 34,000, what is an appropriate label for the vertical axis of the graph?

A) Number of households (in tens)
B) Number of households (in hundreds)
C) Number of households (in thousands)
D) Number of households (in tens of thousands)

2 The table below shows the distribution of age and gender for 100 people who visited an amusement park.

Gender	Under 18	18 or Older	Total
Male	15	40	55
Female	10	35	45
Total	25	75	100

If one out of every 100 amusement park visitors is chosen to win a free ticket, what is the probability that the winner will be either a male or a female over age 18?

A) $\frac{75}{100}$
B) $\frac{40}{100}$
C) $\frac{35}{100}$
D) $\frac{25}{100}$

3 The years that constitute certain milestones in the population of the world are shown in the table below.

Year	Population (millions)
1650	508
1750	790
1800	980
1850	1,260
1900	1,650
1950	2,557
2000	6,088
2050*	9,408

*Estimated. Source: U.S. Census Bureau

Which of the following best describes the relationship between time and the estimated world population during these years?

A) Increasing linear
B) Decreasing linear
C) Exponential growth
D) Exponential decay

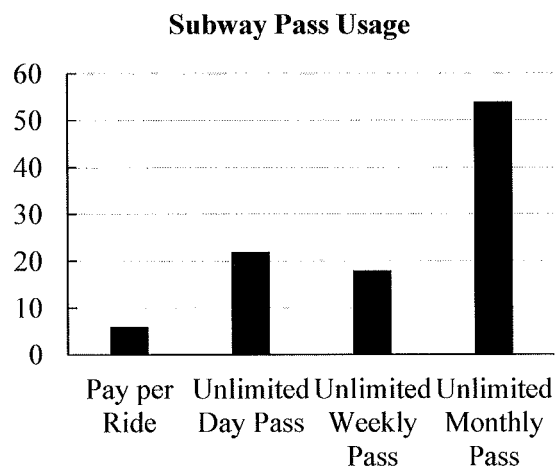
4

Number of Eggs per Carton	Price
1	$ 1.00
6	$ 3.50
12	$ 4.99
18	$ 5.89

Based on the table above, which of the following is the least amount of money needed to purchase exactly 100 eggs?

A) $36.95
B) $43.92
C) $60.00
D) $100.00

5 A researcher polled 100 subway commuters during the morning rush hour to find out which kind of pass they used to pay for their ride. The result of this poll is shown in the chart below.



The sum of the commuters who used Pay Per Ride passes and Unlimited Day Passes is approximately what percent of all commuters polled?

A) 22%
B) 28%
C) 46%
D) 54%

6 A quality control researcher at an electronics company is testing the life of the company's batteries in certain smart phones. The researcher selects 100 batteries at random from the daily output of the batteries and finds that the average life of the batteries has a 95% confidence interval of 8-12 hours with normal phone use. Which of the following conclusions is the most reasonable based on the confidence interval?

A) It is plausible that the true average life of batteries produced by the company that day is between 8 and 12 hours.
B) It is plausible that the true average life of all batteries ever produced by the company is between 8 and 12 hours.
C) 95% of all the batteries produced by the company have a life between 8 and 12 hours.
D) 95% of all the batteries ever produced by the company have a life between 8 and 12 hours.

7 A sports team is deciding whether to implement new security measures to prevent tickets to games from being resold illegally. Capacity at the stadium is 20,000 people, and the security team estimates that there are 45 people per game who purchased tickets illegally. The security manager estimated the results of a new security measures as shown in the table below.

	Illegal Tickets Detected	Tickets Allowed	Total
Illegal Tickets	38	7	45
Legal Tickets	12	19,943	19,955
Total	50	19,950	20,000

According to the manager's estimates, if illegal tickets are detected, what is the probability that the customer did *not* attempt to use an illegal ticket?

A) 0.19%
B) 0.25%
C) 24%
D) 76%

8 A certain calculus professor offers all of the students in his course an additional study session before each exam. The students who attended the additional study session did better on the exam than students who didn't attend. Which of the following is an appropriate conclusion?

A) Attending the additional study session will cause an improvement for any student who takes any math class.
B) Attending the additional study session will cause an improvement for any student who takes a calculus class.
C) Attending the additional study session was the cause of the improvement for the students at this university who took this professor's calculus class.
D) No conclusion about cause and effect can be made regarding students at this university who attended the additional study session and their performance on the exam.

9

Science Class Enrollment by Gender

Class	Male	Female	Total
Biology	45	52	97
Chemistry	39	50	89
Physics	36	38	74
Total	120	140	260

A group of high school students responded to a survey that asked which science course they were currently enrolled in, the results of which were recorded in the table above. Which of the following categories accounts for 15% of all survey respondents?

A) Females taking biology
B) Females taking chemistry
C) Males taking chemistry
D) Males taking biology

10

Heights of Athletes

59	60	60	61	62	63
63	64	65	65	66	67
67	67	68	69	70	78

The table above lists the heights, to the nearest inch, of a random sample of female athletes competing at a track meet. The outlier measurement of 78 inches is an error. Of the mean, median, and range of the listed values, which will change the most if the 78-inch measurement is removed from the data?

A) Mean
B) Median
C) Range
D) The mean, median, and range will all change by the same amount.

Questions 11 and 12 refer to the following information.

Proposed Annual County Budget (millions USD)

Department	2014	2015	2016
General	3.14	3.55	3.05
Health	6.48	7.54	8.56
Public Protection	7.32	7.45	7.63
Public Assistance	6.85	6.54	6.56
Recreation & Culture	0.65	0.7	0.85
Other	1.15	1.25	1.36
Total	25.59	27.03	28.01

The table above lists the annual budget, in millions of dollars, for a county in California.

11 Which of the following best approximates the average rate of change in the annual budget for Public Protection in this county from 2014 to 2016?

A) $130,000 per year
B) $155,000 per year
C) $180,000 per year
D) $1,550,000 per year

12 Of the following, which department's ratio of its 2014 to its 2016 budget is closest to the Recreation & Culture department's ratio of its 2014 to its 2016 budget?

A) Health
B) Public Protection
C) Public Assistance
D) Other

13

The scatterplot above shows home sale prices and square footage for several homes in a certain town. The line of best fit is also shown, and has equation $y = .8529x + 71$. Which of the following best explains how the number 71 in the equation relates to the scatterplot?

A) The smallest house in the town costs \$71 per square foot.
B) The cheapest house in the town costs \$71,000.
C) Land costs about \$71,000 per square foot.
D) Even tiny houses are likely to cost at least \$71,000.

14 A researcher conducted a survey to determine whether people in a certain city preferred shopping at big chains or at independently-owned stores. The researcher surveyed 23 people who were shopping downtown on a Saturday, three of whom refused to respond. Which of the following factors makes it least likely that a reliable conclusion can be drawn about the shopping preferences of all of the people in the city?

A) Sample size.
B) Population size.
C) The number of people who refused to respond.
D) Where the survey was conducted.

15 Eight students each took a shift in the park to count the number of birds spotted in a 1-hour period. Each student was in the park for exactly 1 hour and no shifts overlapped. Their data is shown in the table below.

Student	Number of Birds
A	32
B	27
C	54
D	43
E	35
F	48
G	32
H	29

What is the mean number of birds spotted each hour?

A) 27
B) 32
C) 33.5
D) 37.5

Questions 16 and 17 refer to the following information.

Because the gravitational pull is different on different planets, weight varies depending on the strength of the gravitational pull. The table shows the approximate weight of a 150-pound person on different planets.

Planet	Weight (pounds)
Mars	56.5
Mercury	56.7
Uranus	133.3
Venus	136.0
Saturn	159.6
Neptune	168.7
Jupiter	354.6

16 Based on this data, what is the weight of a 175-pound person on Saturn?

A) 164.5 pounds
B) 184.6 pounds
C) 186.2 pounds
D) 413.7 pounds

17 An object on Earth weighs 100 pounds. On which planet would the same object have an approximate weight of 236 pounds?

A) Venus
B) Jupiter
C) Uranus
D) Neptune

18 In order to determine if a change to a car engine gave the car better gas mileage, a research study was conducted. From all the cars produced by a factory in a given week, 50 were randomly selected. Half of them were tested with the new engines, and the other half kept the old engines. The resulting data showed that cars with the new engines had significantly better gas mileage as compared to the old engines. Based on the design and result of the study, which of the following is an appropriate conclusion?

A) The new engines cause substantial improvements in gas mileage.
B) The new engines will improve the gas mileage of any cars they are used in.
C) The new engines improve gas mileage better than other manufacturing changes.
D) The new engines are likely to improve gas mileage for the car model this factory produces.

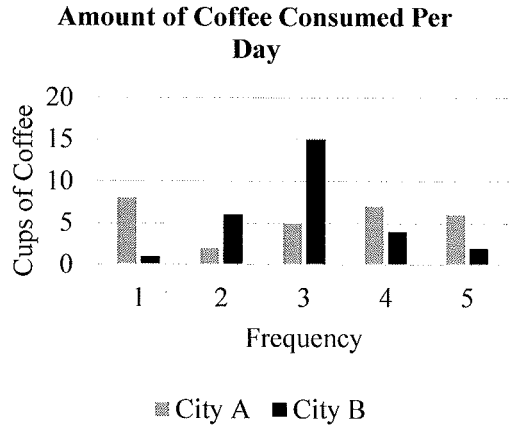
19

Gender	Yes	No
Male		
Female		
Total	110	45

The incomplete table above summarizes, by gender, the number of students who play at least one sport for a tenth-grade class at a high school. There are twice as many female students who play sports than female students who don't and 3 times as many male students who play a sport as those who don't. Which of the following is closest to the probability that a student who plays a sport selected at random is female?

A) 0.23
B) 0.41
C) 0.45
D) 0.55

20 A researcher asked office workers in two different cities about how many cups of coffee they typically drink each day and recorded the results in the chart below.



Which of the following is true about the data shown?

A) The standard deviation of cups of coffee consumed per day in City A is larger than that of City B.
B) The standard deviation of cups of coffee consumed per day in City B is larger than that of City A.
C) The standard deviation of cups of coffee consumed per day City A is the same as that of City B.
D) The standard deviation of cups of coffee consumed per day in these cities cannot be determined with the data provided.

21 A football team scores an average of 11 points per game over 4 games. If the team scored 7, 18, and 6 points at 3 games, respectively, how many points did they score in the 4th game?

A) 11
B) 13
C) 15
D) 17

22

Students Enrolled by Geographic Region

Class	Northeast	Midwest	South	West
Freshman	2,345	1,018	1,164	1,465
Sophomore	1,987	1,672	1,011	1,234
Junior	2,015	1,212	1,332	1,115
Senior	1,874	1,341	1,216	1,102
Total	8,221	5,243	4,723	4,916

The table above shows the number of students enrolled in a large northeastern university, in four geographic regions and class groups. Based on the table, if a student is chosen at random, which of the following is closest to the probability that the student was a junior from the Midwest?

A) 0.05
B) 0.14
C) 0.21
D) 0.25

23

Day	Minutes
1	3.0
2	2.9
3	2.7
4	2.6
5	2.4
6	2.3
7	2.1
8	2.0

Each day for eight days, Wendy ran a lap around the track. The table above shows the time, in minutes, it took Wendy to run each lap. If the track is 0.25 miles around, which of these is the best approximation Wendy's average speed, in miles per hour?

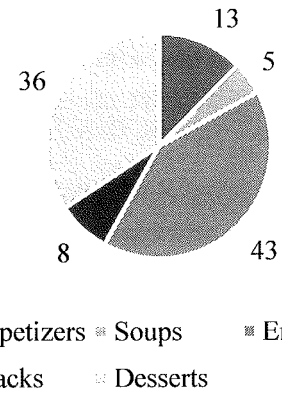
A) 20
B) 10
C) 6
D) 2.8

24 Amar bowled 3 games. During the first two games, he scored 116 and 148 points, respectively. How many points must he have scored in the third round to have an average score of 134?

A) 132
B) 134
C) 138
D) 166

25

Cookbook Recipes



The table above shows the distribution of recipes in a cookbook. Of the mean, median, and range of the values listed, which will change the most if a chapter with 25 side dish recipes is also included in the book?

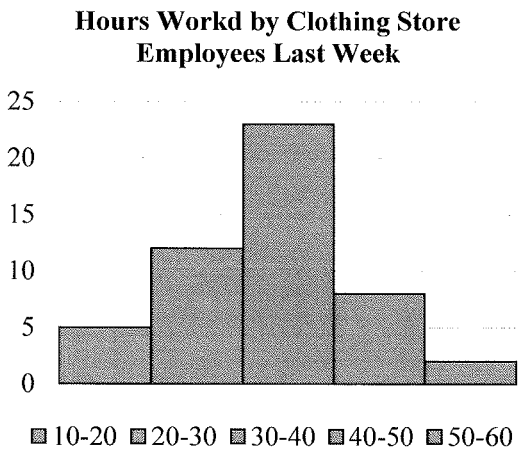
A) Mean
B) Median
C) Range
D) They will all change by the same amount.

26 For a class test, the mean score was 71, the median score was 74, and the standard deviation of the scores was 6. The teacher decided to add 5 points to each score due to a grading error. Which of the following statements must be true for the new scores?

I. The new mean score is 76.
II. The new median score is 79.
III. The new standard deviation of the scores is 11.

A) I only
B) II only
C) I and II only
D) I, II, and III

27



The histogram above shows the number of hours worked last week by 50 employees of a clothing store. In the histogram, the first bar represents all workers who worked at least 10 hours but less than 20 hours; the second represents all workers who worked at least 20 hours but less than 30 hours; and so on. Which of the following could be the median and mean number of hours worked for the 50 employees?

A) Median: 25, Mean: 24
B) Median: 34, Mean: 26
C) Median: 35, Mean: 36
D) Median: 42, Mean: 40

28 At a gym with 121 members, 54 members are enrolled in a martial arts class and 23 are enrolled in a yoga class. Of the members taking kickboxing or yoga, 12 are taking both classes. How many members are not taking either course?

A) 32
B) 44
C) 56
D) 68

Student Produced Response – Calculator

1 The average temperature in Chicago for 6 days is 66°F. If, on the 7th day, the temperature is 52°F, what is the average temperature, in degrees Fahrenheit, for all 7 days?

2

Manatee Length (ft.)
9.5
8.8
8.3
9.2
x

In a study, manatees in one part of Florida were measured and tagged. If the range of data is 2 feet, what is the least possible value of x?

3

Player	Number of Home Runs
Albert Pujols	560
Alex Rodriguez	687
Babe Ruth	714
Barry Bonds	762
Hank Aaron	755
Ken Griffey, Jr.	630
Mark McGwire	583
Mickey Mantle	536
Reggie Jackson	563
Sammy Sosa	609
Willy Mays	660

The table above lists the number of home runs hit over the careers of 11 prominent baseball players at a certain point in time. According to the table, what is the mean number of career home runs of these players? (Round your answer to the nearest whole number.)

4 A product sold through an online store receives customer satisfaction ratings between 0 and 10, inclusive. In the first 5 ratings the store received, the average (arithmetic mean) of the ratings was 7. What is the least value the store can receive for the 6^{th} rating and still be able to have an average of at least 8 for the first 10 ratings

5 The average score for 18 students on an exam is 82. Two more students take the test, averaging a score of 76 between them. What is the total class average (rounded to the nearest tenth) if these two students are added to the 18?

6

T-Shirt Sales

	18-34	35-49	50+	Total
Before	82	23	12	117
During	22	18	10	50
After	108	44	23	175
Total	212	85	45	342

The table above shows the distribution of t-shirt sales at a theater before, during, and after a concert, arranged by age of concert-goer. If one t-shirt customer will be randomly selected to win tickets to the band's next show, what is the probability that the winner will be someone aged 35-49 who purchased a t-shirt after the concert? (Write your answer as a decimal rounded to the nearest hundredth.)

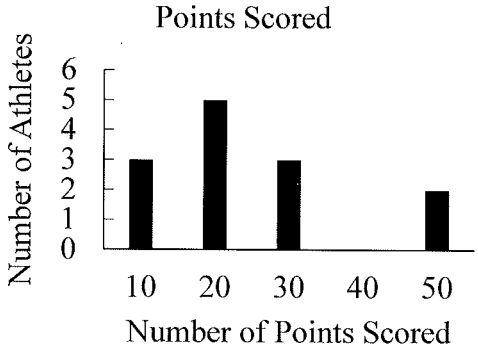
7 The 20 students taking calculus this school year scored an average of 80 points on their midterm exam. If the average score of 19 of the students was 79, what was the score of the 20^{th} students?

Mixed Practice – Data Analysis

(Answers & explanations begin on page 410).

Multiple Choice – Calculator

1 The graph shows the number of points scored by each member of a basketball team during the past season. There is one bar missing.



If the median of the data is 20, which of the following could be the number of athletes that scored 40 points?

A) 2
B) 3
C) 4
D) 5

2 Ignoring air friction, an apple falls from a tree with an acceleration of 9.8 m/s^2 (9.8 meters per second, per second). The speed with which the apple falls (the number of meters traveled per second) can best be described as which of the following?

A) constant
B) constantly increasing
C) constantly decreasing
D) indeterminable

3 Two members on a rowing team, Tom and Everett, are comparing the time it takes them to row 2,000 meters. Tom is faster than Everett, but Everett argues that he has improved his time over the past six months more than Tom. The table below shows the number of minutes each took to row 2,000 meters six months ago and at a recent practice.

	Six months ago	Current
Tom's 2000-meter time	9:00	8:00
Everett 2000-meter time	12:00	10:00

Over the past six months, by what percentage did each runner improve his time?

A) Tom improved his time by 11.1%; Everett improved his time by 20%.
B) Tom improved his time by 12.5%; Everett improved his time by 20%.
C) Tom improved his time by 11.1%; Everett improved his time by 16.7%.
D) Tom improved his time by 12.5%; Everett improved his time by 16.7%.

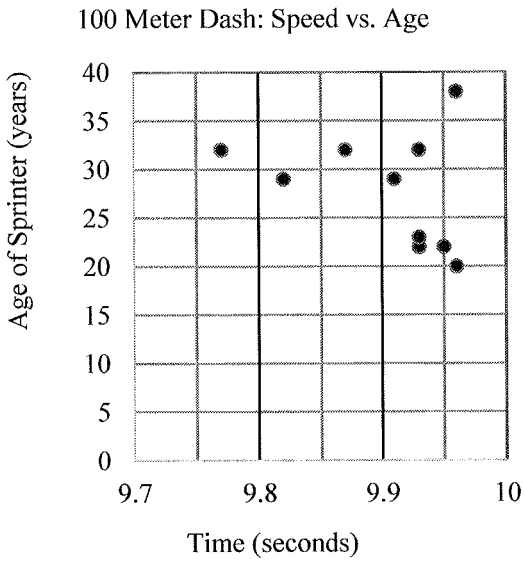
4 The Thompsons are shopping for a new house. They have narrowed their choices down to their four favorites and have recorded the size of each house and the price, as shown below.

House	Size (sq. ft.)	Price (thousands)
Harrison Street	900	$350
Wingspread Road	1,100	$400
Howell Boulevard	1,400	$450
Bennett Avenue	1,500	$500

Based on the information in the table, which house has the lowest cost per square foot?

A) Harrison Street
B) Wingspread Road
C) Howell Boulevard
D) Bennett Avenue

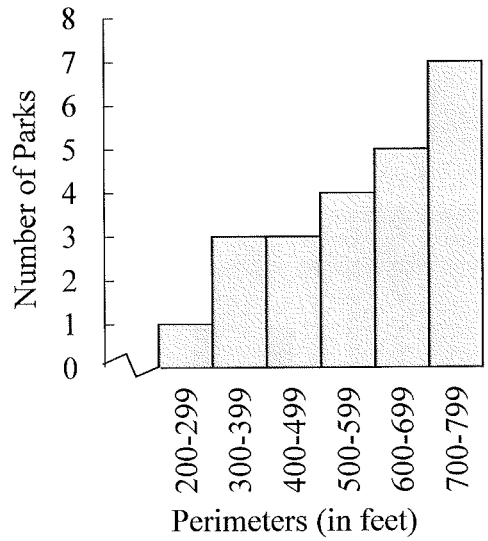
5 The graph below shows the 10 fastest times sprinters ran the 100 meter dash, graphed against the age of each sprinter.



How long did it take the oldest sprinter, in seconds, to finish the 100 meter dash?

A) 9.77
B) 9.87
C) 9.91
D) 9.96

6 The histogram shows the perimeters of all the parks in a certain town.



Which of the following could be the median perimeter?

A) 429
B) 513
C) 602
D) 789

Student Produced Response – Calculator

1 An entomologist is studying a species of fruit fly in a controlled environment and currently has 500 of these fruit flies. The population of this species that the botanist expects to grow next month, $N_{\text{next month}}$, can be estimated from the number of fruit flies this month, $N_{\text{this month}}$, according to the following equation:

$$N_{\text{next month}} = N_{\text{this month}} + 0.2\,(N_{\text{this month}})\left(1 - \frac{1}{K} \cdot N_{\text{this month}}\right)$$

If K is a constant representing the maximum number of fruit flies the environment can support, what is the value of K if the number of fruit flies the following month ($N_{\text{next month}}$) is 580?

2 Jason is planning to drive across the country.

Number of hours Jason plans to drive per day	6
Average speed (miles per hour)	40
Miles per gallon of gas	36
Fuel tank capacity (in gallons)	18
Total Number of Miles	2,522

According to the chart above, if Jason starts the trip with a full tank of gas, how many times will he have to refill the tank?

3 A penguin habitat is 60% female. If there are 300 male penguins, how many female penguins are there?

Advanced Math

Guided Practice – Working with Polynomials

(Answers & explanations begin on page 411).

Multiple Choice – No Calculator

1 $7x^2 - 4x + 8$

$3x^2 - x + 7$

Which of the following is the sum of the two polynomials shown above?

A) $10x^2 - 3x + 15$

B) $10x^2 - 5x + 15$

C) $10x^4 + 4x^2 + 15$

D) $10x^4 - 5x^2 + 15$

2 $(4x+3)(5x-10)$

Which of the following is equivalent to the expression above?

A) $9x - 7$

B) $20x^2 - 30$

C) $20x^2 - 25x - 30$

D) $20x^2 - 55x - 30$

3 $5(3x+1)(2x+1)$

Which of the following is equivalent to the expression above?

A) $60x$

B) $25x^2 + 10$

C) $30x^2 + 5$

D) $30x^2 + 25x + 5$

4 $\frac{1}{2}(2x+4)(x+1)$

Which of the following is equivalent to the expression above?

A) $x^2 + 2$

B) $x^2 + 3x + 2$

C) $x^2 + 6x + 4$

D) $\frac{1}{2}x^2 + \frac{3}{2}x + 1$

5 $7x(2x+1) + 3(2x+1)$

Which of the following is equivalent to the expression shown above?

I. $(7x+3)(2x+1)$

II. $14x^2 + 13x + 3$

A) Only I

B) Only II

C) I and II

D) Neither I nor II

6 If $\frac{20}{ab} = 10$ what is the value of $\frac{32}{a^2b^2}$?

A) 2

B) 4

C) 8

D) 16

7 If $\frac{x}{y} = 3$ what is the value of $\frac{6y}{x}$?

A) 3

B) 2

C) 1

D) 0

8 If $x > 3$, which of the following is equivalent to $\frac{\frac{1}{x+4}+1}{\frac{2}{x-2}}$?

A) $\frac{x-3}{2x+8}$

B) $\frac{x-3}{x+4}$

C) $\frac{2x+10}{x^2+x-12}$

D) $\frac{x^2+2x-10}{2x+8}$

9 If $0 < x < 7$, which of the following is equivalent to $\dfrac{\dfrac{x+5}{x+4}}{\dfrac{x-7}{2x+8}}$?

A) $\dfrac{2x+10}{x-7}$

B) $\dfrac{x+5}{2x-14}$

C) $\dfrac{2x^2+40}{x^2-28}$

D) $\dfrac{x^2-2x-35}{2x^2+16x+32}$

10 $4a^4+20a^2b^2+25b^4$

Which of the following is equivalent to the expression shown above?

A) $(4a+25b)^4$

B) $(2a+5b)^4$

C) $(4a^2+25b^2)^2$

D) $(2a^2+5b^2)^2$

11 $\dfrac{a+3}{x}+\dfrac{b-5}{2x}$

Which of the following is equivalent to the expression above?

A) $\dfrac{a+b-2}{x}$

B) $\dfrac{a+b-2}{2x}$

C) $\dfrac{2a+b+1}{2x}$

D) $\dfrac{a+2b-7}{2x}$

12 The expression $(3x-6)^3$ is equivalent to which of the following?

A) $9x^3-216$

B) $9x^3-36x^2+36$

C) $27x^3-162x^2+324x-216$

D) $27x^3+162x^2-324x+216$

13 The expression $\dfrac{6x-1}{x+4}$ is equivalent to which of the following?

A) $\dfrac{6-1}{4}$

B) $6-\dfrac{1}{4}$

C) $6-\dfrac{1}{x+4}$

D) $6-\dfrac{25}{x+4}$

14 If $x > 0$, which of the following is equivalent to $\dfrac{3}{x+2}-\dfrac{2}{x^2+4x+4}$?

A) $\dfrac{3x+4}{x^2+4x+4}$

B) $\dfrac{1}{x^2+4x+4}$

C) $\dfrac{-1}{x^2+3x+2}$

D) $\dfrac{3x^2+10x+10}{x^2+6x^2+12x+8}$

15 If $\dfrac{y+1}{x}=\dfrac{6}{7}$, which of the following must also be true?

A) $\dfrac{y}{x}=-\dfrac{1}{7}$

B) $\dfrac{y}{x}=\dfrac{6x-7}{7x}$

C) $\dfrac{y}{x}=\dfrac{5}{7}$

D) $\dfrac{y}{x}=\dfrac{5}{7x}$

16 $f(x)=(x-5)^2-1$

Which of the following is an equivalent form of the function f above in which the roots of f appear as constants or coefficients?

A) $f(x)=(x-6)^2$

B) $f(x)=x^2-10x+24$

C) $f(x)=(x-5)(x-1)$

D) $f(x)=(4-x)(6-x)$

Student Produced Response – No Calculator

1 $4x(2x+3)+5(4x+1)=ax^2+bx+c$

In the equation above, a, b, and c are constants. If the equation is true for all values of x, what is the value of b?

2 $x^4-18x^2+81=0$

For what real, positive value of x is the equation above true?

3 $x^3-7x^2+3x-21=0$

For what real value of x is the equation above true?

4 $6x^4(x^4-2)=-6$

If $x>0$, what is one possible solution to the equation above?

Multiple Choice – Calculator

1 The expression $64w^6-v^6$ is equivalent to which of the following?

A) $(8w^3-v^3)^2$

B) $(4w^2-v^2)^3$

C) $(4w^3-v^3)^3$

D) $(8w^3-v^3)(8w^3+v^3)$

2 $f(x)=(x+3)(x-7)$

Which of the following is an equivalent form of the function f above in which the minimum value of f appears as a constant or coefficient?

A) $f(x)=x^2-21$

B) $f(x)=(x-2)^2-25$

C) $f(x)=(x+3)^2-7$

D) $f(x)=x^2-4x-21$

3 $f(x)=x^2$

$g(x)=x^2-6x+19$

What is the difference in the minimum values of the functions $f(x)$ and $g(x)$ above?

A) 3

B) 6

C) 10

D) 19

Student Produced Response – Calculator

1 $(7x^2+2x-4)-5(x^2-3x+2)$

If the expression above is rewritten in the form ax^2+bx+c, where a, b, and c are constants, what is the value of b?

2 $3xy-15x+y^2-c$

In the polynomial above, c is a constant. If the polynomial is divisible by $y-5$, what is the value of c?

3 $x^3=2x(24-x)$

If a, b, and c represent the solutions of the equation above and $a<b<c$, what is the value of c?

Guided Practice – Working with Polynomial Factors in Expressions & Equations

(Answers & explanations begin on page 413).

Multiple Choice – No Calculator

1 If $\frac{t+3}{t-3} = 6$, what is the value of t?

A) 3
B) $\frac{6}{5}$
C) $\frac{15}{7}$
D) $\frac{21}{5}$

2 The function f is defined by a polynomial. Some values of x and $f(x)$ are shown in the table below.

x	$f(x)$
0	2
1	0
2	2
3	10

Which of the following must be a factor of $f(x)$?

A) $x+1$
B) $x-1$
C) $x+2$
D) $x-2$

3 The function f is defined by a polynomial. Some values of x and $f(x)$are shown in the table above.

x	$f(x)$
-3	0
0	-1
1	0
2	6

Which of the following must be a factor of $f(x)$?

A) $x-3$
B) $x-2$
C) $x+1$
D) $x+3$

4 The function f is defined by a polynomial. Some values of x and $f(x)$ are shown in the table above.

x	$f(x)$
0	1
2	3
4	7
6	0

Which of the following must be a factor of $f(x)$?

A) $x-6$
B) $x-4$
C) $x-2$
D) $x-1$

5 What is the remainder when $4x^2 - 2x + 9$ is divided by $(x-1)$?

A) 0
B) 11
C) 13
D) 15

6 What is the remainder when $9x^2 + 5x - 2$ is divided by $(x+3)$?

A) 0
B) 4
C) 34
D) 64

7 For a polynomial $p(x)$, the following information is given: $p(-3) = 0$ and $p(2) = 4$. Which of the following must be true about $p(x)$?

A) $x-3$ is a factor of $p(x)$
B) $x-6$ is a factor of $p(x)$
C) The remainder when $p(x)$ is divided by $x-2$ is 4.
D) The remainder when $p(x)$ is divided by $x-4$ is 2.

8 If $\frac{2}{x}=\frac{10}{x+24}$, what is the value of $\frac{x}{2}$?

A) $\frac{1}{3}$
B) 3
C) 5
D) 6

9 If $\frac{x}{x+6}=\frac{1}{4}$, what is the value of $\frac{1}{x+6}$?

A) $\frac{1}{2}$
B) $\frac{1}{4}$
C) $\frac{1}{6}$
D) $\frac{1}{8}$

Student Produced Response – No Calculator

1 When $8x^2-20x+40$ is divided by $4x-6$ the result is $2x-2+\frac{R}{4x-6}$ where R is a constant. What is the value of R?

2 $\frac{1}{t+2}=\frac{-2}{t+10}+\frac{1}{4}$

If t is a solution to the equation above and $t>0$, what is the value of t?

Multiple Choice – Calculator

1 If $\frac{t-3}{t-5}=7$, what is the value of t?

A) $\frac{8}{3}$
B) $\frac{16}{3}$
C) $\frac{21}{5}$
D) $\frac{3}{35}$

2 If $\frac{3t}{t+6}=\frac{1}{10}$, what is the value of t?

A) $-\frac{60}{7}$
B) $\frac{1}{20}$
C) $\frac{6}{29}$
D) 5

3 What is the remainder when $x^2-7x+12$ is divided by $x-3$?

A) 0
B) 3
C) 4
D) –4

4 For a polynomial $p(x)$, the value of $p(4)$ is –1. Which of the following must be true about $p(x)$?

A) $x-5$ is a factor.
B) $x-1$ is a factor.
C) $x+1$ is a factor.
D) The remainder when $p(x)$is divided by $x-4$ is –1.

5 $\frac{x+2}{x+11}=\frac{2y}{5y}$

The equation above is true for all values of $y\neq 0$. What must be the value of $x+11$?

A) 5
B) 10
C) 15
D) 20

6 $\frac{30x^2-26x-19}{ax-6}=6x+2-\frac{7}{ax-6}$

The equation above is true for all values of $x\neq\frac{1}{6}$. What is the value of a?

A) 5
B) –5
C) 24
D) –24

7 $\frac{8x^2-22x-9}{2x+b}=4x+3+\frac{12}{2x+b}$

The equation above is true for all values of $x\neq\frac{b}{2}$, where b is a constant. What is the value of b?

A) −11
B) −7
C) −3
D) 1

Student Produced Response – Calculator

1 $\frac{2}{t-1}+\frac{1}{6}=\frac{5}{t+2}$

If t is a solution to the equation above and $t<12$, what is the value of t?

2 When $7x^2-38x+88$ is divided by $7x+4$ the result is $x-6+\frac{R}{7x+4}$, where R is a constant. What is the value of R?

3 $\frac{10x^2-34x-6}{5x+8}=2x-10+\frac{R}{5x+8}$

What is the value of the constant R in the equation above?

Guided Practice – Quadratic Functions & Equations

(Answers & explanations begin on page 415).

Multiple Choice – No Calculator

1 Which of the following does NOT have the same solutions as the other three equations?

A) $x^2+x-20=0$
B) $x^2-x-20=0$
C) $(x-4)(x+5)=0$
D) $(x-4)(2x+10)=0$

2 What are the solutions to $x^2+2x-35=0$?

A) –7,5
B) –5,7
C) –5,–7
D) –2,35

3 What are the solutions to $2x^2+3x+1=0$?

A) 1,2
B) $1,\frac{1}{2}$
C) –1,–2
D) $-1,-\frac{1}{2}$

4 What are the solutions to $x^2+5x+2=0$?

A) $\frac{5\pm\sqrt{17}}{2}$
B) $\frac{-5\pm\sqrt{17}}{2}$
C) $\frac{5\pm\sqrt{33}}{2}$
D) $\frac{-5\pm\sqrt{33}}{2}$

5 What are the solutions to $x^2+1=-4x$?

A) $2\pm\sqrt{3}$
B) $-2\pm\sqrt{3}$
C) $\frac{1\pm\sqrt{17}}{2}$
D) $\frac{-1\pm\sqrt{17}}{2}$

6 $x^2-8x+16=0$

How many real solutions does the equation above have?

A) 0
B) 1
C) 2
D) 4

7 What are the solutions to $3x(x+2)=1$?

A) $\frac{-1\pm\sqrt{2}}{3}$
B) $\frac{-3\pm\sqrt{17}}{2}$
C) $-1\pm\frac{\sqrt{6}}{3}$
D) $-1\pm\frac{2\sqrt{3}}{3}$

8 If $(ax+4)(bx+5)=6x^2+cx+20$ for all values of x, and $a+b=5$, what are two possible values for c?

A) 2 and 3
B) 8 and 15
C) 10 and 12
D) 22 and 23

9 $(x+a)(x+b)=x^2+cx-7$

In the equation above, a, b and c are integers and the equation is true for all values of x. What are two possible values for c?

A) –1 and 7
B) –1 and –7
C) –6 and 6
D) –8 and 8

10 If $(x+a)(x+b)=(x+3)^2+11$ for all values of x, what is the value of ab?

A) 11
B) 14
C) 20
D) 33

11 The function $f(x) = x^2 - 10x + 25$ passes through the points $(a,0)$ and $(0,b)$. What is the value of $a + b$?

A) -10
B) 10
C) 20
D) 30

12 The function $f(x) = 25x^2 - 9$ passes through the points $(-k,0)$ and $(k,0)$. What is the value of k?

A) $\frac{3}{5}$
B) $\frac{5}{3}$
C) $\frac{9}{25}$
D) $\frac{25}{9}$

13 $(3x + 2)(ax + b) = cx^2 + d$

In the equation above, a, b, c and d are non-zero integers and the equation is true for all values of x. Which of the following MUST be negative?

A) a
B) b
C) $a+b$
D) ab

14 The functions $f(x) = -x^2 + 8x - 7$ and $g(x) = x^2 - 8x + 17$ are graphed on the xy-plane. The graphs of f and g intersect at the points (a,k) and (b,k). What is the value of k?

A) 2
B) 5
C) 6
D) 7

15 The functions $f(x) = 16 - x^2$ and $g(x) = x^2$ are graphed on the xy-plane. The graphs of f and g intersect at the points $(-a,b)$ and (a,b). What is the value of a?

A) 4
B) 8
C) $2\sqrt{2}$
D) $2\sqrt{3}$

16 $g(x) = \frac{1}{x^2 - 4x + 8}$

For how many values of x is the function $g(x)$ above undefined?

A) 0
B) 1
C) 2
D) There are infinitely many values.

17 A swimming pool is 3 meters deep. Its length is 4 meters longer than twice its width. If the pool can hold 60 cubic meters of water, which of the following equations can be used to find the width, x, of the pool?

A) $x^2 + 2x + 10 = 0$
B) $x^2 + 4x + 20 = 0$
C) $x^2 + 2x - 10 = 0$
D) $x^2 + 12x - 60 = 0$

Student Produced Response – No Calculator

1 What is the product of the solutions of $(x+4)^2 = 1$?

2 $f(x) = \frac{1}{x^2 - 5x - 14}$

For what positive value of x is the function $f(x)$ above undefined?

Multiple Choice – Calculator

1 Which of the following has NO real solution?

I. $x^2+7x-3=0$

II. $4x^2+4x+1=0$

III. $x^2+2x+5=0$

A) I only
B) II only
C) III only
D) I and II

2 What is the sum of all values of r that satisfy $3r^2+21r+36=0$?

A) -7
B) 7
C) -21
D) 12

3 Three functions, f, g, and h are given below. Which of these functions is/are undefined when $x=2$?

$$f(x)=\frac{1}{(x-2)^2+(x-4)^2}$$

$$g(x)=\frac{3x}{x^2-6x+8}$$

$$h(x)=\frac{x-2}{x^2+10x+16}$$

A) f only
B) g only
C) h only
D) g and h

4 A hiker walks at a speed of $\frac{x}{2}$ miles per hour for $x+2$ hours. At that time, he reaches a mountain, where he hikes for 3 additional hours at half of his original speed. If the hiker walked a total of 15 miles, which equation can be used to find the value of x?

A) $x^2+5x-30=0$
B) $x^2+5x-15=0$
C) $2x^2+7x-56=0$
D) $2x^2+7x-60=0$

5 A woman left her home and drove north for $3x$ hours at a speed of $\frac{2x}{3}$ miles per hour. Then she realized she had forgotten something at home. She began driving south at her original speed. After driving south for 30 minutes, she was 50 miles from her home. Which equation could be used to find the value of x?

A) $x^2-15x-25=0$
B) $x^2-45x-25=0$
C) $6x^2-x-150=0$
D) $6x^2+x-150=0$

Student Produced Response – Calculator

1 If $x>0$ and $3x^2+14x-5=0$, what is the value of x?

2 What is the sum of the solutions of $2x^2+35=17x$?

3 $$h(x)=\frac{1}{(x-7)^2+2(x-7)+1}$$

For what value of x is the function $h(x)$ above undefined?

Guided Practice – Exponents & Radicals

(Answers & explanations begin on page 418).

Multiple Choice – No Calculator

1 Which of the following is equivalent to $a^{\frac{4}{5}}$, for all values of a?

A) $\sqrt{a^{\frac{2}{5}}}$

B) $\sqrt{a^5}$

C) $\sqrt[5]{a^{\frac{1}{4}}}$

D) $\sqrt[5]{a^4}$

2 If $2x - y = 18$, what is the value of $\frac{4^x}{2^y}$?

A) 2^2

B) 2^{18}

C) 4^2

D) The value cannot be determined from the information given.

3 $\sqrt{3k^2 + 9} - x = 0$

If $k > 0$ and $x = 6$ in the equation above, what is the value of k?

A) 1

B) 2

C) 3

D) 6

4 A news website with is gaining subscribers at the rate of 3% per year. The company had 4 million subscribers at the start of 2014. Assume the company continues to gain new subscribers at the same rate and that it does not lose subscribers. Which of the following functions, s, models the number of subscribers (in millions) the website gains t years after the start of 2014?

A) $s(t) = 4(1.03)^t$

B) $s(t) = 4(0.97)^t$

C) $s(t) = 4(0.03)^t$

D) $s(t) = 4(1.03)t$

5 If $\frac{x^a}{x^b} = x$, $x > 1$, what is the value of $b - a$?

A) -1

B) 3

C) 5

D) 7

6 If $10^{ab} = 1{,}000$, where a and b are positive integers, which of the following is a possible value of a?

A) 2

B) 3

C) 6

D) 10

7 A radioactive substance decays 3% each day. If the initial amount of the substance was 400 grams, which of the following functions f models the remaining amount of the substance, in grams, t days later?

A) $f(t) = 400(0.03)^t$

B) $f(t) = 0.03(400)^t$

C) $f(t) = 400(0.97)^t$

D) $f(t) = 0.97(400)^t$

8 $x - 6 = \sqrt{x - 6}$

What is the solution set for the above equation?

A) {6}

B) {7}

C) {–6,7}

D) {6,7}

9 If m and k are positive integers and $m^2k^{-1} = 10m$, what is m^{-1} in terms of k?

A) $\frac{k}{10}$

B) $\frac{\sqrt{k}}{10}$

C) $\frac{1}{10k}$

D) $\frac{10}{k}$

Student Produced Response – No Calculator

1 If $5x^2 = 4y = 20$, what is the value of x^2y?

2 If $\frac{x^{a^2}}{x^{b^2}} = x^{36}$, $x > 1$, and $a + b = 4$, what is the value of $a - b$?

3 If $a = 3\sqrt{3}$ and $2a = \sqrt{3x}$, what is the value of x?

Multiple Choice – Calculator

1 $P(t) = 2{,}000 \times 2^{\frac{t}{2}}$

Scientists study cultures of organisms in a lab. The population P of the organisms in the culture t days after the culture began is modeled by the function above. By how many organisms does the population increase from $t = 2$ to $t = 4$?

A) 2,000
B) 4,000
C) 8,000
D) 16,000

2 If a, b, and c are different positive integers and $2^a \times 2^b \times 2^c = 64$, then $2^a + 2^b + 2^c =$

A) 12
B) 14
C) 17
D) 32

3 $\sqrt{x+16} = x - 4$

For all values of x greater than 4, the equation above is equivalent to which of the following?

A) $x = x^2$
B) $x = x^2 + 16$
C) $x = x^2 - 8x$
D) $x = x^2 - 8x + 16$

4 Greg purchased a car for \$20,000. The value of the car decreases 20% every year. The value, in dollars, of the car n years from the date of purchase is given by the function V, where $V(n) = 20{,}000(0.8)^n$. How many years from the date of purchase will the value of the car be \$10,240?

A) 1
B) 2
C) 3
D) 4

5 If k is a positive integer, what is the least value of k for which $\sqrt{\frac{5k}{4}}$ is an integer?

A) 4
B) 5
C) 20
D) 80

6 $A = \sqrt{s(s-a)(s-b)(s-c)}$

To determine the area of a triangle using the lengths of each side of the triangle, use Heron's Formula (above). The length of each side of the triangle is represented by a, b, and c. s is equal to $\frac{1}{2}$ of the triangle's perimeter. What is the area of a triangle with side lengths of 6, 10, 12 units?

A) $4\sqrt{14}$
B) $8\sqrt{14}$
C) $64\sqrt{28}$
D) $421\sqrt{8}$

7 $\sqrt{4x^2+1} = 2x+1$

Which values of x make the above equation true?

A) $x = 0$
B) $x > 0$
C) $x < 0$
D) The value cannot be determined with the information given.

8 The administration at a large university implemented a new policy to make the university more exclusive, which resulted in a decrease in student population of 10 percent every 4 years. If the population at the university when this policy was adopted was 25,000 students, which of the following expressions represents an estimate of the student population t years from the time of adoption?

A) $25{,}000(0.1)^{4t}$
B) $25{,}000(0.9)^{4t}$
C) $25{,}000(0.1)^{\frac{t}{4}}$
D) $25{,}000(0.9)^{\frac{t}{4}}$

9 The population of bees in a major city is in decline. If the city had an estimated 500,000 bees and they are dying off at the rate of 5% per year, approximately how many bees will there be in the city in ten years?

A) 30,000
B) 100,000
C) 300,000
D) 450,000

Student Produced Response – Calculator

Questions 1 and 2 refer to the following information.

The price of one share of stock in a certain company is worth \$270 today. A stock analyst believes the stock will lose 24% of its value each week for the next three weeks. The analyst uses the equation $V = 270(r)^t$ to model the value V, of the stock after t weeks.

1 How many weeks will it take for the value of the stock to decrease by \$114?

2 If this trend continues, how many weeks will it take for $V \leq \$50$?

3 $\sqrt{x^2 - t^2} = 2t - x$

If x and t are positive numbers that satisfy the equation above, what is the value of $\frac{x}{t}$?

Guided Practice – Systems of Equations

(Answers & explanations begin on page 419).

Multiple Choice – No Calculator

1 $y = 2x - 3$

$y = x^2 + 6x$

Which of the following ordered pairs represents a solution (x,y) to the system of equations above?

A) $(1,-1)$
B) $(1,3)$
C) $(-1,-3)$
D) $(-1,-5)$

2 $y = x^2 + ax + b$

$x = -3$

In the system of equations above, a and b are constants. For which of the following values of a and b does the system of equations have exactly two real solutions?

A) $a = 5, b = 6$
B) $a = 6, b = 9$
C) $a = -1, b = -6$
D) None of these

3 $x^2 + y^2 = r^2$

$y = |rx|$

In the system of equations above, r is a constant and $r \neq 0$. The solution(s) of this system can be found in

A) Quadrant I only
B) Quadrants I and II
C) Quadrants III and IV
D) Cannot be determined without knowing the value of r.

4 Which of the following is a system of equations with exactly two solutions (x_1, y_1) and (x_2, y_2) where $y_1 = y_2$?

A) $x + y = 5$ and $x + 2y = 5$
B) $x + y = 5$ and $2x + y = 5$
C) $x^2 + y^2 = 9$ and $y = 0$
D) $x^2 + y^2 = 9$ and $x = 0$

5 $y = 2x - 10$

$x = y^2 - 2y$

How many ordered pairs (x,y) satisfy the system of equations shown above?

A) 0
B) 1
C) 2
D) There are infinitely many ordered pairs.

6 $y = x^2 + 4$

$y = ax^2 + b$

In the system of equations above, a and b are constants. For which of the following values of a and b does the system of equations have no real solution?

A) $a = \frac{1}{2}, b = 2$
B) $a = 2, b = 2$
C) $a = \frac{1}{2}, b = 6$
D) $a = -1, b = 4$

7 $y = x^3 + 15x$

$y = cx^2$

In the system of equations above, c is a constant. For which of the following values of c does the system of equations have exactly three real solutions?

A) -5
B) 3
C) 8
D) None of these

8 $3x+4y=0$

$y=\frac{9}{4}x^2-6x+3$

If (x,y) is a solution of the system of equations above, what could be the value of y?

A) $-\frac{3}{4}$

B) $\frac{3}{4}$

C) $-\frac{4}{3}$

D) $\frac{4}{3}$

Student Produced Response – No Calculator

1 $y=x^2-x-35$

$x=y$

If (x,y) is a solution of the system of equations above and $x>0$, what is the value of x?

2 $y+4=|x-2|$

$x+2y=200$

If (x,y) is a solution of the system of equations above and $x<0$, what is the value of y?

Multiple Choice – Calculator

1 Which of the following does NOT represent the same system of equations as the other three choices?

A) $y=x^2+2x+1$

$y=-2x+6$

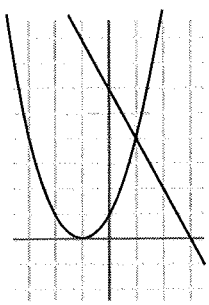
B) A system of equations with a unique solution at (4,1)

C)

Function I	
x	y
–1	0
0	1
1	4

Function II	
x	y
–1	8
0	6
1	4

D)



2 $y=(x-3)(x-7)$

$4x+y=13$

Which of the following ordered pairs represents a solution (x,y) to the system of equations above?

A) (3,0)

B) (2,4)

C) (2,5)

D) (3,7)

3 $y=(x-2)(x+12)$

$y=(x+5)^2-49$

How many ordered pairs (x,y) satisfy the system of equations shown above?

A) 0
B) 1
C) 2
D) There are infinitely many ordered pairs.

4 $y=5x^2+x+8$

$y=3x^2+4$

How many ordered pairs (x,y) satisfy the system of equations shown above?

A) 0
B) 1
C) 2
D) There are infinitely many ordered pairs.

5 $4x+y=0$

$y=mx^2+n$

In the system of equations above, m and n are constants. For which of the following values of m and n does the system of equations have exactly one real solution?

A) $m=2, n=8$
B) $m=2, n=2$
C) $m=-2, n=2$
D) $m=-1, n=4$

Student Produced Response – Calculator

1 $x=-4$

$y=x^3+300$

If (x,y) is a solution of the system of equations above, what is the value of y?

2 $y=5(x^2-4x+4)$

$y=x^2-1$

If (x,y) is a solution of the system of equations above, what could be the value of x?

Guided Practice – Function Notation

(Answers & explanations begin on page 421).

Multiple Choice – No Calculator

1 If $f(x)=5g(x)$ and $g(x)=2x+1$, what is $f(4)$?

A) 9
B) 20
C) 41
D) 45

2 A function f satisfies $f(7)=6$ and $f(6)=2$.
A function g satisfies $g(6)=7$ and $g(2)=0$.
What is the value of $f(g(6))$?

A) 0
B) 2
C) 6
D) 7

3 What is the value of $f(f(10))$ in the table below?

x	1	10	100	1,000	10,000
$f(x)$	1,000	10,000	10	100	1

A) 1
B) 100
C) 1000
D) 10,000

4 If $g(x)=2x+7$ and $f(g(x))=\sqrt{2x+3}$, which of the following describes $f(x)$?

A) $\sqrt{x+4}$
B) $\sqrt{x-4}$
C) $\sqrt{2x+4}$
D) $\sqrt{2x-4}$

5 $g(x)=ax^2-2ax+6$
For the function g defined above, a is a constant. What is the value of $g(2)$?

A) 0
B) 4
C) 6
D) Cannot be determined from the information given.

6 A function f satisfies $f(a)=b$ and $f(b)=c$.
A function g satisfies $g(a)=c$ and $g(b)=a$.
What is the value of $g(f(a))$?

A) a
B) b
C) c
D) bc

7 If $f(x)=x^2-9$ and $g(x)=x+5$, what is $f(g(x))$ equal to?

A) $x^3+5x^2-9x-45$
B) x^3-45
C) $x^2+10x+16$
D) x^2+x-4

8 The table below shows some values of the linear function f. Which of the following defines f?

n	1	2	3	4
$f(n)$	1	3	5	7

A) $f(n)=n+2$
B) $f(n)=2n-1$
C) $f(n)=3n-2$
D) $f(n)=4n-5$

9 The table below shows some values of the linear function f. Which of the following defines f?

n	1	2	3	4
$f(n)$	1	4	7	10

A) $f(n)=n+4$
B) $f(n)=2n-1$
C) $f(n)=3n-2$
D) $f(n)=4n-6$

10 If $f(x)=2x+4$, which of the following is equal to $4x+8$?

A) $f(2)$
B) $f(2x)$
C) $f(2x+2)$
D) $f(2x+4)$

11 If $f(x) = x^2 + 2$ and $g(x) = 5x$, which of the following is equal to $f(g(x)) - g(f(x))$?

A) 0
B) $x^2 - 5x + 2$
C) $5x^3 + 10x$
D) $20x^2 - 8$

12 If $y(x) = \sqrt{x^2 - 6}$ which of the following is equivalent to $y(y(x))$?

A) $\sqrt{x^2 - 6}$
B) $\sqrt{x^2 - 6} - 6$
C) $\sqrt{x^2 - 12}$
D) $\sqrt{(x^2 - 6)^2 - 6}$

13 The function $f(x)$ is graphed on the xy-plane. The graph $y = g(x)$ is equivalent to the graph $y = f(x)$ reflected over the y-axis. Which of the following correctly relates $f(x)$ and $g(x)$?

A) $f(x) = g(x)$
B) $f(x) = g(-x)$
C) $f(x) = -g(x)$
D) $f(x) = -g(-x)$

14 The function $p(x)$ is graphed on the xy-plane. The graph $y = q(x)$ is equivalent to the graph $y = p(x)$ translated 4 units up and then reflected over the x-axis. The graph $y = r(x)$ is equivalent to the graph $y = p(x)$ reflected over the x-axis and then translated 4 units up. Which of the following correctly relates $q(x)$ and $r(x)$?

A) $q(x) = r(x)$
B) $q(x) = -r(x - 4)$
C) $q(x) = -r(x) - 4$
D) $q(x) = r(x) - 8$

Student Produced Response – No Calculator

1 If $g(x) = 3x + 4$ and $f(x) = g(x) + 10$, what is $f(3)$?

2 $f(x) = x^2 - 13x + 40$

For the function f above, $f(0) = f(a)$ where a is a constant and $a > 0$. What is the value of a?

Multiple Choice – Calculator

1 $g(x) = cx^2 - 6$

For the function g defined above, c is a constant and $g(2) = 6$. What is the value of $g(-2)$?

A) 6
B) 3
C) -6
D) -18

2 $f(x) = ax - 4$

For the function f defined above, a is a constant and $f(2) = 16$. What is the value of $f(3)$?

A) 26
B) 24
C) 10
D) -1

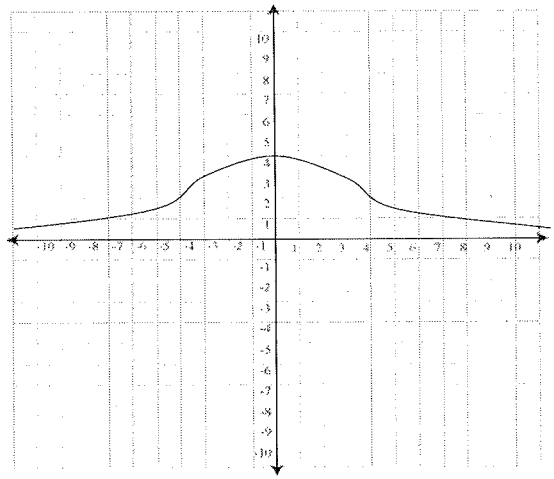
3 If $f(x) = x^2 - 3$ and $g(x) = 4x - 4$, which of the following is true for all values of x?

A) $f(x) = -f(x)$
B) $f(x) = f(-x)$
C) $g(x) = -g(x)$
D) $g(x) = g(-x)$

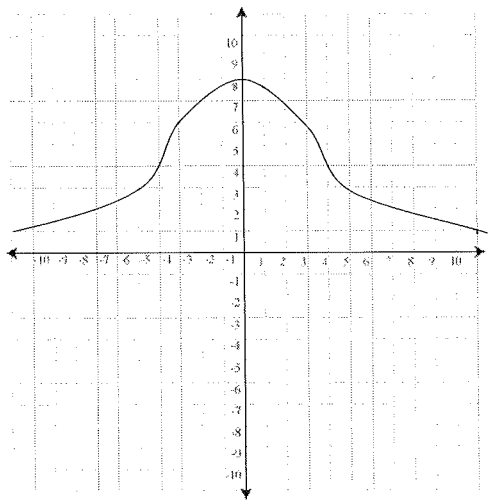
4 If $f(x) = 3x - 4$ and $g(x) = 2x + 2$, what is the value of $f(g(3))$?

A) 40
B) 20
C) 12
D) 8

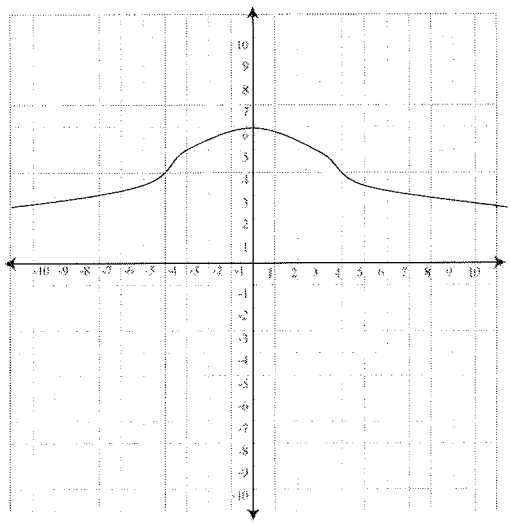
5 The graph of $y = f(x)$ is shown below. If $g(x) = 2f(-x)$, which of the following is the graph of $y = g(x)$?



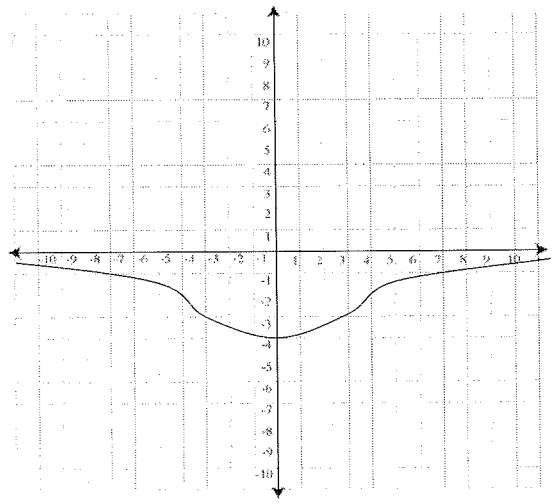
A)



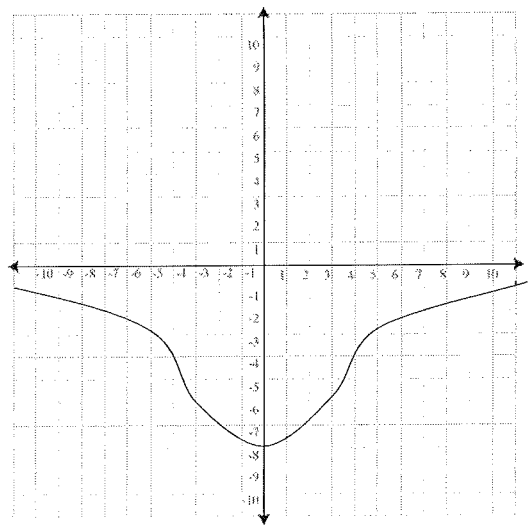
B)



C)



D)



6 If $f(x) = 4x + 6$, what is $f(-5x)$ equal to?

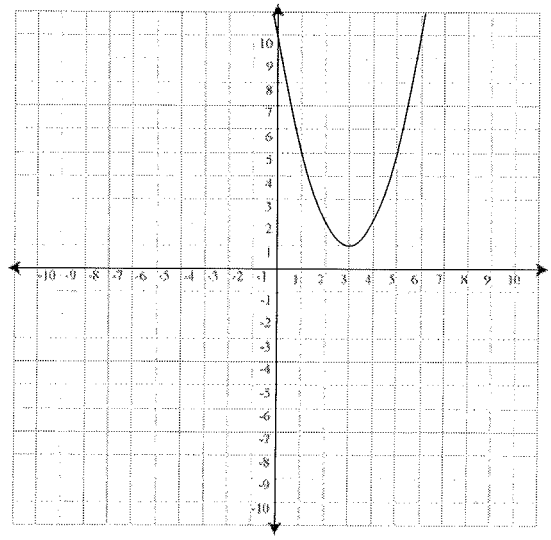
A) $-x + 6$
B) $-20x + 6$
C) $-20x - 30$
D) $-20x^2 - 30x$

Student Produced Response – Calculator

1 If $f(x) = 2x - 4$ and $g(x) = 3f(x)$, what is $g(20)$?

2 If $h(x) = f(x) + g(x)$ and $g(x) = 3x + 7$ and $f(x) = 2x + 5$, what is $h(2)$?

3 The graph of $y = f(x + 6)$ is shown below.



For which value of x, rounded to the nearest whole number, must $f(x) = 1$?

Guided Practice – Graphs of Functions

(Answers & explanations begin on page 423).

Multiple Choice – No Calculator

1 The graph of which of the following functions on the *xy*-plane has *x*-intercepts at –3 and 2?

A) $f(x)=(x+3)(x-2)$

B) $g(x)=(x-3)(x+2)$

C) $h(x)=(x-3)^2+2$

D) $k(x)=(x+2)^2-3$

2 Which of the following equations has a graph on the *xy*-plane for which y is always greater than or equal to 0?

A) $y=|x|-1$

B) $y=x^2-1$

C) $y=(x-1)^2$

D) $y=x^3-1$

3 On the *xy*-plane, the parabola with the equation $y=(x-5)^2$ intersects the line with equation $y=4$ at two points, A and B. What is the length of $\overline{AB}$?

A) 4
B) 6
C) 8
D) 10

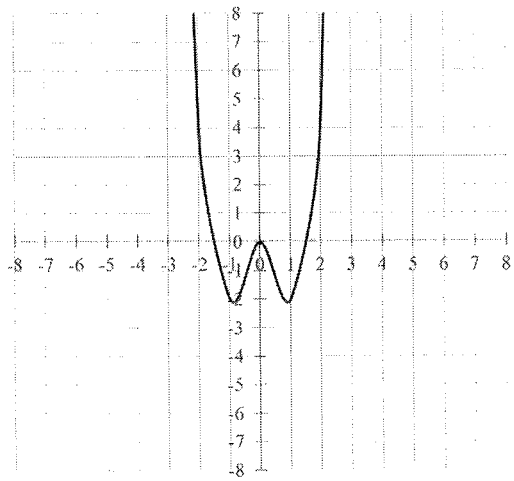
4 If $f(x)=\sqrt{2x-8}$, the domain of the function is equal to

A) all real numbers.
B) all real numbers greater than or equal to 8.
C) all real numbers greater than or equal to 4.
D) all real numbers greater than or equal to 2.

5 What is the range of the function $f(x)=x^2+2x+2$?

A) All real numbers.
B) $\{-1, 1\}$
C) $y\geq 1$
D) The range is undefined.

6



The function $f(x)=x^4-3x^2$ is graphed on the *xy*-plane as shown above. If k is a constant such that the equation $f(x)=k$ has 4 solutions, which of the following could be a value of k?

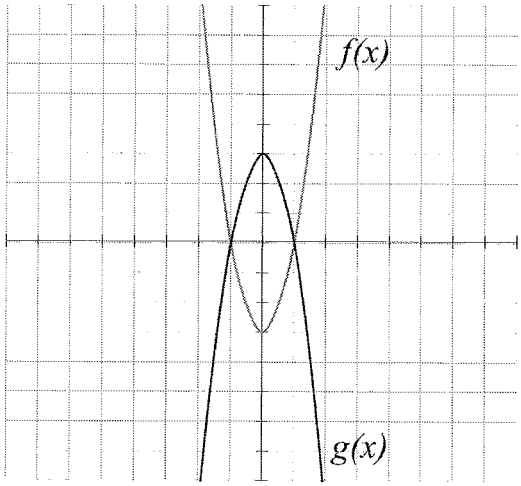
A) 3
B) 1
C) 0
D) –1

7 $y=a(x-2)(x+6)$

In the quadratic equation above, a is a nonzero constant. The graph of the equation on the *xy*-plane is a parabola with vertex (c,d). Which of the following is equal to d?

A) $-4a$
B) $-8a$
C) –12a
D) –16a

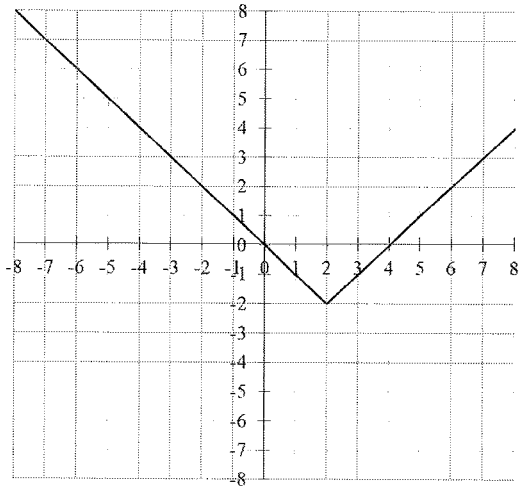
8



The functions f and g are defined by $f(x)=3x^2-3$ and $g(x)=-3x^2+3$, and graphed on the xy-plane above. The graphs of f and g intersect at the points $(k,0)$ and $(-k,0)$. Which of the following could be the value of x?

A) $\frac{1}{2}$
B) 0
C) 1
D) 2

9



The graph above shows $f(x)$. Which equation is equal to $f(x)$?

A) $f(x)=|x|$
B) $f(x)=|x-2|$
C) $f(x)=|x-2|-2$
D) $f(x)=(x-2)-2$

Student Produced Response – No Calculator

1 On the xy-plane (1,5) lies on the graph of the function $f(x)=bx+2$. What is the value of b?

2 On the xy-plane, the parabola with the equation $y=(x-4)^2$ intersects the line with equation $y=25$ at 2 points, A and B. What is the length of $\overline{AB}$?

3 $y=a(x+1)(x-3)$

In the quadratic equation above, a is a nonzero constant. The graph of the equation on the xy-plane is a parabola with vertex $(1,-8)$. What is the value of a?

Student Produced Response – Calculator

1 On the xy-plane, (3,2) lies on the graph of the function $f(x)=2x^2-bx+2$. What is the value of b?

2 The function f is defined by $f(x)=x^3+3x^2+kx-6$, where k is a constant. On the xy-plane, the graph of f intersects the x-axis at a single point, namely $(1,0)$. What is the value of k?

3 Suppose the graph of $f(x)=x^2$ is translated 2 units left and 1 unit up. If the resulting graph represents $g(x)$, what is the value of $g(3)$?

Multiple Choice – Calculator

1 $y = x^2 - 7x + 12$

The equation above represents a parabola on the xy-plane. Which of the following equivalent forms of the equation displays the x-intercepts of the parabola as constants or coefficients?

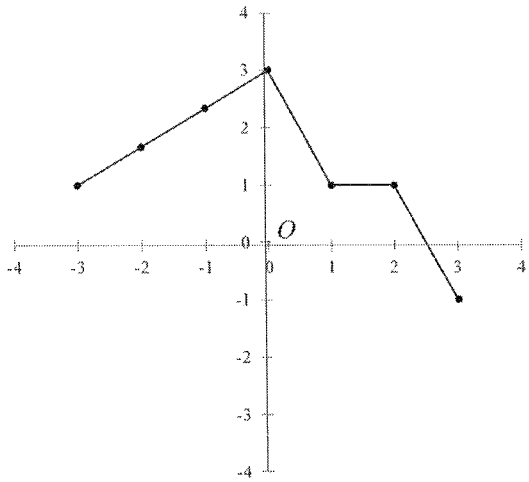
A) $y - 12 = x^2 - 7x$

B) $y - 1 = (x - 3)^2$

C) $y = x(x - 7) + 12$

D) $y = (x - 3)(x - 4)$

2



The complete graph of the function f is shown on the xy-plane above. Which of the following are equal to 1?

I. $f(-3)$

II. $f(1)$

III. $f(2)$

A) I only

B) II only

C) II and III only

D) I, II, and III

3 $y = x^2 - 8x + 12$

The equation above represents a parabola on the xy-plane. Which of the following equivalent forms of the equation displays the xy-coordinates of the vertex of the parabola as constants or coefficients?

A) $y - 12 = x^2 - 8x$

B) $y = (x - 10)(x + 2)$

C) $y = (x - 2)(x - 6)$

D) $y = (x - 4)^2 - 4$

4 If $f(-x) = f(x)$ for all real numbers x and if $(1,4)$ is a point on the graph of f, which of the following points must also be on the graph of f?

A) $(-4,-1)$

B) $(-1,-4)$

C) $(-1,4)$

D) $(1,-4)$

5 The function f is defined by $f(x) = 2x^3 + x^2 + kx + 4$, where k is a constant. On the xy-plane, the graph of f intersects the x-axis at a single point, namely $(-1,0)$. What is the value of k?

A) -6

B) 1

C) 3

D) 4

6 $y = x^2 + 6x + 9$

The equation above represents a parabola on the xy-plane. Which of the following equivalent forms of the equation correctly displays the x-intercepts of the parabola as constants or coefficients?

A) $y - 9 = x^2 + 6x$

B) $y - 1 = (x + 2)(x + 4)$

C) $y = x(x + 6) + 9$

D) $y = (x + 3)^2$

7 For a rational function $\frac{x^2 + 4x + 2}{x + 2}$, there is a vertical asymptote at $x = a$. What is the value of a?

A) -2

B) 0

C) 2

D) 4

8 What is the smallest value that belongs to the range of the function $f(x) = 2|x - 4| + 2$?

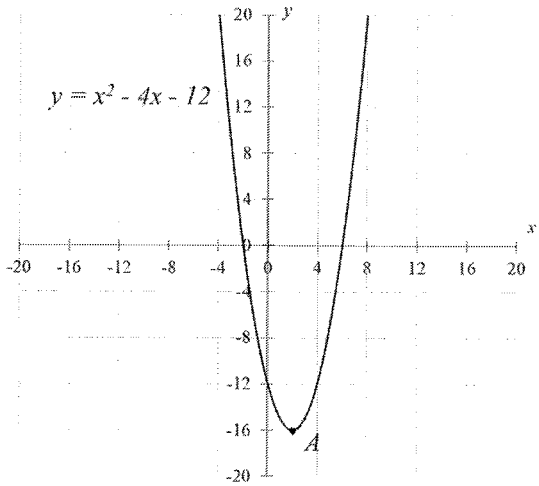
A) -2

B) 0

C) 2

D) 4

9



Which of the following is an equivalent form of the equation of the graph shown on the xy-plane above, from which the coordinates of vertex A can be identified as constants in the equation?

A) $y = (x-2)(x+6)$

B) $y = (x+2)(x-6)$

C) $y = (x-4)^2$

D) $y = (x-2)^2 - 16$

10 On the xy-plane, the graph of function f has x-intercepts at -2, -1, and 2. Which of the following could define f?

A) $f(x) = (x-2)^2(x-1)$

B) $f(x) = (x-2)(x-1)(x+2)$

C) $f(x) = (x+2)(x+1)(x-2)$

D) $f(x) = (x+2)^2(x+1)$

11 Suppose the graph of $f(x) = -x^2$ is translated 3 units left and 1 unit up. If the resulting graph represents $g(x)$, what is the value of $g(-2)$?

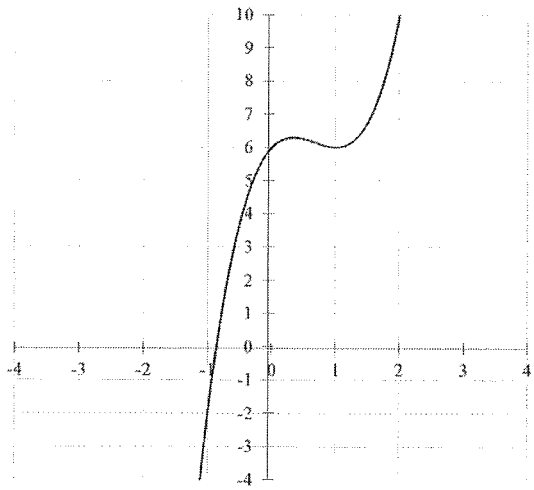
A) -1

B) 0

C) 1

D) 2

12



If $f(x) = 2x^3 - 4x^2 + 2x + 6$, which of the following statements are true?

I. The function f is increasing for $x \geq 1$.

II. The equation $f(x) = 0$ has two real solutions.

III. $f(x) \geq 6$ for all $x \geq 0$.

A) I only

B) II only

C) I and II

D) I and III

13 What is the range of the function defined by $f(x) = \frac{1}{x} - 2$?

A) All real numbers.

B) All real numbers except -2.

C) All real numbers except 0.

D) All real numbers except 2.

14 Which of the following values of x is in the domain of the function $f(x) = \frac{x-2}{x^2+3x-10}$?

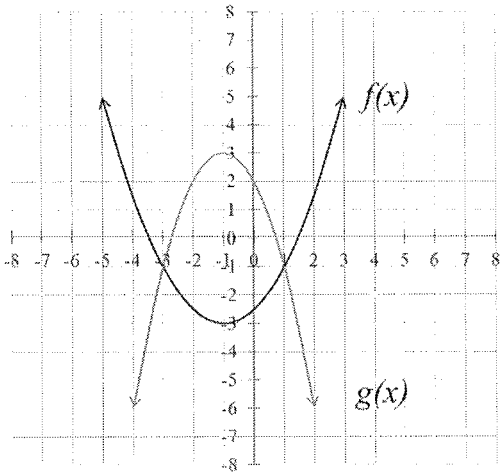
A) -5

B) 0

C) 2

D) The domain includes all real numbers.

15

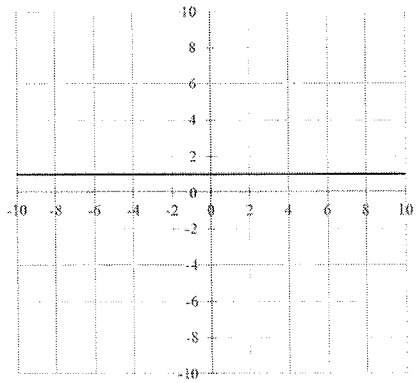


Graphs of the functions f and g are shown on the xy-plane above. For which of the following values of x does $f(x)+g(x)=0$?

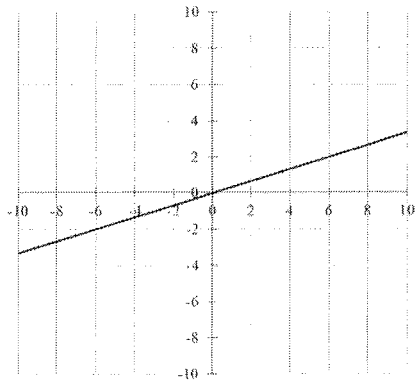
A) –3
B) –1
C) 1
D) 3

16 If k is a positive constant and $k \neq 1$, which graph could be the graph of $y+x=k(x-y)$ on the xy-plane?

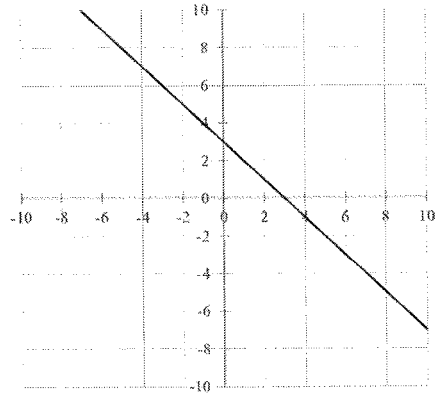
A)



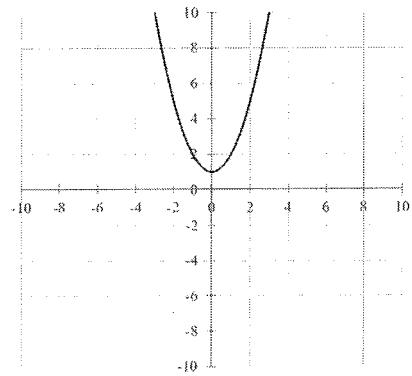
B)



C)



D)



Guided Practice – More Word Problems

(Answers & explanations begin on page 425).

Multiple Choice – No Calculator

1 The bill, b, for a particular cell phone plan is calculated using the formula $b = 0.45m + 40$ where m is the number of extra minutes that the person has used above their monthly contract. Which of the following expresses the number of extra minutes in terms of the bill?

A) $m = \dfrac{b+40}{0.45}$

B) $m = \dfrac{b-40}{0.45}$

C) $m = \dfrac{40-b}{0.45}$

D) $m = \dfrac{0.45}{b+40}$

2 At a particular fitness center, customers get their first two yoga classes for free, and after that the classes cost \$12 each. The cost, c, of taking y yoga classes can be modeled by the equation $c = 12y - 24$ for all $y \geq 2$. Which of the following expresses the number of yoga classes in terms of the cost?

A) $y = \dfrac{c}{12} + 2$

B) $y = \dfrac{c}{12} + 24$

C) $y = \dfrac{c}{12} - 2$

D) $y = \dfrac{c}{12} - 24$

3 $w = -0.5y^2 + ay + b$

The equation above gives the hourly wage, w, of an employee who has worked at the company for y years. Which of the following gives a in terms of w, y, and b?

A) $a = w + b - 0.5y$

B) $a = \dfrac{w - b + 0.5}{y}$

C) $a = \dfrac{w-b}{y} + 0.5y$

D) $a = \dfrac{w+b}{y} - 0.5y$

The next two questions refer to the below information:

$C(t) = 100 - t$

$L(t) = 2t + 33$

Two customers at a restaurant order coffee and lemonade. As the beverages sit on the table, their temperatures change. The function $C(t)$ gives the temperature of the coffee t minutes after it is served. The function $L(t)$ gives the temperature of the lemonade t minutes after it is served.

4 How will the temperature of the lemonade change over a period of 10 minutes?

A) The temperature will increase by 2 degrees.

B) The temperature will decrease by 10 degrees.

C) The temperature will increase by 20 degrees.

D) The temperature will increase by 53 degrees.

5 If the temperature of the lemonade increases by 10 degrees, the temperature of the coffee will

A) increase by 10 degrees.

B) decrease by 5 degrees.

C) decrease by 10 degrees.

D) decrease by 20 degrees.

The next two questions refer to the below information:

$$R = \frac{\sqrt{P}}{2}$$

A company uses the formula above to calculate an employee's rating, R, based on the number of projects, P, that he or she completes.

6 Which of the following expresses the number of projects an employee completed in terms of their rating?

A) $P = 2R$
B) $P = 4R$
C) $P = 2R^2$
D) $P = 4R^2$

7 Anna completed 9 times as many projects as Brenda. Brenda's rating will be what fraction of Anna's rating?

A) $\frac{1}{3}$
B) $\frac{1}{9}$
C) $\frac{2}{3}$
D) $\frac{2}{9}$

8 An animal's height, h, in inches, when it is m months old is modeled by the function $h(m) = \frac{1}{6}m + 20$. How does the animal's height change over the course of one year?

A) Its height increases by 2 inches.
B) Its height increases by 12 inches.
C) Its height increases by 20 inches.
D) Its height increases by 22 inches.

The next two questions refer to the below information:

$$h = \frac{w}{5r^2}$$

The formula above is used to calculate the number of hours, h, that it takes for a circular field of radius r to absorb w gallons of water.

9 Which of the following expresses the square of the radius in terms of the number of hours and gallons of water?

A) $r^2 = \frac{5h}{w}$
B) $r^2 = \frac{w}{5h}$
C) $r^2 = \frac{h}{5w}$
D) $r^2 = \frac{hw}{5}$

10 The same amount of water is poured onto Field A and Field B, but on Field B, it takes 4 times as long for the water to be absorbed. The radius of Field B is what fraction of the radius of Field A?

A) $\frac{1}{2}$
B) $\frac{1}{4}$
C) $\frac{1}{8}$
D) $\frac{1}{16}$

11 A factory builds hollow plastic cubes. The mass, m, of each cube can be calculated using the formula $m = 6ds^2$ where d is the density of the plastic and s is the length of each side of the cube. If the density of the plastic is doubled and the length of each side is halved, how does the mass change?

A) The mass is doubled.
B) The mass is unchanged.
C) The mass is halved.
D) The mass is quartered (divided by 4).

12 $h = -3t^2 - 3t + 60$

The equation above expresses the height, h, of a rock in feet above the bottom of a lake t seconds after it is thrown into the lake. How many seconds will pass from when the rock is thrown until it hits the bottom of the lake?

A) 4
B) 5
C) 9
D) 10

13 $c = dpk$

The formula above gives the total cost, c, of gasoline when a car drives a distance of d miles and pays a price of p dollars per gallon of gasoline. The quantity k in this formula could represent which of the following?

A) The amount of gasoline required for the trip.
B) The amount of gasoline required per mile.
C) The number of miles driven per gallon of gasoline.
D) The number of times the car stops to refill the gasoline.

Student Produced Response – No Calculator

1 The formula $PV = nRT$ is known as the ideal gas law, and is useful to predict the behavior of a gas under ideal conditions. In this equation, P represents the pressure of the gas, V represents the volume, n represents the amount of gas, R is the universal gas constant, and T is the temperature. In a particular lab experiment, the volume of a gas is doubled, the temperature is multiplied by 1.5, and the amount of gas is multiplied by 20. Under ideal conditions, the pressure of this gas will be multiplied by what number?

2 $m = 12t^2l$

If a metal rod has length l and thickness t, then its mass, m, is calculated using the formula above. Two rods have the same mass, but one has thickness t and the other has thickness $\frac{3}{2}t$. What is the ratio of the length of the thicker rod to the length of the thinner rod? (Grid your answer in fraction form.)

Multiple Choice – Calculator

1 A truck is carrying boxes of bricks. The total weight of the truck's cargo is calculated by the expression $n(bK + w)$ where n is the number of boxes, b is the number of bricks per box, K is a constant, and w is the weight of an empty box. If the brick company decides to start making their bricks with a heavier material, which of the quantities in the expression would change?

A) n
B) b
C) K
D) w

2 $h = -16t^2 + 65t$

The equation above expresses the height, h, in feet, of a ball t seconds after it is launched from the ground with an upward velocity of 65 feet per second. After approximately how many seconds will the ball hit the ground?

A) 3.5
B) 4.0
C) 4.5
D) 5.0

3 A sprinter finds that his time, t, in seconds, for the 100-meter dash depends on the hours of sleep, s, that he gets the night before the race. This relationship is modeled by the function $t(s) = s^2 - 16s + 75$. Which of the following equivalent forms of $t(s)$ shows, as constants or coefficients, the minimum possible time that he can run the race and the amount of sleep that results in that minimum time?

A) $t(s) = (s-8)^2 + 11$

B) $t(s) = s(s-1) - 15(s-5)$

C) $t(s) = s(s-10) - 3(2s-25)$

D) $t(s) = s(s-16) + 75$

4 $\frac{\pi d}{4} = \sqrt{\frac{w}{p}}$

The equation above shows the relationship between the diameter, d, of a cylindrical column of liquid, its weight, w, and the average pressure, p, exerted by the liquid. Which of the following correctly expresses w in terms of d and p?

A) $w = \frac{\pi d\sqrt{p}}{4}$

B) $w = \frac{\pi d^2 p}{4}$

C) $w = \frac{\pi d^2}{16p}$

D) $w = \frac{\pi^2 d^2 p}{16}$

5 $P = \frac{r}{r+b}$

The probability, P, of picking a red marble out of a bag containing r red marbles and b blue marbles is calculated using the formula above. Let D represent the probability of picking a red marble if r is doubled but b remains the same. Which of the following equations expresses D in terms of P?

A) $D = P$

B) $D = 2P$

C) $D = \frac{2P}{P+1}$

D) $D = \frac{2P}{2P+1}$

6 A vehicle manufacturer estimates that if a particular vehicle is driven at a velocity of v miles per hour, then its gas mileage, m, in miles per gallon is $m(v) = -\frac{1}{10}v^2 + 4v$.

Which of the following equivalent forms of $m(v)$ shows, as constants or coefficients, the maximum possible gas mileage and the velocity that results in that maximum gas mileage?

A) $m(v) = 4v - \frac{1}{10}v^2$

B) $m(v) = -\frac{1}{10}(v^2 - 40v)$

C) $m(v) = 4v\left(1 - \frac{1}{40}v\right)$

D) $m(v) = -\frac{1}{10}(v-20)^2 + 40$

7 The number of cells, c in a test tube after t minutes is modeled by the equation $c = ke^{rt}$. If r in the equation is replaced with $2r$, how will the value of c change?

A) The value of c will be doubled.

B) The value of c will be squared.

C) The value of c will be multiplied by e.

D) The value of c will be multiplied by e^{rt}.

Student Produced Response – Calculator

The next two questions refer to the below information:

At the beginning of the week a certain pond had 270 cubic meters of water. Evaporation caused the amount of water in the pond to decrease by 12% each day. The equation $V = 270(r)^t$ models the volume, V, of the pond after t days.

1 What is the value of r?

2 To the nearest cubic meter, what is the volume of the pond after 3 days?

3 $B = \frac{m}{h^2}$

The body mass index (B) of a person who has mass m and height h is calculated using the formula above. If two people have the same mass, but one has height h and the other has height $0.9h$, what is the ratio (to the nearest hundredth) of the taller person's body mass index to the shorter person's body mass index?

Mixed Practice – Advanced Math

(Answers & explanations begin on page 428).

Multiple Choice – No Calculator

1 $9\sqrt{x^3}+3=x-6$

For all values of x greater than 0, the equation above is equivalent to which of the following?

A) $x^3=\dfrac{x-9}{9}$

B) $x^3=x^2-18x$

C) $x^3=\dfrac{x^2-18x+81}{81}$

D) $x=\sqrt{\dfrac{x^2-18x+81}{81}}$

2 $c=kx^2+20x+b$

The equation above gives the cost of tiling a bathroom that is x feet long and x feet wide. Which of the following gives k in terms of x, b, and c?

A) $k=\dfrac{c-b-20}{x^2}$

B) $k=\dfrac{c+b+20x}{x^2}$

C) $k=\dfrac{c-b}{x^2}-20$

D) $k=\dfrac{c-b}{x^2}-\dfrac{20}{x}$

3 $y=3x^2-8x+36$

$y=2x^2+2x+11$

If (x,y) is a solution of the system of equations above, what is the value of $x+y$?

A) 10
B) 25
C) 47
D) 76

4 A science student determines that the amount of energy, E, required to carry a water balloon a distance d can be represented by the equation $E=kdr^3$ where r is the radius of the water balloon and k is a constant. If the distance and radius are both doubled, the energy will be multiplied by

A) 2
B) 4
C) 8
D) 16

5 A function f satisfies $f(1)=0$ and $f(0)=-1$.

A function g satisfies $g(-1)=2$ and $g(0)=1$.

What is the value of $g(f(0))$?

A) -1
B) 0
C) 1
D) 2

6 Which of the following equations has exactly one real solution?

I. $x^2-6x+9=0$

II. $x^3+5x=0$

III. $x^2-4=0$

A) I only
B) I and II
C) II and III
D) III only

7 $y=x^2+4x+4$

The equation above represents a parabola on the xy-plane. Which of the following equivalent forms of the equation displays the x-intercepts of the parabola as constants or coefficients?

A) $y-5=x^2+4x$

B) $y=(x+2)^2$

C) $y=x(x+2)+4$

D) $y=(x+4)^2$

Multiple Choice – Calculator

1 $\frac{t}{t-2}+\frac{1}{t-4}=\frac{2}{t^2-6t+8}$

If t is a solution to the equation above, what is the value of t?

A) 6
B) 4
C) 2
D) –1

2 If $\frac{4}{x-1}=\frac{7}{x+8}$, what is the value of x?

A) $\frac{3}{25}$
B) $\frac{25}{3}$
C) 13
D) $\frac{1}{7}$

3 If $f(x)=2x-7$, what is $f(x+4)$ equal to?

A) $2x+1$
B) $2x-3$
C) $8x-7$
D) $2x^2+x-28$

4 A teacher bought 30 books. Out of those 30 books, there were x books that cost 10 dollars each. The rest of the books cost $\frac{x}{2}$ dollars each. If the total cost of the 30 books was \$200, which equation could be used to find the value of x?

A) $x^2-50x+400=0$
B) $x^2+20x-400=0$
C) $2x^2+3x-40=0$
D) $3x^2+2x-40=0$

5 $y-6=-(x+2)^2$
$2x+y=3$

How many ordered pairs (x,y) satisfy the system of equations shown above?

A) 0
B) 1
C) 2
D) There are infinitely many ordered pairs.

6 Which of the following is NOT divisible by $x-8$?

A) $x^4(x-8)+17(x-8)$
B) $(x-8)(x+2)+6$
C) x^3-13x^2+40x
D) x^2y-64y

Student Produced Response – Calculator

1 Joe's savings account has a balance of \$1,325. The account earns 2% interest compounded annually. If Joe has left the money in the account for 5 years with no additional deposits or withdrawals, how much money did he initially deposit into the account?

2 Suppose the graph of $f(x)=x^3$ is translated 3 units left and 2 unit up. If the resulting graph represents $g(x)$, what is the value of $g(2)$?

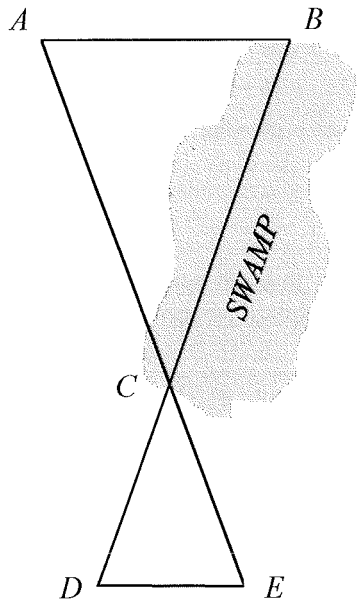
Additional Math Topics

Guided Practice – Geometry

(Answers & explanations begin on page 429)

Multiple Choice – No Calculator

1 In the figure below, line segments AB and DE are parallel. A surveyor needs to find the distance BC across a swamp.



Note: Figure not drawn to scale.

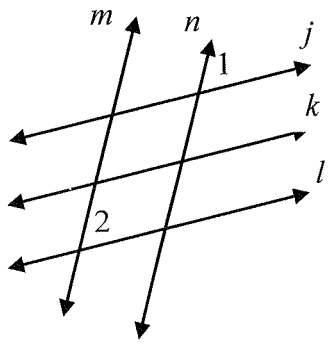
The length of DE is 4 meters, AB is 12 meters, DC is 5 meters, and EC is 6 meters. What is the length of BC?

A) 12 meters
B) 15 meters
C) 18 meters
D) 21 meters

2 An oil storage container is a right circular cylinder that has a diameter of 20 meters and height of 8 meters. What is the volume of the oil storage container in cubic meters?

A) 640π
B) 800π
C) $1{,}280\pi$
D) $3{,}200\pi$

3

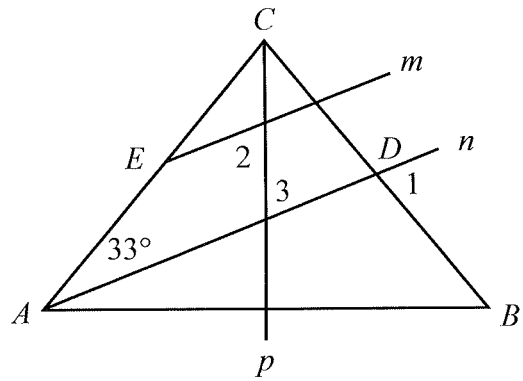


Note: Figure not drawn to scale.

In the figure above, lines j, k, and l are parallel to each other and lines m and n are parallel to each other. If the measure of $\angle 1$ is 65°, what is the measure of $\angle 2$?

A) 25°
B) 65°
C) 115°
D) 135°

4 In the figure below, lines m and n are parallel, and line n intersects segment BC at point D.

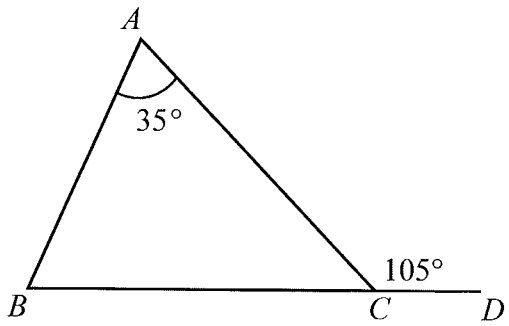


Note: Figure not drawn to scale.

If angle $CAD = 33°$ and $\angle 1 = 78°$, what is the measure of angle ACB?

A) 69°
B) 73°
C) 75°
D) 78°

5 In the figure below, $\angle DCA = 105°$ and $\angle BAC = 35°$.

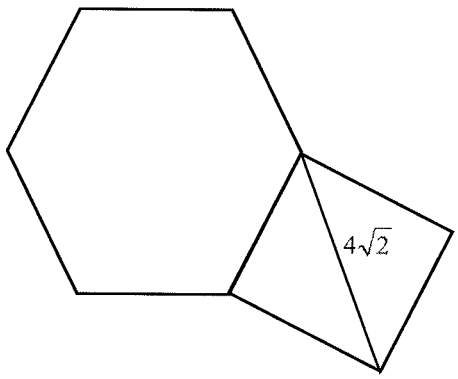


Note: Figure not drawn to scale.

What is the measure of $\angle ABC$?

A) 60°
B) 65°
C) 70°
D) 75°

6

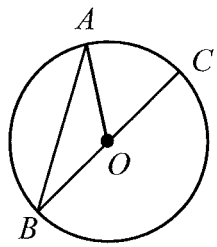


Note: Figure not drawn to scale.

In the picture above, a regular hexagon is attached to a square with diagonal distance of $4\sqrt{2}$. One side of the square is equal to one side of the hexagon. Find the area of the hexagon.

A) $96\sqrt{3}$
B) $48\sqrt{3}$
C) $24\sqrt{3}$
D) $18\sqrt{3}$

Questions 7 and 8 refer to the following image.



Note: Figure not drawn to scale.

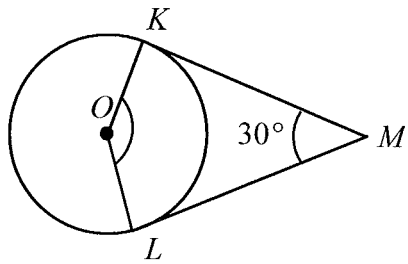
7 In the figure above, if the central angle $AOC = 60°$, find the measure of angle ABC.

A) 15°
B) 20°
C) 30°
D) 60°

8 In the figure above, if central angle $AOC = 56°$, what is the angle OAB?

A) 14°
B) 28°
C) 56°
D) 62°

9 In the figure below, point O is the center of the circle, line segments LM and KM are tangent to the circle at points L and K, respectively, and the segments intersect at point M.

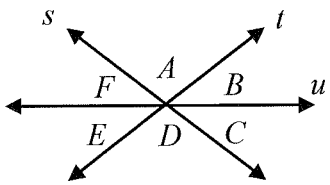


Note: Figure not drawn to scale.

If the circumference of the circle is 720, what is the length of major arc LK?

A) 300
B) 480
C) 600
D) 660

10 In the figure below, lines s, t, and u intersect at a point.

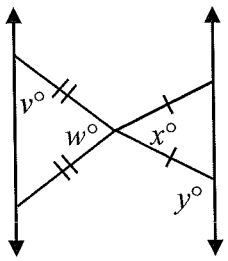


If the measures of angles $A + B = C + D$, which of the following must be true?

I. $B + C + D = A + F + B$
II. $E = C$
III. $A = D$

A) III only
B) I and III only
C) II and III only
D) I, II, and III

11

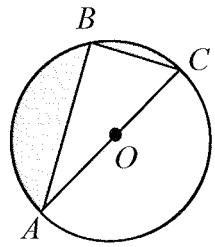


Note: Figure not drawn to scale.

In the figure above, the two vertical lines are parallel, angle $v = 40°$ and angle $w = 2x°$. What is angle $y°$?

A) 110°
B) 115°
C) 125°
D) 140°

12 In the figure below, triangle ABC is inscribed in a semicircle with a diameter of 20. Angle BAC subtends arc BC, which is 60°.

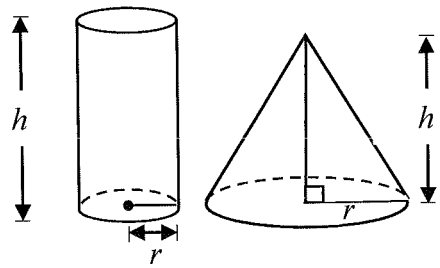


Note: Figure not drawn to scale.

What is the area of the shaded region?

A) $50\pi - 10\sqrt{2}$
B) $50\pi - 50\sqrt{3}$
C) $100\pi - 10\sqrt{2}$
D) $100\pi - 50\sqrt{3}$

13 In the figure below, a right circular cylinder and a right circular cone are drawn, but not to scale.



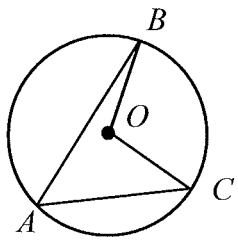
Which of the following conditions will always result in the cone and cylinder having equal volumes?

I. The cone has a base radius one third the radius of the cylinder and the height of the cone is nine times the height of the cylinder.
II. The cylinder has a base radius twice the radius of the cone and the cone's height is twelve times the cylinder's height.
III. The cone and cylinder have the same base radius and the height of the cylinder is one third the height of the cone.

A) I and II only
B) I and III only
C) II and III only
D) I, II, and III

Student Produced Response – No Calculator

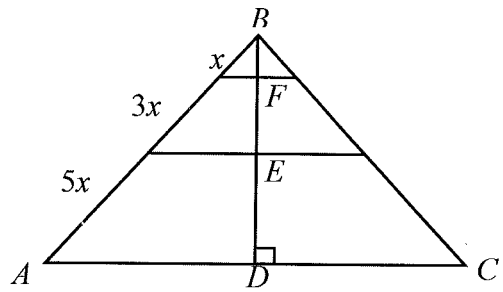
1



Note: Figure not drawn to scale.

In the figure above, angle BAC is 43° and subtends arc BC. What is the measure, in degrees, of central angle BOC?

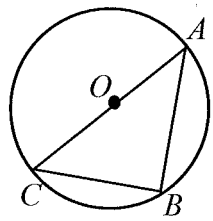
2 In the figure below, the height of the triangle ABC is the length of the segment BD.



Note: Figure not drawn to scale.

If $BD = 90$ and segment AB is split into parts with proportions as shown, find the length of DE.

3 In the figure below, isosceles triangle ABC is inscribed in a semicircle.

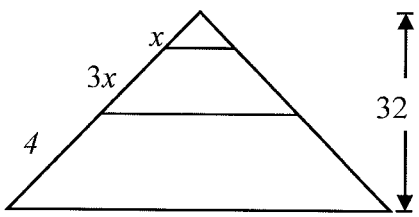


Note: Figure not drawn to scale.

If the circumference of the circle is 8π, what is the area of the triangle ABC in square units?

Multiple Choice – Calculator

1 As shown on the figure below, a cabin is designed so that the roof rests on the ground. The intersections of the roof lines and first floor, second floor, and attic are in the proportions shown. The entire structure is 32 feet tall.



Note: Figure not drawn to scale.

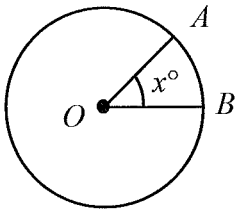
What is the height of the first floor?

A) 28 feet
B) 24 feet
C) 20 feet
D) 16 feet

2 A bush and a tree cast shadows 2 meters and 15 meters long, respectively. The height of the bush is measured at 0.7 meters. What is the height of the tree?

A) 1.75 meters
B) 5.25 meters
C) 10.5 meters
D) 17.3 meters

3 In the figure below, the minor arc length $AB = 1.58$ cm and angle $x = 8°$.

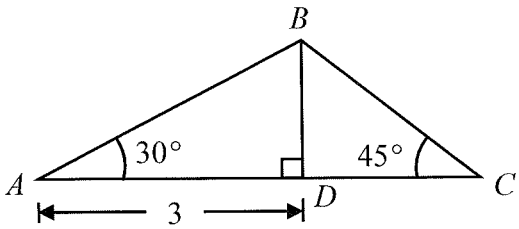


Note: Figure not drawn to scale.

Which of the following is the closest to the radius of the circle?

A) 3.06 cm
B) 6.10 cm
C) 9.13 cm
D) 11.32 cm

4

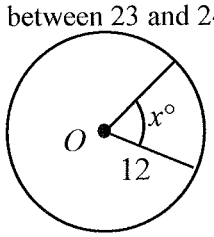


Note: Figure not drawn to scale.

Find the length of side BC in the figure above.

A) $\sqrt{2}$
B) $3\sqrt{2}$
C) $\sqrt{6}$
D) $3\sqrt{6}$

5 The figure below shows a circle with radius 12. A central angle $x°$ subtends an arc length between 23 and 24.

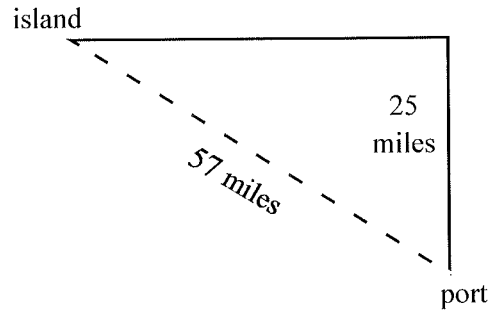


Note: Figure not drawn to scale.

Find the range of integer answers for x.

A) $105 \le x \le 109$
B) $110 \le x \le 114$
C) $115 \le x \le 119$
D) $120 \le x \le 124$

6 Captain Tom is sailing his ship from a port to an island. The direct route is 57 miles. When a storm comes, he decides to take a detour by heading 25 miles north until he is directly east of the island, then sailing west directly towards the island, as shown below.

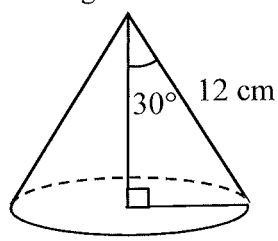


Note: Figure not drawn to scale.

To the nearest mile, how many miles west does Captain Tom have to sail to reach the island?

A) 13
B) 32
C) 51
D) 62

7 In the figure below, a right circular cone has a slant height of 12 cm and the altitude makes a 30° angle with the slant height.



Note: Figure not drawn to scale.

Which of the following is closest to the volume of the cone, in cubic centimeters?

A) 65.3
B) 130.6
C) 195.9
D) 391.8

8 Two geometric solids, a pyramid with square base and a rectangular solid, have equal volumes. Which of the following pairs have equal volume?

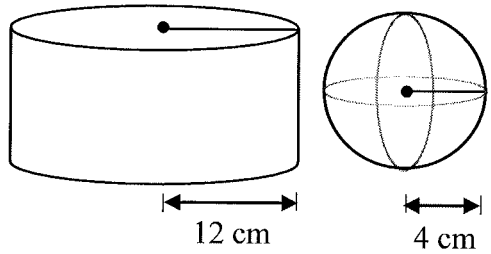
A) A pyramid with base sides 6 and height 4; a rectangular solid with dimensions $9 \times 3 \times 2$

B) A pyramid with base sides 3 and height 4; a rectangular solid with dimensions $3 \times 3 \times 2$

C) A pyramid with base sides 9 and height 3; a rectangular solid with dimensions $3 \times 3 \times 9$

D) A pyramid with base sides 6 and height 8; a rectangular solid with dimensions $3 \times 4 \times 6$

9



Note: Figure not drawn to scale.

A sphere with a radius of 4 centimeters is filled with liquid and then poured into an empty right circular cylinder with a radius of 12 centimeters. Which of the following is closest to the height of the liquid in the cylinder?

A) 0.593 cm

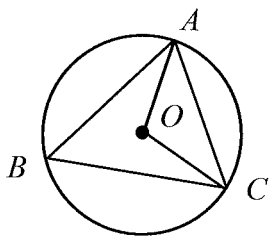
B) 21.3 cm

C) 67.0 cm

D) 85.3 cm

Student Produced Response – Calculator

1 In the figure below, the measure of $\angle ABC$ is 20° and the measure of $\angle BAC$ is 80°.



If the circumference of the circle is 720, what is the arc length of AB?

2 A spherical volleyball, when fully inflated, holds 288π cubic inches of air. What is the diameter of the volleyball, in inches?

Guided Practice – Equations of Circles

(Answers & explanations begin on page 432)

Multiple Choice – Calculator

1 $(x-6)^2+(y+8)^2=16$

A circle on the xy-plane has the equation shown above. Which of the following correctly describes the location of the center of the circle and the length of its radius?

A) Center: (6,–8)
Radius: 4

B) Center: (6,–8)
Radius: 8

C) Center: (–6,8)
Radius: 4

D) Center: (–6,8)
Radius: 8

2 A circle on the xy-plane has its center at (5,–4) and radius 8. Which of the following is an equation of the circle?

A) $(x+5)^2+(y-4)^2=16$
B) $(x-5)^2+(y+4)^2=16$
C) $(x+5)^2+(y-4)^2=64$
D) $(x-5)^2+(y+4)^2=64$

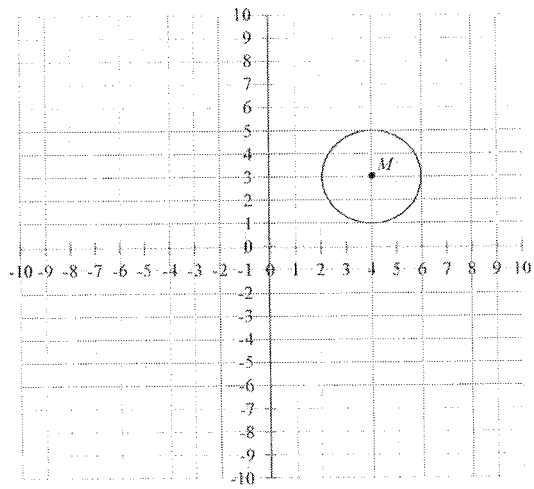
3 A circle on the xy-plane has its center at (–3.6,–13.4) and radius $\sqrt{8}$. Which of the following is an equation of the circle?

A) $(x+3.6)^2+(y+13.4)^2=2$
B) $(x-3.6)^2+(y-13.4)^2=2$
C) $(x+3.6)^2+(y+13.4)^2=8$
D) $(x-3.6)^2+(y-13.4)^2=8$

4 A circle on the xy-plane has its center at $(\frac{1}{6},\frac{5}{7})$ and radius $\frac{2}{3}$. Which of the following is an equation of the circle?

A) $(x+\frac{1}{6})^2+(y+\frac{5}{7})^2=\frac{4}{9}$
B) $(x-\frac{1}{6})^2+(y-\frac{5}{7})^2=\frac{4}{9}$
C) $(x+\frac{1}{6})^2+(y+\frac{5}{7})^2=\frac{4}{6}$
D) $(x-\frac{1}{6})^2+(y-\frac{5}{7})^2=\frac{4}{6}$

5 A circle with center M is graphed on the xy-plane.



Which of the following is an equation of the circle?

A) $(x+4)^2+(y+3)^2=2$
B) $(x-4)^2+(y-3)^2=2$
C) $(x+4)^2+(y+3)^2=4$
D) $(x-4)^2+(y-3)^2=4$

6 A circle on the xy-plane has its center at (–8,11). If the point (4,11) lies on the circle, which of the following is an equation of the circle?

A) $(x+8)^2+(y-11)^2=12$
B) $(x-8)^2+(y+11)^2=12$
C) $(x+8)^2+(y-11)^2=144$
D) $(x-8)^2+(y+11)^2=144$

7 $x^2+(y+3)^2=16$

The graph of the equation above on the xy-plane is a circle. If the center of the circle is translated 1 unit up and the radius is increased by 1, which of the following is an equation of the resulting circle?

A) $(x-1)^2+(y+3)^2=17$
B) $(x-1)^2+(y+3)^2=25$
C) $x^2+(y+2)^2=25$
D) $x^2+(y+4)^2=17$

8 $x^2 + y^2 = 25$

The graph of the equation above on the xy-plane is a circle. If the center of the circle is translated 2 units up and the radius is decreased by 1, which of the following is an equation of the resulting circle?

A) $x^2 + (y+2)^2 = 16$

B) $x^2 + (y+2)^2 = 36$

C) $x^2 + (y-2)^2 = 16$

D) $x^2 + (y-2)^2 = 36$

9 Which of the following is an equation of a circle on the xy-plane with center (2,5) and a radius with endpoint (2,2)?

A) $(x-2)^2 + (y-5)^2 = 9$

B) $(x+2)^2 + (y+5)^2 = 16$

C) $(x+2)^2 + (y-2)^2 = 9$

D) $(x-2)^2 + (y-5)^2 = 16$

10 A circle in the xy-plane has a diameter with endpoints at (–4,–2) and (8,–2). Which of the following is an equation of the circle?

A) $(x-2)^2 + (y+2)^2 = 36$

B) $(x+2)^2 + (y-2)^2 = 36$

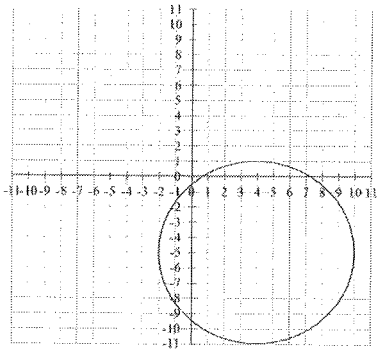
C) $(x-2)^2 + (y+2)^2 = 64$

D) $(x+2)^2 + (y-2)^2 = 64$

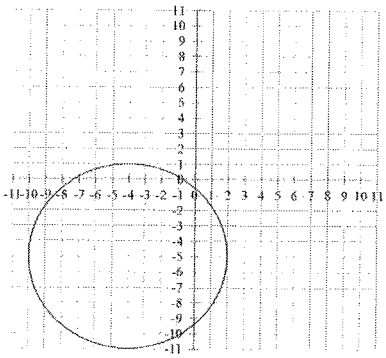
11 $(x-4)^2 + (y-5)^2 = 36$

Which of the following circle graphs is best represented by the equation shown above?

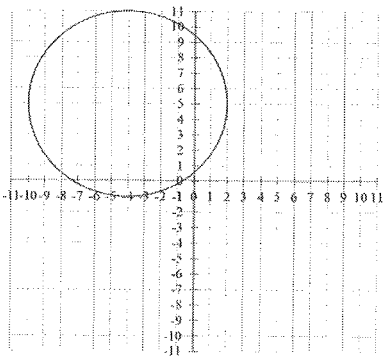
A)



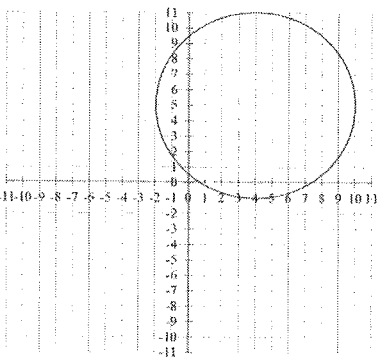
B)



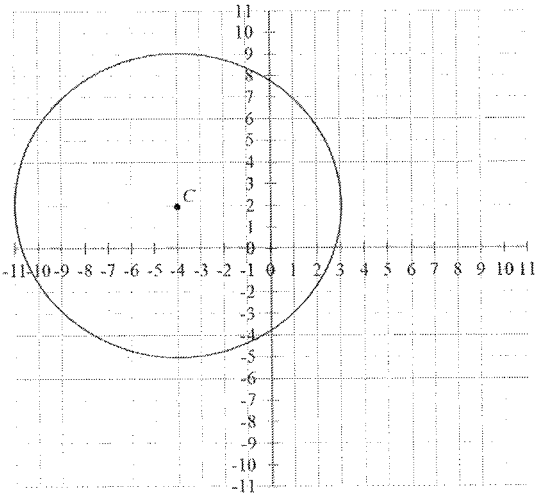
C)



D)



12 A circle with center C is graphed on the xy-plane.



Which of the following is an equation of the circle?

A) $(x+4)^2+(y-2)^2=14$

B) $(x-4)^2+(y+2)^2=14$

C) $(x+4)^2+(y-2)^2=49$

D) $(x-4)^2+(y+2)^2=49$

13 $x^2+(y-7)^2=9$

The graph of the equation above on the xy-plane is a circle. If the center of the circle is translated 2 units to the left and the radius is increased by 7, which of the following is an equation of the resulting circle?

A) $(x-2)^2+(y-7)^2=16$

B) $(x+2)^2+(y-7)^2=16$

C) $(x-2)^2+(y-7)^2=100$

D) $(x+2)^2+(y-7)^2=100$

14 Which of the following could be an equation of a circle in the xy-plane with a diameter having one endpoint at (6,–6) and a length of 14?

A) $(x+1)^2+(y+6)^2=49$

B) $(x+6)^2+(y-1)^2=49$

C) $(x-1)^2+(y+6)^2=196$

D) $(x-6)^2+(y-1)^2=196$

15 $(x-3)^2+(y+2)^2=25$

The graph of the equation above in the xy-plane is the resulting circle after another circle was translated 3 units to the right and the radius was increased by 3. Which of the following is an equation of the original circle before the transformation?

A) $x^2+(y+2)^2=4$

B) $x^2+(y+2)^2=64$

C) $(x-6)^2+(y+2)^2=4$

D) $(x-6)^2+(y+2)^2=64$

Student Produced Response – Calculator

1 A circle on the xy-plane has center (4,4) and a radius with endpoint (0,7). What is the radius of the circle?

2 $x^2+4x+y^2-12y=41$

The graph of the equation above on the xy-plane is a circle. What is the diameter of the circle?

3 $x^2-6x+y^2-16y=27$

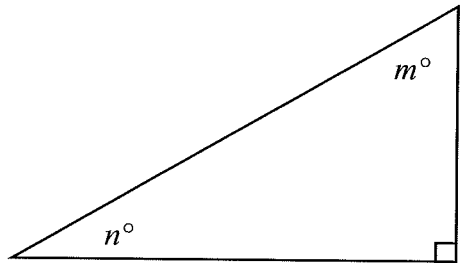
The graph of the equation above on the xy-plane is a circle. What is the diameter of the circle?

Guided Practice – Trigonometry

(Answers & explanations begin on page 434)

Multiple Choice – No Calculator

1

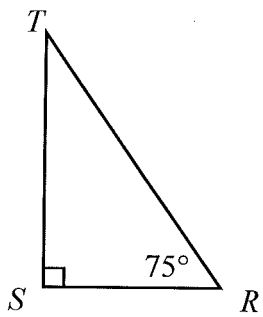


Note: Figure not drawn to scale.

In the right triangle above, $\cos(n) = 0.8$. What is $\sin(m)$?

A) 0.2
B) 0.4
C) 0.6
D) 0.8

2



Note: Figure not drawn to scale.

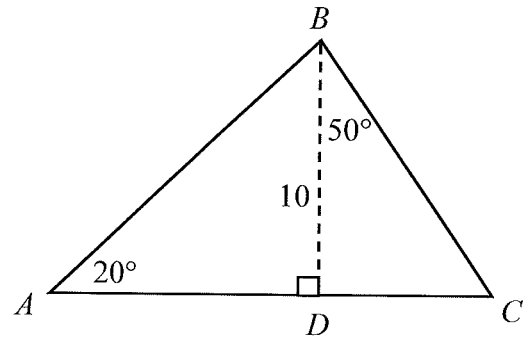
Which expression is equivalent to $\sin(75°)$?

A) $\cos(T)$
B) $\cos(R)$
C) $\sin(T)$
D) $\sin(S)$

3 Two acute angles, A and B, have measures $a°$ and $b°$ respectively and $\sin(a°) = \cos(b°)$. If $a = k + 6$ and $b = 3k + 4$, what is the value of k?

A) 1
B) 7
C) 20
D) 45

4

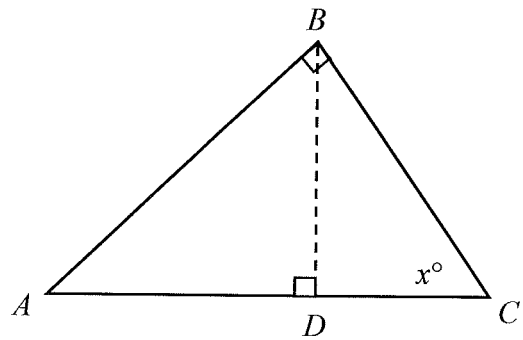


Note: Figure not drawn to scale.

Which of the following is equal to the length of AD?

A) $10\tan(70°)$
B) $10\tan(50°)$
C) $10\tan(20°)$
D) $10\sin(20°)$

5



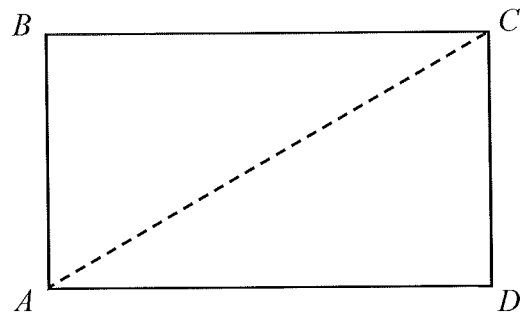
Note: Figure not drawn to scale.

In the figure above, ABC and BDC are both right triangles. Which of the following expressions must be equal to the length of the segment BD?

A) $\tan(x°)AD$
B) $\dfrac{1}{\tan(x°)}AD$
C) $\sin(x°)DC$
D) $\dfrac{1}{\tan(x°)}DC$

Student Produced Response – No Calculator

1

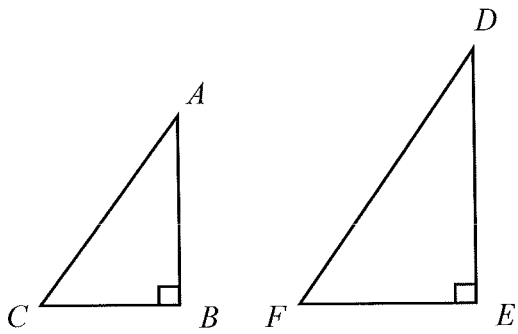


Note: Figure not drawn to scale.

In the rectangle $ABCD$ above, the length of AB is 6 and the length of BC is $6\sqrt{3}$. What is the measure, in degrees, of $\angle CAD$?

Multiple Choice – Calculator

1

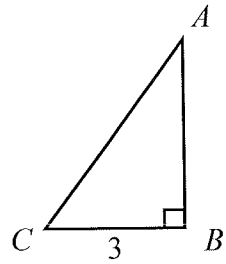


Note: Figure not drawn to scale.

In the figure above, right triangle ABC is similar to right triangle DEF. Vertices A, B, and C correspond to vertices D, E, and F respectively. If $\sin(A) = 0.45$, what is the value of $\sin(D)$?

A) 0.89
B) 0.55
C) 0.45
D) 0.20

2

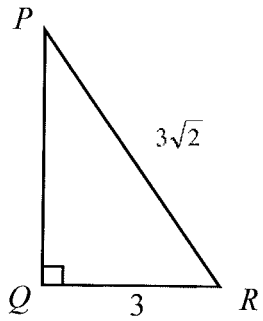


Note: Figure not drawn to scale.

It is given that $\sin(65°) \approx 0.9063, \cos(65°) \approx 0.4226$, and $\tan(65°) \approx 2.1445$. The length of CB is 3 and the measure of $\angle BCA$ is 65°. Which of the following is closest to the length of AC?

A) 1.26
B) 3.29
C) 6.42
D) 7.14

3

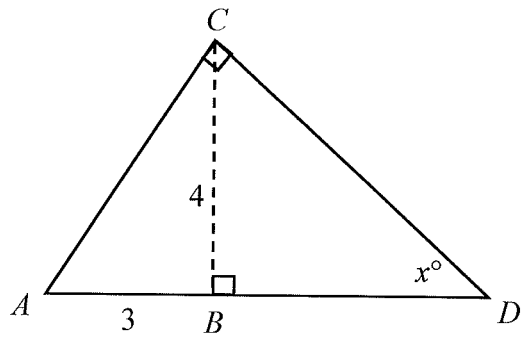


Note: Figure not drawn to scale.

What is the measure in degrees of $\angle PRQ$ in the figure shown?

A) 60
B) 50
C) 45
D) 30

4



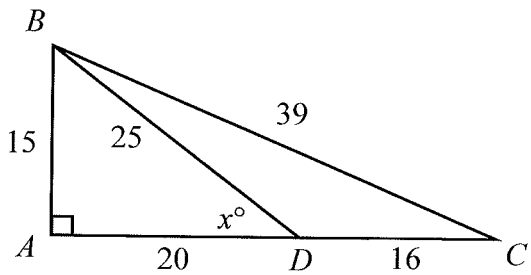
Note: Figure not drawn to scale.

In the figure above, right triangle ABC is similar to right triangle CBD. The lengths of segments AB and BC are 3 and 4 respectively. Which of the following inequalities is true about the value of $\sin(x°)$?

A) $0 \le \sin(x°) < 0.5$
B) $0.5 \le \sin(x°) < 0.7$
C) $0.7 \le \sin(x°) < 1$
D) $1 \le \sin(x°)$

Student Produced Response – Calculator

1

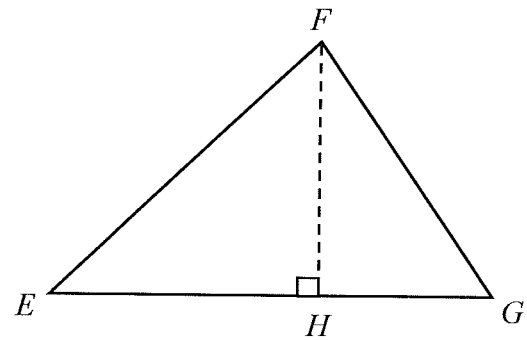


Note: Figure not drawn to scale.

In the figure above, what is $\cos(x°)$ as a fraction in simplest form?

2 Right triangle ABC has sides 5 m., 12 m., and 13 m. If the side opposite to angle A is 12 m., then what is $\cos A$?

3



Note: Figure not drawn to scale.

The measure of angle FEH is 30°, the measure of angle FGH is 45° and the height FH of the triangle is 14 inches. Given that $\sqrt{2} \approx 1.41$ and $\sqrt{3} \approx 1.73$, what is the length of EG to the nearest inch?

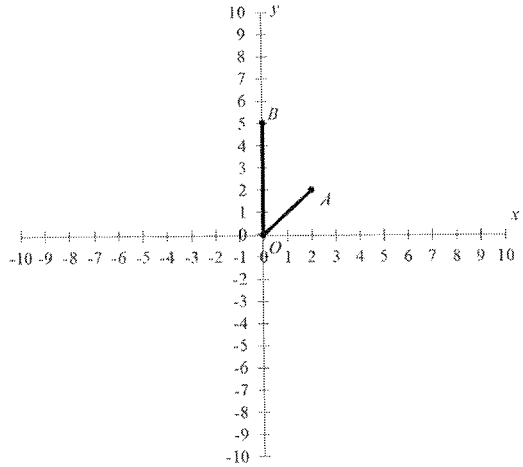
4 In triangle XYZ, the measure of $\angle Y$ is 90°, the measure of $\angle X$ is $x°$, $YZ = 12$ and $XZ = 13$. What is the value of $\tan(x°)$?

Guided Practice – Radians

(Answers & explanations begin on page 435)

Multiple Choice – No Calculator

1



In the figure above, the coordinates of point A are (2,2). What is the measure, in radians, of angle AOB?

A) $\frac{\pi}{8}$

B) $\frac{\pi}{4}$

C) $\frac{\pi}{3}$

D) $\frac{\pi}{2}$

2 $\sin(C) = \cos(C)$

The equation above is true for some $\angle C$. What is the measure of $\angle C$ in radians?

A) $\frac{\pi}{4}$

B) $\frac{\pi}{3}$

C) $\frac{\pi}{2}$

D) π

3 Which of the following is true for any acute angle x?

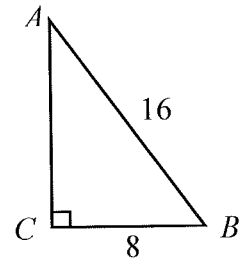
A) $\sin(x) = \cos(\pi - x)$

B) $\sin(x) = \cos(x - \pi)$

C) $\sin(x) = \cos\left(\frac{\pi}{2} - x\right)$

D) $\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$

4



Note: Figure not drawn to scale.

What is the measure in radians of $\angle A$ in the figure above?

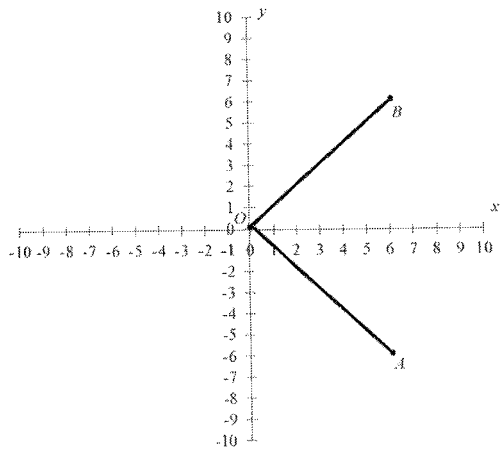
A) $\frac{\pi}{6}$

B) $\frac{\pi}{4}$

C) $\frac{\pi}{3}$

D) $\frac{\pi}{2}$

5



In the figure, the coordinates of point A are (6,–6) and the coordinates of point B are (6,6). What is the measure in radians of $\angle AOB$?

A) $\frac{\pi}{4}$

B) $\frac{\pi}{3}$

C) $\frac{\pi}{2}$

D) $\frac{2\pi}{3}$

6 In a unit circle centered on the origin, the point A has coordinates (1,0) and point B lies on the terminal side of angle AOB, where O is the origin. The measure of angle AOB is 210°. What is the measure of angle AOB in radians?

A) $\frac{\pi}{3}$

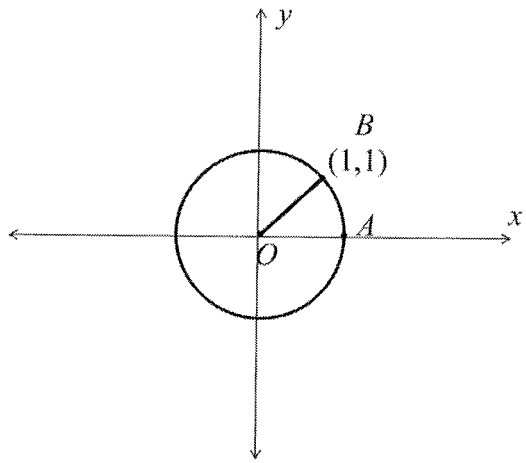
B) $\frac{2\pi}{3}$

C) $\frac{5\pi}{6}$

D) $\frac{7\pi}{6}$

Student Produced Response – No Calculator

1

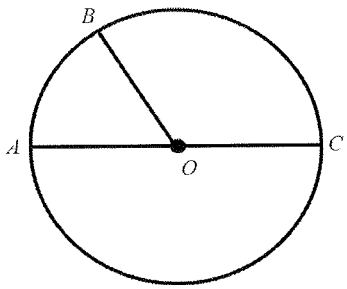


On the xy-plane above, O is the center of the circle, and the measure of $\angle AOB$ is $\frac{\pi}{a}$ radians. What is the value of a?

2 In a circle with center O, central angle AOB has a measure of $\frac{7\pi}{8}$ radians. The area of the sector formed by central angle AOB is what fraction of the area of the circle?

Multiple Choice – Calculator

1



The measure of $\angle BOA$ in the figure above is $\frac{\pi}{6}$. What is the measure of $\angle COB$?

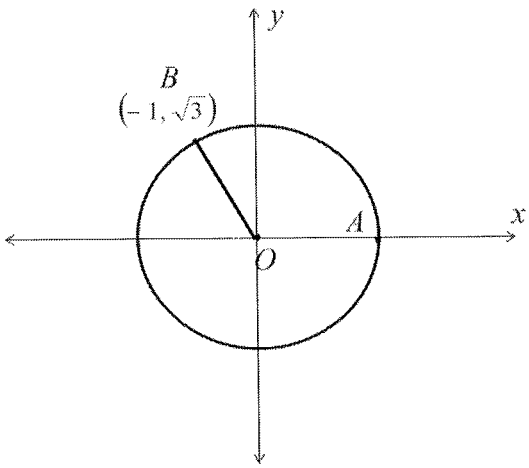
A) $\frac{\pi}{3}$

B) $\frac{5\pi}{6}$

C) $\frac{7\pi}{6}$

D) $\frac{11\pi}{6}$

2



On the xy-plane above, O is the center of the circle. What is the measure in radians of $\angle AOB$?

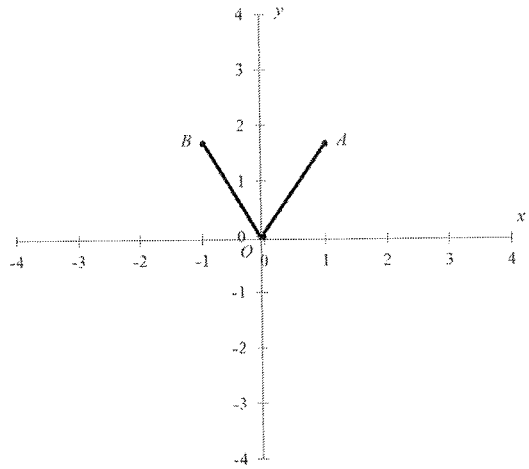
A) $\frac{\pi}{3}$

B) $\frac{2\pi}{3}$

C) $\frac{3\pi}{4}$

D) $\frac{5\pi}{6}$

3



In the figure above, the coordinates of point A are $\left(1,\sqrt{3}\right)$ and the coordinates of point B are $\left(-1,\sqrt{3}\right)$. What is the measure in radians of $\angle AOB$?

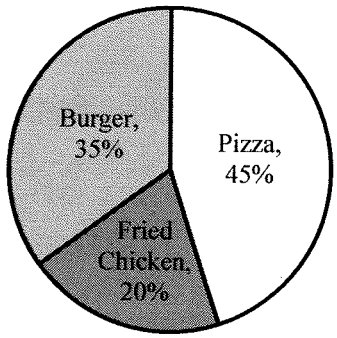
A) $\frac{\pi}{6}$

B) $\frac{\pi}{4}$

C) $\frac{\pi}{3}$

D) $\frac{2\pi}{3}$

4



Students were asked which fast food they preferred and the results are shown in the pie chart above. What is the measure in radians of the central angle of the sector representing pizza?

A) $\frac{2\pi}{45}$

B) $\frac{9\pi}{40}$

C) $\frac{9\pi}{20}$

D) $\frac{9\pi}{10}$

Student Produced Response – Calculator

1 In a circle with center O, central angle AOB has a measure of $\frac{3\pi}{5}$ radians. The length of the arc intercepted by angle AOB is what fraction of the circumference of the circle?

5 Angles A and B are supplementary. The measure of angle A is twice the measure of angle B. What is the measure of angle B in radians?

A) $\frac{\pi}{3}$

B) $\frac{\pi}{2}$

C) $\frac{2\pi}{3}$

D) π

Guided Practice – Imaginary & Complex Numbers

(Answers & explanations begin on page 437)

Multiple Choice – No Calculator

1 Which of the following is equal to $i^2 + i^2$? (Note: $i = \sqrt{-1}$)
A) i^4
B) $2i^4$
C) 0
D) –2

2 Which of the following is equal to $(3 + 4i) + (7 + 6i)$? (Note: $i = \sqrt{-1}$)
A) $7 + 13i$
B) $10 + 10i$
C) $10 - 10i$
D) $10 - 10i^2$

3 Which of the following is equal to $(4 + 2i) - (7 - 2i)$? (Note: $i = \sqrt{-1}$)
A) –3
B) $-3i$
C) $-3 + 4i$
D) $-3 - 4i$

4 Which of the following is equal to $-6i(3 - 2i)$? (Note: $i = \sqrt{-1}$)
A) $12 - 18i$
B) $-12 - 18i$
C) $-18 + 12i$
D) –30

5 Which of the following is equal to $12i(4 + 3i)$? (Note: $i = \sqrt{-1}$)
A) $36 - 48i$
B) $-36 + 48i$
C) $-36 - 48i$
D) 84

6 Which of the following is equal to $\frac{12 + 8i}{2}$?
(Note: $i = \sqrt{-1}$)
A) 10
B) $10i$
C) $6 + 4i$
D) $6 + 8i$

7 Which of the following is equal to $5i^4 - 5i^2 - 5$? (Note: $i = \sqrt{-1}$)
A) –5
B) 0
C) 1
D) 5

8 Which of the following is equal to $(-5i^2) - (-3 - 7i^2)$? (Note: $i = \sqrt{-1}$)
A) 1
B) $-3 - 12i^2$
C) $3 - 12i^2$
D) $3 + 2i^2$

9 Which of the following is equal to $5i^3(4 - 2i^2)$? (Note: $i = \sqrt{-1}$)
A) $10i$
B) $20 - 10i$
C) $30i$
D) $-30i$

10 Which of the following is equal to $8i(7 - 3i)$? (Note: $i = \sqrt{-1}$)
A) $-24 + 56i$
B) $24 + 56i$
C) $56 + 24i$
D) $32 + 56i$

11 Which of the following is equal to $(4 - i)^2$? (Note: $i = \sqrt{-1}$)
A) $15 - 8i$
B) $15 + 8i$
C) $17 - 8i$
D) $17 + 8i$

12 Which of the following is equal to $(5 - 5i)(4 + 4i)$? (Note: $i = \sqrt{-1}$)
A) 40
B) $40 - 40i$
C) $20 - 20i$
D) $20 - 40i - 20i^2$

13 Which of the following is equal to i^{32}?
(Note: $i = \sqrt{-1}$)
A) i
B) $-i$
C) 1
D) -1

14 Which of the following is equal to i^{47}?
(Note: $i = \sqrt{-1}$)
A) i
B) $-i$
C) 1
D) -1

15 Which of the following is equal to $(4+2i)(3-7i)$? (Note: $i = \sqrt{-1}$)
A) $12-14i^2$
B) -2
C) $-2-22i$
D) $26-22i$

16 Which of the following is equal to $(8-3i)(8+3i)$? (Note: $i = \sqrt{-1}$)
A) 55
B) 64–6i
C) $64-9i$
D) 73

17 Which of the following is equal to $(i+1)(i-1)$? (Note: $i = \sqrt{-1}$)
A) 0
B) $-2+i$
C) $-2-i$
D) -2

18 Which of the following is equal to $\frac{7}{2i}$?
(Note: $i = \sqrt{-1}$)
A) $\frac{7}{4}$
B) $\frac{-7}{4}$
C) $\frac{7i}{2}$
D) $\frac{-7i}{2}$

19 Which of the following is equal to $\frac{9i}{1+8i}$?
(Note: $i = \sqrt{-1}$)
A) $\frac{9+72i}{9}$
B) $\frac{72+9i}{65}$
C) $\frac{9-72i}{9}$
D) $\frac{72-9i}{65}$

20 Which of the following is equal to $\frac{9-4i}{-5i}$?
(Note: $i = \sqrt{-1}$)
A) $\frac{9i+4}{5}$
B) $\frac{13}{5}$
C) $\frac{9i-4}{5}$
D) $\frac{-9i+4}{5}$

21 Which of the following is equal to $i + i^2 + i^3 + i^4$? (Note: $i = \sqrt{-1}$)
A) 0
B) $2+2i$
C) $2-2i$
D) i^{10}

22 Which of the following is equal to $\frac{4}{2+3i}$?
(Note: $i = \sqrt{-1}$)
A) $\frac{8+12i}{5}$
B) $\frac{8-12i}{13}$
C) $\frac{8+12i}{13}$
D) $\frac{-4}{5}$

23 $\frac{8-3i}{2+i}$

If the expression above is rewritten in the form $a + bi$, where a and b are real numbers, what is the value of a? (Note: $i = \sqrt{-1}$)

A) $\frac{13}{5}$

B) $\frac{19}{5}$

C) $\frac{13}{3}$

D) $\frac{14}{3}$

24 Which of the following is equal to $\frac{5}{5+i} + \frac{i}{5-i}$? (Note: $i = \sqrt{-1}$)

A) $\frac{12}{13}$

B) $\frac{1}{4}$

C) $5 + i$

D) $1 - 5i$

Student Produced Response – No Calculator

1 What is the value of the following expression: $(5 + 6i) + (4 - 6i)$? (Note: $i = \sqrt{-1}$)

2 What is the value of the following expression: $i^4 - 6i^2 + 9$? (Note: $i = \sqrt{-1}$)

3 What is the value of the following expression: $(4 + 5i)(4 - 5i)$? (Note: $i = \sqrt{-1}$)

4 What is the value of the following expression: $\frac{i^3}{-5i}$? (Note: $i = \sqrt{-1}$)

5 What is the value of the following expression: $(-2 - 8i)(-2 + 8i)$? (Note: $i = \sqrt{-1}$)

6 What is the value of the following expression: $\frac{i^7}{-i}$? (Note: $i = \sqrt{-1}$)

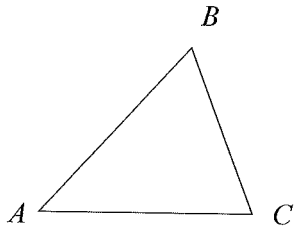
7 What is the value of the following expression: $\frac{2-i}{2+i} + \frac{2+i}{2-i}$? (Note: $i = \sqrt{-1}$)

Mixed Practice – Additional Math Topics

(Answers & explanations begin on page 439)

Multiple Choice – No Calculator

1



Note: Figure not drawn to scale.

In the figure above, A and C are acute angles and $\sin(A) = \cos(C)$. What is the measure, in degrees, of angle B?

A) 120
B) 90
C) 60
D) 45

2 Which of the following is equal to $(12 + 2i)^2$?

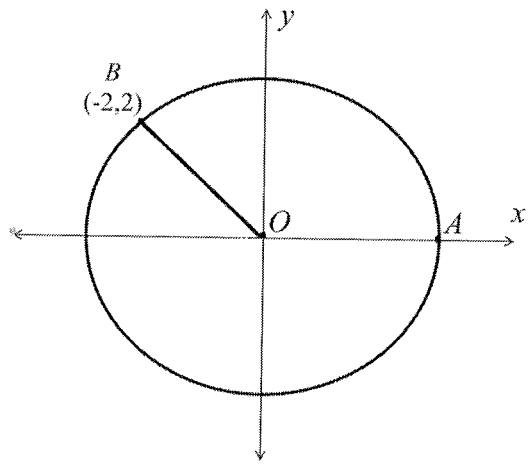
(Note: $i = \sqrt{-1}$)

A) 140
B) $148 + 48i$
C) $144 + 48i$
D) $140 + 48i$

3 Which of the following is an equation of a circle on the xy-plane with center (2,4) and a radius with endpoint (8,–4)?

A) $(x+2)^2 + (y+4)^2 = 36$
B) $(x-2)^2 + (y-4)^2 = 64$
C) $(x-2)^2 + (y-4)^2 = 100$
D) $(x-2)^2 + (y-4)^2 = 196$

4



On the xy-plane above, O is the center of the circle. What is the measure in radians of angle AOB?

A) $\frac{\pi}{4}$
B) $\frac{\pi}{3}$
C) $\frac{3\pi}{2}$
D) $\frac{3\pi}{4}$

5 $\frac{10 + 40i}{-3 + 5i}$

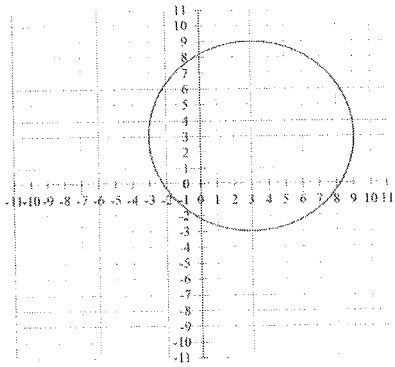
If the expression above is rewritten in the form $a + bi$, where a and b are real numbers, what is the value of b? (Note: $i = \sqrt{-1}$)

A) –170
B) –5
C) 5
D) 170

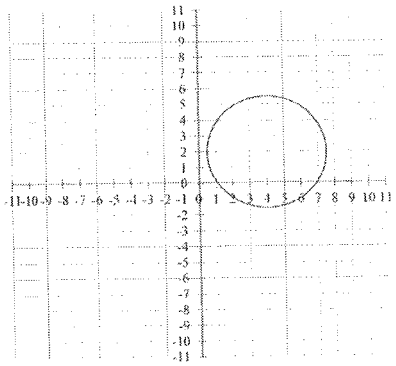
6 $(x-4)^2+(y-2)^2=49$

Which of the following circle graphs is best represented by the equation shown above?

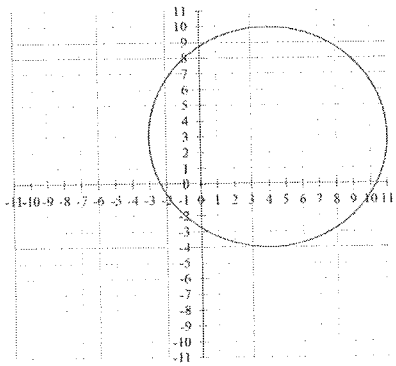
A)



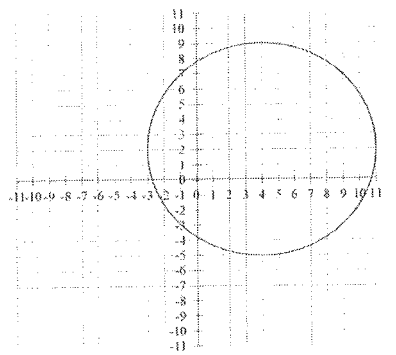
B)



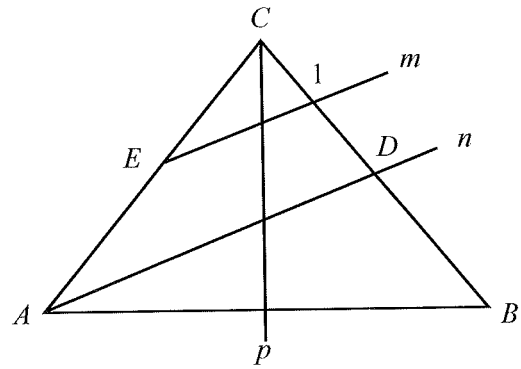
C)



D)



7



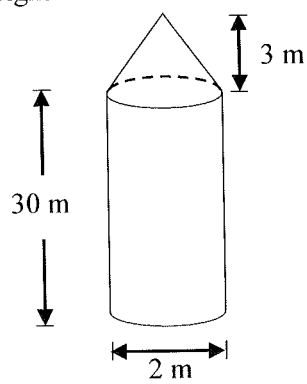
Note: Figure not drawn to scale.

In the above figure, lines m and n are parallel, and line n intersects segment BC at point D. If angle 1 = 98° and angle DAB = 12°, what is angle DBA?

A) 55°
B) 70°
C) 86°
D) 90°

Multiple Choice – Calculator

1 In the figure below, a rocket engine is constructed from a right circular cylinder with a right circular cone attached to the top.



Note: Figure not drawn to scale.

Both the cone and cylinder have diameters of 2 meters, the height of the cylinder is 30 meters, and the height of the cone is 3 meters. Find the volume of the rocket engine in cubic meters.

A) 30π
B) 31π
C) 33π
D) 62π

Scoring

To score the practice test, use the following instructions (refer to the *Scoring & Reporting* section of this workbook for more information on scoring, terminology, and interpretation).

Step 1: Using the answer key at the end of this workbook, determine which questions were answered correctly.

Step 2: Add together the total number of questions answered correctly in each of the four sections. Enter those totals in the table below in the "# Correct Answers" column.

Step 3: For both the Reading Test and Writing & Language Test, consult the table on the next page to determine the Test Score based on the number of questions answered correctly. Enter the two Test Scores in the "Test Score" column below.

For example, if the # of Correct Answers on the Reading Test is 40, then the Reading Test Score would be 33; if the # Correct Answers on the Writing & Language Test is 31, then the Writing & Language Test Score would be 30.

Practice Test Section	**# Correct Answers**	**Test Score**
Reading Test	_____	_____
Writing & Language Test	_____	_____
Math Test – No Calculator	_____	N/A
Math Test – Calculator	_____	N/A

Step 4: Add together the Reading Test Score and the Writing & Language Test Score and multiply that number by 10 to receive your Reading & Writing Section Score. Input that number in the "Section Score" column below.

Continuing the example from Step 3: a Reading Test score and Writing & Language Test score of 33 and 30, respectively, would result in a total of 63. $63 \times 10 = 630$, the Reading & Writing Section Score.

Step 5: Add together the total number of correct answers from both Math Tests (No Calculator + Calculator). Consult the table on the next page to determine the Math Section Score based on the total number of correct answers. Record the resulting Math Section Score in the "Section Score" column below.

For example, if the total number of correct answers on both Math Tests is 35, then the Math Section Score is 600.

Step 6: Add together both the Reading & Writing Section Score and the Math Section Score. The sum is the total score (out of a 400-1600 range).

Practice Test Section	**Section Score**
Reading & Writing Section	_____
Math Section	_____
Total Score	_____

Note that this practice test does not provide a breakdown of subscores or cross-test scores.

# Correct Answers	Math Section Score	Reading Test Score	Writing Test Score
0	200	10	10
1	200	10	10
2	210	10	10
3	230	11	11
4	250	12	12
5	270	13	13
6	290	14	14
7	300	15	14
8	320	15	15
9	330	16	16
10	340	17	17
11	360	18	17
12	370	18	18
13	380	19	19
14	390	19	19
15	410	20	20
16	420	21	21
17	430	21	22
18	440	22	22
19	450	22	23
20	460	23	24
21	470	23	24
22	480	24	25
23	490	24	26
24	500	25	26
25	510	25	27
26	520	26	27
27	530	26	28
28	540	27	28
29	550	28	29

# Correct Answers	Math Section Score	Reading Test Score	Writing Test Score
30	560	28	30
31	570	29	30
32	580	29	31
33	580	29	32
34	590	30	32
35	600	30	33
36	610	31	33
37	620	31	34
38	630	32	35
39	630	32	35
40	640	33	36
41	650	33	37
42	660	34	38
43	670	34	39
44	680	35	40
45	680	36	
46	690	36	
47	700	37	
48	700	38	
49	710	38	
50	720	39	
51	730	40	
52	740	40	
53	580		
54	760		
55	770		
56	780		
57	790		
58	800		

Math – No Calculator

Directions for questions 1-15

Solve each question by choosing the best answer from the choices provided. Fill in the appropriate circle on the answer sheet. Writing in the test booklet is allowed.

Directions for questions 16-20

Answer each question by solving the problem and entering the answer in the provided grid on the answer sheet. Though it is recommended that you write the answer in the boxes at the top of each column, you will receive credit only if the circles are appropriately filled in. Do not mark more than one circle in any column. No question has a negative answer. Some questions may have multiple answers, in which case you need only provide one answer. Mixed numbers such as $2\frac{1}{2}$ must be gridded either as 2.5 or 5/2; do not grid the answer as 21/2. Decimal answers may be rounded or truncated. Answers may be entered starting in any column. Writing in the test booklet is allowed.

Notes

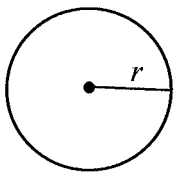
1. You **may not** use a calculator.
2. The variables and expressions used are real numbers unless otherwise indicated.
3. All figures shown in this test are drawn to scale unless otherwise indicated.
4. Figures lie in a plane unless otherwise indicated.
5. The domain of a given function f is the set of all real numbers x for which $f(x)$ is a real number, unless otherwise indicated.

Reference

A circle has 360 degrees of arc.

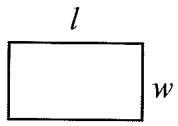
A circle has 2π radians of arc.

The sum of the measures of all angles in a triangle is 180.

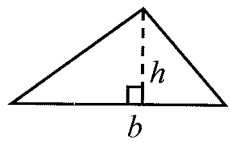


$A = \pi r^2$

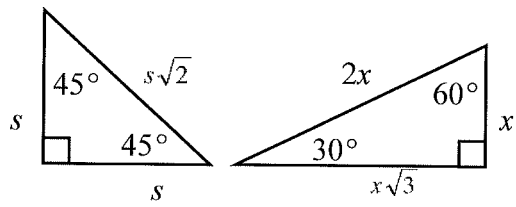
$C = 2\pi r$



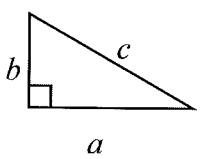
$A = lw$



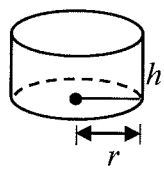
$A = \frac{1}{2}bh$



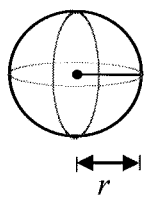
Special Right Triangles



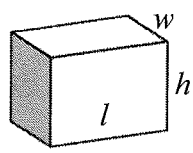
$c^2 = a^2 + b^2$



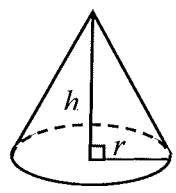
$V = \pi r^2 h$



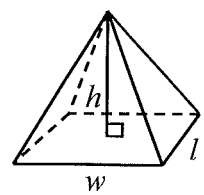
$V = \frac{4}{3}\pi r^3$



$V = lwh$



$V = \frac{1}{3}\pi r^2 h$



$V = \frac{1}{3}lwh$

1

If $\frac{2}{3}w = \frac{5}{4}$, what is the value of w?

A) $\frac{8}{15}$

B) $\frac{7}{12}$

C) $\frac{5}{6}$

D) $\frac{15}{8}$

2

$y = a(x+2)(x-4)$

In the quadratic equation above, a is a nonzero constant. The graph of the equation on the xy-plane is a parabola with vertex (c,d). Which of the following is equal to d?

A) $-9a$

B) $-6a$

C) $-3a$

D) $-2a$

3

The relationship between a reading C on the Celsius temperature scale and a reading F on the Fahrenheit temperature scale is $C = \frac{5}{9}(F-32)$, and the relationship between a reading on the Celsius temperature scale and a reading K on the Kelvin temperature scale is $K = C+273$. Which of the following expresses the relationship between readings on the Kelvin and Fahrenheit temperature scales?

A) $K = \frac{5}{9}(F-231)$

B) $K = \frac{5}{9}(F-32)+273$

C) $K = \frac{5}{9}(F-32)-273$

D) $K = \frac{5}{9}(F+32)+273$

4

The population of a small town from 2000 to 2010 can be modeled by the equation $y = 12x + 13{,}580$, where x represents the number of years since 2000, and y represents the total population. Which of the following best describes the meaning of the number 0.12 in the equation?

A) The total population in the year 2000.

B) The estimated increase in the population each year.

C) The estimated difference between the population in 2010 and 2000.

D) The total population in the year 2010.

5

$y = 5x + b$

The above equation represents a line on the xy-plane with y-intercept b. The line passes through the point $(-1,-2)$. What is the value of b?

A) -7

B) 3

C) 7

D) 9

6

Let $g(x) = x^2 - 3$.

If $f(g(x)) = \sqrt{x^2-6}$, which of the following describes $f(x)$?

A) $f(x) = \sqrt{x-3}$

B) $f(x) = \sqrt{x^2-1}$

C) $f(x) = \sqrt{x^2-3}$

D) $f(x) = \sqrt{x^4-6x^2+15}$

7

$(6x^2+2x-3)-(10x^2-2x+5)$

Which of the following is equivalent to the expression above?

A) $-4x^2+2$

B) $-4x^2-8$

C) $-4x^2+4x-8$

D) $-4x^2+4x+2$

CONTINUE

8

What is the sum of all values of m that satisfy $25m^2 - 4 = 0$?

A) 0

B) $\frac{2}{5}$

C) $\frac{4}{5}$

D) $\frac{4}{25}$

9

The population of a certain city increases by 6% each year. If the city's current population is 5 million, which function models the population, P, measured in millions, as a function of the number of years, t, into the future?

A) $P(t) = 5(1.06)^t$

B) $P(t) = 5(0.94)^t$

C) $P(t) = 1.06(5)^t$

D) $P(t) = 0.94(5)^t$

10

For a polynomial $p(x)$, the value of $p(-2)$ is 6. Which of the following must be true about $p(x)$?

A) The remainder when $p(x)$ is divided by $x + 2$ is 6.

B) The remainder when $p(x)$ is divided by $x - 2$ is 6.

C) The remainder when $p(x)$ is divided by $x + 6$ is –2.

D) The remainder when $p(x)$ is divided by $x - 6$ is –2.

11

$$R = \frac{T}{T + L}$$

An airline uses the formula above to calculate their on-time arrival rate, R, based on the number on-time arrivals, T, and the number of late arrivals, L. Which of the following expresses the number of on-time arrivals in terms of the other variables?

A) $T = \frac{L}{R - 1}$

B) $T = \frac{L}{1 - R}$

C) $T = \frac{RL}{R - 1}$

D) $T = \frac{RL}{1 - R}$

12

$f(x) = |3x - 15|$

For the function defined above, what is a possible value of a for which $f(a) < a$?

A) –6

B) –4

C) 0

D) 6

13

If $f(x) = x^2 + 1$ and $g(x) = 3x$, what is $f(g(x))$ equal to?

A) $3x^2 + 1$

B) $9x^2 + 1$

C) $3x^2 + 3$

D) $9x^2 + 3$

CONTINUE ➡

14

If $\frac{a+b}{a-b}=\frac{9}{7}$, which of the following must also be true?

A) $\frac{a}{b}=8$

B) $\frac{a}{b}=-8$

C) $\frac{a+2b}{a-b}=\frac{10}{7}$

D) $b-a=-7$

15

$$\frac{-16+4i}{5+3i}$$

If the expression above is rewritten in the form $a + bi$, where a and b are real numbers, what is the value of a? (Note: $i=\sqrt{-1}$)

A) −68
B) −2
C) 2
D) 68

Student Produced Responses

16

$ax+by=12$

$2x+5y=72$

In the system of equations above, a and b are constants. If the system has infinitely many solutions, what is the value of ab?

17

Steve has a budget of $1,000 for a home improvement project. He buys cans of paint for $50 each and bundles of lumber for $100 each. If he buys at least one can of paint and one bundle of lumber, what is the greatest number of cans of paint he can buy?

18

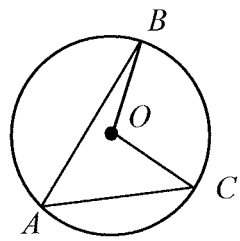
$x^3-4x^2+9x-36=0$

For what real value of x is the equation above true?

19

In a right triangle, one angle measures $x°$, where $\cos(x°)=\frac{3}{5}$. What is $\sin(90°-x°)$?

20



Note: Figure not drawn to scale.

In the figure above, central angle BOC is 114° and subtends arc BC. What is the measure, in degrees, of angle BAC?

STOP. Do not go on until instructed to do so.

STOP

Math – Calculator

Directions for questions 1-30

Solve each question by choosing the best answer from the choices provided. Fill in the appropriate circle on the answer sheet. Writing in the test booklet is allowed.

Directions for questions 31-38

Answer each question by solving the problem and entering the answer in the provided grid on the answer sheet. Though it is recommended that you write the answer in the boxes at the top of each column, you will receive credit only if the circles are appropriately filled in. Do not mark more than one circle in any column. No question has a negative answer. Some questions may have multiple answers, in which case you need only provide one answer. Mixed numbers such as $2\frac{1}{2}$ must be gridded either as 2.5 or 5/2; do not grid the answer as 21/2. Decimal answers may be rounded or truncated. Answers may be entered starting in any column. Writing in the test booklet is allowed.

Notes

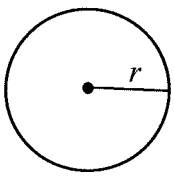
1. You **may** use a calculator.
2. The variables and expressions used are real numbers unless otherwise indicated.
3. All figures shown in this test are drawn to scale unless otherwise indicated.
4. Figures lie in a plane unless otherwise indicated.
5. The domain of a given function f is the set of all real numbers x for which $f(x)$ is a real number, unless otherwise indicated.

Reference

A circle has 360 degrees of arc.

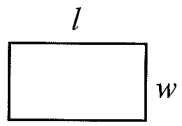
A circle has 2π radians of arc.

The sum of the measures of all angles in a triangle is 180.

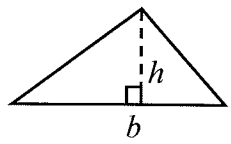


$A = \pi r^2$

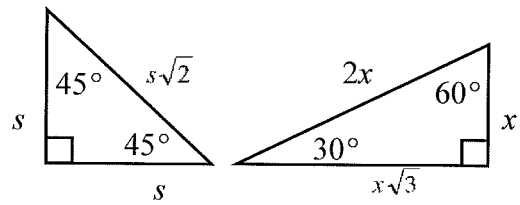
$C = 2\pi r$



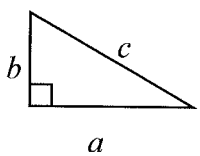
$A = lw$



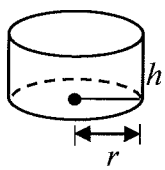
$A = \frac{1}{2}bh$



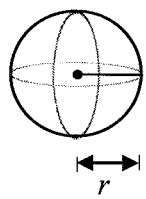
Special Right Triangles



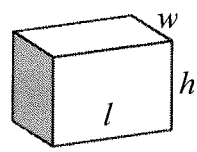
$c^2 = a^2 + b^2$



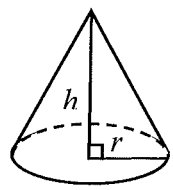
$V = \pi r^2 h$



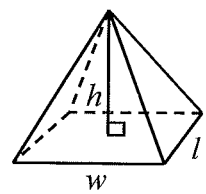
$V = \frac{4}{3}\pi r^3$



$V = lwh$



$V = \frac{1}{3}\pi r^2 h$



$V = \frac{1}{3}lwh$

1

If $12 < 6 + 9mx$, which inequality represents the possible range of values of $2 + 3mx$?

A) $2 + 3mx < 4$
B) $2 + 3mx > 4$
C) $2 + 3mx < 8$
D) $2 + 3mx > 8$

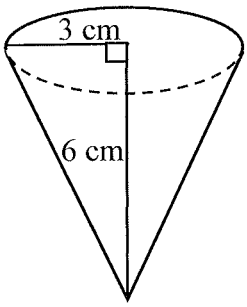
2

The function f is defined by $f(x) = x^3 + 2x^2 + kx + 2$, where k is a constant. On the xy-plane, the graph of f intersects the x-axis at the point $(-2,0)$ and the y-axis at $(0,2)$. What is the value of k?

A) -1
B) 0
C) 1
D) 2

3

1,000 cubic centimeters of water is to be poured into right circular conical cups with a radius of 3 centimeters and a height of 6 centimeters, as depicted below.

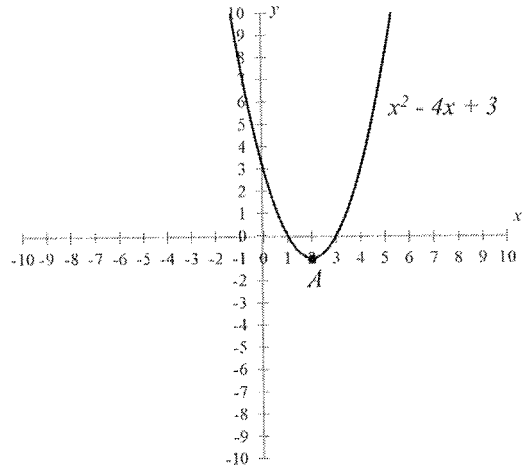


Note: Figure not drawn to scale.

What is the greatest number of cups that can be completely filled?

A) 5
B) 17
C) 18
D) 26

4



Which of the following is an equivalent form of the equation of the graph shown on the xy-plane above, from which the coordinates of the vertex A can be identified as constants in the equation?

A) $y = (x-1)(x-3)$
B) $y = (x+1)(x+3)$
C) $y = x(x-2)^2 - 3$
D) $y = (x-2)^2 - 1$

5

In a triangle ABC, the measure of angle A is $\frac{\pi}{4}$ and the measure of angle B is $\frac{\pi}{3}$. What is the measure of angle C?

A) $\frac{5\pi}{12}$
B) $\frac{7\pi}{12}$
C) $\frac{11\pi}{12}$
D) $\frac{17\pi}{12}$

CONTINUE

6

Mike and Dylan own a total of 128 comic books. If Dylan owns 14 more comic books than Mike, how many comic books does Dylan own?

A) 14
B) 57
C) 71
D) 89

7

In a random sample of certain species of peas, 12 out of every 700 peas have a genetic characteristic that turns them red. What would be the expected number of peas with this characteristic in a random sample of 210,000?

A) 360
B) 3,600
C) 840
D) 8,400

8

$x = 2y + 4$

$y = x^2 + \frac{1}{2}x - 2$

If (x,y) is a solution of the system of equations above, what is the value of y?

A) -2
B) 0
C) $\frac{1}{2}$
D) 2

9

The force a spring exerts on an object can be represented by $F = kx$ where k is the spring constant, which measures how strong the spring is and x is the distance the spring is stretched or compressed away from its rest position. The force applied to a spring in the suspension of a certain car is 2,450 N. What is the approximate spring constant when the spring is compressed 1.5 feet, in N/ft.?

A) 1,225.5
B) 1,633.3
C) 3,675.0
D) 4,900.0

10

The retail price for a book was $30. It was marked down 20% on Black Friday. The next day, it was marked back up 30%. By what percentage did the price of the book increase from the original price to the final price?

A) 0%
B) 4%
C) 6%
D) 10%

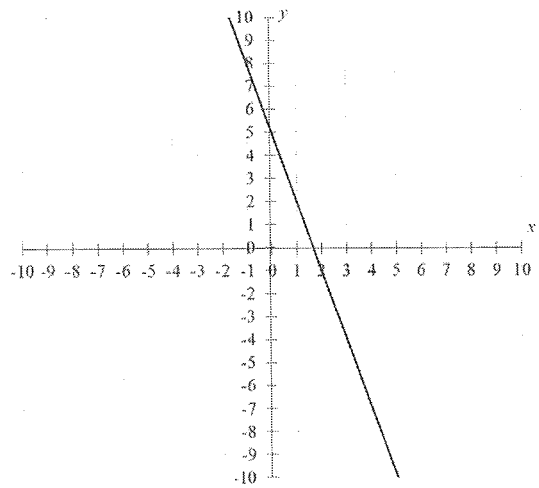
11

The cost for vendors of using the Wi-Fi at a convention center is $0.15 per minute. Which of the following equations represents the total cost c, in dollars, for h hours of Wi-Fi use?

A) $c = 0.15(60h)$
B) $c = 0.15h + 60$
C) $c = \frac{60h}{0.15}$
D) $c = \frac{0.15h}{60}$

CONTINUE ➡

12



The graph above shows a line on the xy-plane. Which of these is the equation of a parallel line?

A) $3x+y=4$
B) $-3x+y=4$
C) $x-3y=5$
D) $3y=x+5$

13

A certain lake contains 420,000 cubic meters of water. A government environmental agency finds a toxic level of waste in the lake and decides to drain the entire lake. The water is pumped out at a rate of 240 cubic meters per minute. How much water, in cubic meters, remains in the lake after 2 hours?

A) 60,000
B) 369,600
C) 391,200
D) 419,520

14

The average number of students per classroom at Western High School from 2000 to 2010 can be modeled by the equation $y=0.45x+22.5$ where x represents the number of years since 2000 and y represents the average number of students per classroom. The average number of students at Eastern High School can be similarly modeled using the function $y=0.55x+22.1$. In what year did the two schools have the same average number of students per classroom?

A) 2002
B) 2004
C) 2005
D) 2008

15

When 5 times the number x is added to 7, the result is -13. What number results when 3 times x is added to 25?

A) -13
B) -3
C) 13
D) 37

CONTINUE

Questions 16 & 17 refer to the following information:

A fruit company is concerned because some bananas become rotten while being transported to the store. The company uses the formula $S = B(0.95)^t$ to predict how many bananas will be fresh enough to sell by the time they reach the store. B represents the number of bananas that are transported and t represents the number of days that it takes for them to be transported to the store.

16

If it takes one week for the bananas to reach the store, what percentage of them are no longer fresh enough to sell when they arrive at the store?

A) 5%
B) 7%
C) 30%
D) 35%

17

Last year it took d days to transport the bananas. This year it takes $2d$ days to transport the same number of bananas. How does the value of S change from last year to this year?

A) It is multiplied by 0.95
B) It is multiplied by 0.95^d
C) It is multiplied by 0.95^2
D) It is multiplied by 0.95^{2d}

18

A circle on the xy-plane has its center at (7,1). If the point (10,5) lies on the circle, which of the following is an equation of the circle?

A) $x^2 + y^2 - 14x - 2y = -25$
B) $x^2 + y^2 - 14x + 2y = -25$
C) $x^2 + y^2 + 14x - 2y = -25$
D) $x^2 + y^2 + 14x + 2y = -25$

19

If $f(x) = 14x - 7$ and $g(x) = 10x - 5$, which of the following is NOT divisible by $2x - 1$?

A) $f(x)$
B) $f(g(x))$
C) $f(x)g(x)$
D) $f(x) + g(x)$

20

The table below summarizes the results of 150 students in the Biology 101 class at a university.

Biology 101 Exam Results

	Passed	Did Not Pass
Attended Review Session	82	18
Did Not Attend Review Session	33	17

If one of the students who passed is chosen at random for an interview, what is the probability that the person chosen did *not* attend the review session?

A) $\frac{17}{72}$
B) $\frac{33}{50}$
C) $\frac{33}{82}$
D) $\frac{33}{115}$

CONTINUE ➡

Questions 21 & 22 refer to the following information.

A sociologist chose 200 students at random from each of two schools and asked how many pets he or she has. The results are shown in the table below.

Students' Pets Survey

Number of Pets	Pembroke School	Elkwood School
0	60	80
1	50	40
2	40	30
3	25	25
4	25	25

There are a total of 1,800 students at Pembroke school and 2,000 students at Elkwood School.

21

What is the median number of pets for all the students surveyed?

A) 0
B) 1
C) 2
D) 3

22

Based on the survey data, which of the following most accurately compares the expected total number of students with 4 pets at the two schools.

A) The total number of students with 4 pets is expected to be equal at the two schools.
B) The total number of students with 4 pets at Elkwood School is expected to be 25 more than at Pembroke School.
C) The total number of students with 4 pets at Pembroke school is expected to be 25 more than at Elkwood School.
D) The total number of students with 4 pets at Elkwood School is expected to be 200 more than at Lincoln School.

23

The expression $(2x+5)^3$ is equivalent to which of the following?

A) $8x^3+125$
B) $4x^3+20x^2+25$
C) $8x^3+20x^2+50x+125$
D) $8x^3+60x^2+150x+125$

24

A cable company has been losing approximately 25% of its subscribers each year for the past several years, as shown in the chart below:

Year	Average annual subscribership (in thousands)
2010	318.9
2011	240.5
2012	179.4
2013	134.6
2014	100.9
2015	75.7
2016	56.8
2017 (estimated)	42.6

Which of the following best describes the relationship between time and the cable company's average annual subscribership during these years?

A) Increasing linear
B) Decreasing linear
C) Exponential growth
D) Exponential decay

CONTINUE

25

$g = 60 + 0.3x$

$q = 62.25 + 0.15x$

In the equations above, g and q represent the price per square foot in dollars, of granite and quartz, respectively, at a home improvement store, plus an additional surcharge for x weeks after delivery to the store. What was the price per square foot of granite when it was the same as the price per square foot of quartz?

A) $60.00
B) $62.25
C) $64.50
D) $65.15

26

The table below gives the initial weight of a certain radioactive substance observed in a lab and allowed to decay.

Time (Days)	Weight of Sample (ounces)
0	5.22
5	4.07
10	2.92
15	1.77
20	0.62

Which of the following functions models the weight of the substance after t days?

A) $f(t) = 5.22 - 1.15t$
B) $f(t) = 5.22 - 0.23t$
C) $f(t) = 5.22 - 0.23^t$
D) $f(t) = 5.22 - 0.23^{5t}$

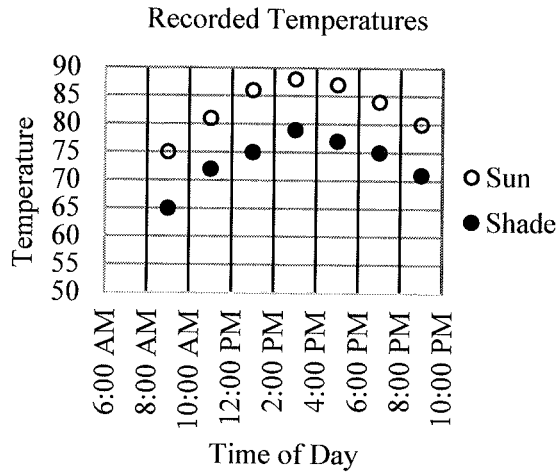
27

Which of the following situations describes exponential growth?

A) An employee gets a job that pays $1000 per day.
B) An employee's pay increases from $800 per week to $825 per week.
C) An employee has a job that pays $1 the first day, $2 the second day, $4 the third day, and doubles the amount per day each day.
D) The number of employees in a certain company starts at 100 and increases by 50 each year.

28

Mel placed one thermometer in direct sunlight and one in the shade and recorded the temperatures on both over a 12-hour period in the graph below.



Which of the following statements correctly compares the average rates at which the temperatures of the two samples change?

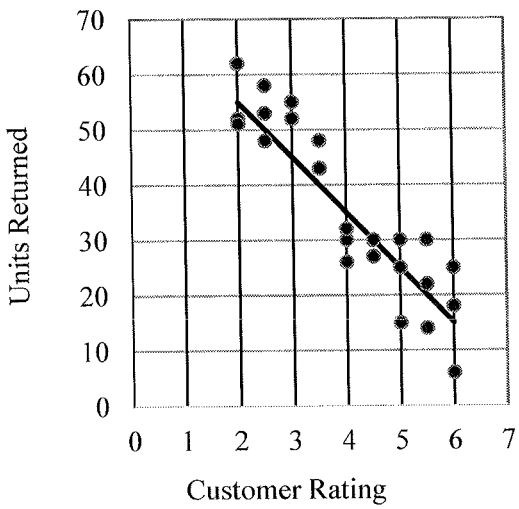
A) In every 2-hour interval, the magnitude of the rate of change of temperature recorded by the thermometer in the sun is greater than that of the thermometer in the shade.
B) In every 2-hour interval, the magnitude of the rate of change of temperature recorded by the thermometer in the shade is greater than that of the thermometer in the sun.
C) In the intervals from 9:00 to 11:00 and 11:00 to 13:00, the rates of change of temperature recorded by the thermometer in the shade are of greater magnitude, whereas in the intervals between 15:00 and 17:00 and 17:00 and 19:00, the rates of change of the temperature recorded by the thermometer in the sun are of greater magnitude.
D) The rates of change of temperature recorded by the thermometer in the sun are roughly the same as the rates of change of temperature recorded by the thermometer in the shade.

CONTINUE ➡

Questions 29 & 30 refer to the following information.

A home improvement e-store sells 27 different kinds of hammers. The store tracked the number of hammers that were returned over the course of one year as a function of each hammer's average customer rating on a scale of 0–7. The scatterplot below shows the customer rating and the number of each type of hammer that was returned, along with the line of best fit. The equation for the line of best fit is $y = 75 - 10x$.

Hammer Return Information



29

According to the line of best fit, how many hammers would the e-store expect to be returned if its customer rating was 4.5?

A) 45
B) 39
C) 30
D) 25

30

For how many of the 27 hammers tracked was the number of hammers returned greater than the number predicted by the line of best fit?

A) 2
B) 10
C) 12
D) 27

Student Produced Responses

31

If $y = kx$, where k is a constant, and $y = 128$ when $x = 16$, what is the value of y when x is increased by 25%?

32

Aaron calculates, using a certain font and line spacing, that each page of a typed document contains at least 240 words and at most 260 words. What is the least number of pages needed for a 12,000-word document?

33

$$B = \frac{m}{h^2}$$

The body mass index B of a person who has mass m and height h is calculated using the formula above. If two people have the same mass, but one has height h and the other has height $\frac{3}{4}h$, what is the ratio of the taller person's body mass index to the shorter person's body mass index? *(Grid your answer as a fraction.)*

CONTINUE

34

Sadie kept track of how far she ran each day and recorded her data in a table.

Day	Distance on Street (miles)	Distance in Park (miles)	Total
Monday		3	
Tuesday	2		
Total			10

On Monday, $\frac{3}{4}$ of the distance Sadie ran was in the park. What distance, in miles, did she run in the park on Tuesday?

35

John usually gets to work by driving 29 miles down Greenwich Street, which goes from his house to his office in a straight line. One week, Greenwich Street is closed due to construction, so John gets to work by driving 13 miles west on 57th Street, and then turning onto Harrison Street and driving north to his office. Approximately how many more miles does John have to drive by taking the detour? (*Round your answer to the nearest tenth of a mile.*)

36

There are 3 feet in one yard. A man is walking at a speed of 90 yards per minute. What is the man's speed, in feet per second?

37

From 1980 to 2015, the student body at University A grew approximately 3% per year. By contrast, during the same period, the student body at University B grew by approximately 300 students per year. If both universities had exactly 10,000 students in 1980, what was the difference between the student bodies of University A and University B in 2015?

38

$y^2 = \sqrt{x}$

If $y = 6$, what is the value of x?

STOP. Do not go on until instructed to do so.

Explanations: Part Three – Math

Algebra

Guided Practice – Solving Linear Equations & Inequalities

Multiple Choice – No Calculator

1 D. If $k = 4$, then $\frac{x-1}{2} = 4$. Multiply both sides by 2 to get $x - 1 = 8$, so $x = 9$.

2 D. Solving the first equation yields $x = 8$, so $4(8) + 5 = 37$.

3 B. Combing like terms on each side results in $\frac{4}{6}x = \frac{12}{6}$. Multiplying all terms by 6 yields $4x = 12$, so $x = 3$.

4 C. Multiplying both sides by 4 cancels out the 4 on the left side, creating $x + 3 = 5 \cdot 4$, so $x = 20 - 3 = 17$.

5 D. If we distribute, we get $8x - 8 = ax + 3x - 8$, which we can simplify to $5x = ax$, which means $a = 5$.

Student Produced Response – No Calculator

1 0. Start by distributing: $-6x + 7.5 = -15x + 7.5$. Adding $15x$ to both sides and subtracting 7.5 from both sides results in $9x = 0$, so $x = 0$.

2 4. Since $5c - 12$ must be greater than or equal to 8, the least possible value of c is when $5c - 12$ is equal to 8. If we solve the equation $5c - 12 = 8$, we get $c = 4$.

3 22. If $3x + 7 = 22$, then $x = 5$. Therefore, $6(5) - 8 = 22$.

4 5. Combining like terms results in $\frac{2}{8}x = \frac{5}{4}$. We can cancel out the coefficient of x by multiplying both sides by its reciprocal, resulting in $x = \frac{5}{4} \cdot \frac{8}{2} = \frac{40}{8} = 5$.

5 1/40 or 0.025. Solve for p by first distributing the 2: $1 - 2p = \frac{3}{4} + 8p$, then combine like terms, resulting in $\frac{1}{4} = 10p$. Finally, divide both sides by 10, resulting in $\frac{1}{40} = p$.

6 0.7. We could distribute and solve for x, but we don't have to. The structure of this equation reveals that a quantity, $4x - 2.8$, multiplied by –3 is equal to that same quantity multiplied by –5. This is only possible if $4x - 2.8 = 0$. Therefore, $4x = 2.8$ and $x = 0.7$.

Multiple Choice – Calculator

1 D. Simplify the inequality: $2b > 10$, so $b > 5$, which means b can be any value greater than 5.

2 C. 10 more than 17 is 27; therefore $12 + 3x = 27$, which leads to $3x = 15$ and finally $x = 5$. But that's not the answer. You must find $4x$, so $4(5) = 20$.

3 A. $g = \frac{3}{5} \div \frac{1}{2} = \frac{3}{5} \times \frac{2}{1} = \frac{6}{5}$.

4 A. If $\frac{1}{4}a > -\frac{5}{8}$, then $a > -\frac{5}{8} \times \frac{4}{1}$, which means $a > -\frac{5}{2}$.

5 B. 20 more than 4 is 22, so $6 + 9x = 24$, which leads to $9x = 18$, and finally $x = 2$. But that's not the final answer because we are asked to find the value of $5x$, which is $5(2) = 10$.

6 C. Solving $7x + 3 = 38$ yields $x = 5$. But that's not the answer, because the questions asks us to find $4.5x$, which is $4.5(5) = 22.5$.

7 B. Multiplying both sides of the equation by $(x + 1)$ yields $x + 2 = 5x + 5$, which leads to $-4x = 3$, so $x = -\frac{3}{4}$.

8 B. Simplify the equation by applying the distributive property and combining like terms. The right side of the equation becomes $-15x+12-6$, which simplifies to $-15x+6$. Thus, $3x+3=-15x+6$, and $18x=3$, so $x=\frac{1}{6}$.

9 D. Isolating the variable yields $x \leq 5$ so 6 cannot be a possible solution to the inequality.

10 C. For the equation to have infinitely many solutions, both sides of the equation must be equivalent. Simplifying the left side of the equation yields $8x-1$ and simplifying the right side yields $ax+x-1$. So, $8x-1=ax+x-1$. Combine like terms to simplify: $7x=ax$, so $a=7$.

11 C. If $2a-3 \geq 1$, then $2a \geq 4$ and $a \geq 2$. So the least possible value of $2a+3$ occurs when $a=2$: $2(2)+3=7$.

12 C. Statement I is not correct because x has to be between the positive and negative values of a number, but we do not know whether x itself is positive or negative. (For example, if y and $-y$ are 5 and –5, then x could be any number in between them, including a negative number.) Statement II is incorrect because the given information states that $x < y$. Only statement III is correct, because regardless of whether x is a positive or negative number, it must be less than y.

13 C. We could solve for x, but we don't need to. If you look closely, you can notice that $-10x-25$ is just $-2x-5$ multiplied by 5. If we multiply the whole inequality by 5, we get $20 > -10x-25$, which can be flipped to read $-10x-25 < 20$.

14 D. We start by distributing: $6x+4<6x-3$. If we try to solve further, the $6x$ on both sides cancels out, leaving us with $4<-3$, which is not true, so there is no solution.

15 A. If we combine like terms, we get $11x+5>11x-6$. This is always true, because no matter what number we plug in for x, adding 5 to a value is always greater than subtracting 6 from that value. Therefore, all real numbers satisfy the inequality.

16 B. If x is 50% more than the sum of the other two numbers (y and z), then $x=1.5(y+z)$ or $y+z=\frac{x}{1.5}$. We can rewrite $\frac{x}{1.5}$ as $\frac{2x}{3}$ and substitute it in the equation for $y+z$ such that $x+y+z=x+\frac{2}{3}x=670$. Combine terms to find $\frac{5x}{3}=670$. So $x=670\times\frac{3}{5}=402$.

17 A. We want to write each of the three averages as algebraic equations, then find the average of those three equations. First, we write $x=\frac{1}{2}(m+8)$, $y=\frac{1}{2}(2m+12)$, and $z=\frac{1}{2}(3m+16)$. Then, to find the average of x, y, and z, we add $\frac{1}{2}m+4+m+6+\frac{3}{2}m+8=3m+18$, and then divide $3m+18$ by 3, which yields $m+6$.

Guided Practice – Solving Systems of Equations

Multiple Choice – No Calculator

1 A. Isolate one variable to solve. In the second equation, $-x=-6-2y$, so $x=6+2y$. Substitute this in the first equation: $3(6+2y)+2y=2$, so $18+6y+2y=2$ and $8y=-16$. Therefore, $y=-2$. $3x+2(-2)=2$, so $3x=6$ and $x=2$.

2 B. Since the second equation has been solved for x, we can use substitution by plugging in $3(y+1)$ for x in the first equation: $\frac{3(y+1)}{y}=4$, which can be simplified to $3y+3=4y$, and finally $y=3$.

3 C. We can use elimination to solve. If we multiply the top equation by 2, we get $2x+2y=6$. If we subtract this equation from the second, we are left with $x=6$. Plugging that back into the second equation, we get $3(6)+2y=12$, in which case $2y=-6$ and $y=-3$.

4 C. First, find the values of x and y. One way to solve is to add the equations, yielding $5x+y=11$, so $y=11-5x$. Substitute this value of y into the first equation: $2x+3(11-5x)=-6$, which simplifies to $2x+33-15x=-6$ and then $33-13x=-6$. Therefore, $x=3$. We can then plug 3 back into either equation.

If we use the first equation: $2(3) + 3y = -6$, which leads to $6 + 3y = -6$. Ultimately, $y = -4$. But we're not done, because we want to find the value of $x - y$, which is $3 - (-4) = 7$.

5 C. One way to solve is by using elimination. The top equation can be rewritten as $3x - 2y = 8$. If we add the two equations, we get $4x = 16$, which results in $x = 4$. Plugging that into the second equation gives us $4 + 2y = 8$, which turns into $2y = 4$, so $y = 2$. But we're not done, because we are asked to find the product of xy, which is $4(2) = 8$.

Multiple Choice – Calculator

1 B. Linear functions have 0 solutions if the slopes are equal (because that means the lines are parallel), 1 solution if they intersect at 1 point, and infinite solutions if the equations are equivalent (meaning the lines overlap). Solving the first equation in terms of y yields $y = \frac{1}{2}x$. This means the two functions have different slopes, so the lines must intersect at exactly one point, meaning there is one solution.

2 C. The system has no solutions when the lines are parallel, meaning they will have the same slope. Multiplying the first equation by 2 yields $6x + 4y = -4$, which shows a different y-intercept but the same slope as the second equation, so a must be equal to 4.

3 B. The number of solutions is equal to the number of times the lines that represent these equations intersect. If we solve the second equation for y, we get $y = 0.5x - 0.5b$. Since the two equations do not have the same slope (indicated by the x coefficient), they are not parallel and do not overlap, which eliminates choices A and D. The equations represent lines that only intersect at one point, so B.

4 B. If we solve the top equation for x, we get $x = b + 5$. If we solve the bottom equation for y, we get $y = c + 5$. Since we've been told that $b = c + \frac{1}{2}$, we can substitute and get $x = c + \frac{1}{2} + 5$, or $x = c + 5 + \frac{1}{2}$. Now we can substitute y in for $c + 5$ and get $x = y + \frac{1}{2}$, which is answer choice B.

5 A. We can use substitution by first solving the top equation for x. If we divide both sides by –3, we get $x = -2y - 4$. We can then plug that into the second equation: $2(2y + 3) = 3(-2y - 4) - 2$, which simplifies to $4y + 6 = -6y - 14$. Solving for y results in $y = -2$. If we plug that back into the top equation, we get $-3x = 6(-2) + 12$, so $-3x = 0$, and $x = 0$. Thus, the solution is (0,–2).

6 A. A system with infinite solutions will overlap, so the value of a that makes this true can be found by finding the value of a that makes both lines have the same graphs, meaning both lines have the same slope and y intercept. In this case, multiplying both sides of the first equation by 2 yields $6x + 10y = -4$, so the value of a that makes the second equation equivalent is –10.

7 B. Linear functions have 0 solutions if the slopes are equal (because that means the lines are parallel), 1 solution if they intersect at 1 point, and infinite solutions if the equations are equivalent (meaning the lines overlap). We can solve both equations for y so that they are in the same $y = mx + b$ format. In the top equation, we can subtract 15 from both sides and then divide the equation by –5, which gives us $y = 2x + 3$. In the bottom equation, we can subtract 3 from both sides, and then divide by 4, which gives us $y = 3x - 2$. Since the lines have different slopes, they must intersect at exactly one point, meaning there is one solution.

8 A. The system will have no solutions when the lines are parallel, meaning the lines will have the same slope. To determine the slope, rewrite the equations in the format $y = mx + b$, where m is the slope. If $-2x = 6y + 8$, then $-6y = 2x + 8$ and $y = \frac{2x + 8}{-6}$ and $y = -\frac{1}{3}x - \frac{4}{3}$. In the second equation, $3ay + 15 = 4x - 3$, so $3ay = 4x - 18$. Thus, $y = \frac{4x - 18}{3a}$ or $y = \frac{4}{3a}x - \frac{6}{a}$. If these equations have the same slope, then $-\frac{1}{3} = \frac{4}{3a}$. Solve for a by multiplying both sides of the equation by 3: $-1 = \frac{4}{a}$. Multiplying both sides by a yields $-a = 4$, so $a = -4$.

Student Produced Response – Calculator

1 3/4 or 0.75: The two equations must be equivalent. Multiplying both sides of the first equation by 2 yields $2ax+2by=20$. Since both equations now equal 20, equating the coefficients gives $2a = 3$ and $2b = 4$.

Therefore, $a=\frac{3}{2}$ and $b = 2$. $\frac{a}{b}=\frac{3}{2}\div 2=\frac{3}{4}$.

2 9. The graphs of these inequalities are represented by the given half-planes and include the boundary lines $y = -15x + 54$ and $y = 3x$, respectively. The solution set of the system of inequalities will be the intersection of these half-planes, including the boundary lines, and the solution (a,b) with the greatest possible value of b being the point of intersection of the boundary lines. The boundary lines can be found by setting them equal to each other: $3x = -15x + 54$, which has solution $x = 3$. Thus, the x coordinate at the point of intersection is 3. Therefore, the y coordinate of the point of intersection of the boundary lines is $3(3) = -15(3) + 54 = 9$. This is the maximum possible value of b in the solution set of the system of inequalities.

3 1. The number of solutions of a system of equations is equal to the number of times the system intersects on the xy-plane. A system has zero solutions if the lines are parallel, or if they have the same slope. Find the slope by rewriting both equations in the form $y = mx + b$, where m is the slope. The first equation can be rewritten as $2y = -3x - 4$ or $y=-\frac{3}{2}x-2$. The second equation can be rewritten as $2x - 8 = 3y$ or $y=\frac{2}{3}x-\frac{8}{3}$. The equations do not have the same slope, and they are lines, so they will intersect at only one point on the xy-plane, and thus have only one solution.

Guided Practice – Linear Equations, Inequalities, & Systems in Word Problems

Multiple Choice – No Calculator

1 C. The number of emails Jack and Amy each sent can be represented by the number of message per hour multiplied by the number of hours, and then Jack's ($3m$) and Amy's ($4a$) messages should be added together to find the total number of emails sent ($3m + 4a$).

2 D. The coefficient 4.5 can be interpreted as the rate of change in this equation, so the kitten's weight will increase by 4.5 ounces each week.

3 D. If x represents the number of 4-person tables, then $15 - x$ represents the number of 6-person tables. The number of people at each 4-person table is $4x$, and the number of people at each 6-person table is $6(15 - x)$. The total number of people can be represented by $4x + 6(15 - x) = 86$. Solving for x yields 2, which is the number of 4-person tables. Therefore, the number of 6-person tables must be 13.

4 B. To solve, find the number of weeks after delivery when $a = s$ by solving for x, so $1 + 0.2x = 0.4 + 0.4x$. Simplifying yields $0.6 = 0.2x$, so $x = 3$. The price of aluminum 3 weeks after delivery is $1 + 0.2(3)$, so the price is \$1.60.

5 C. Toby must increase his swim distance by 600 meters over the course of 12 weeks, which is equal to 50 meters per week. That is not an answer choice, but it is equal to 100 meters every 2 weeks, which is choice C.

6 C. Each volunteer has 3 hours to set up 6 tables per hour, so each volunteer will set up 18 tables. Therefore, if s is equal to the number of set-up volunteers, $18s = 180$. So the organization needs 10 set-up volunteers. If a volunteer hands out 20 flyers each hour for 3 hours, each volunteer will hand out 60 flyers. Therefore, if f is equal to the number of volunteers handing out flyers, $60f = 300$. So the organization needs 5 volunteers to hand out flyers. The total number of volunteers is equal to $s + f = 10 + 5 = 15$.

7 D. To solve, find the number of days after Labor Day when $d = p$ by solving for x, so $145 + 0.1x = 75 + 0.2x$. Simplifying yields $70 = 0.1x$, so $x = 700$. The price of both coats 700 days after Labor Day is $145 + 0.1(700) = 145 + 70 = 215$.

8 D. Let p represent the cost of a paperback, in dollars, and h represent the cost of a hardcover, in dollars. The question is asking for the sum of $h + p$. From the problem, we know that: $\begin{matrix}4p+2h=36\\3p+5h=69\end{matrix}$. One way to

solve this equation is to add both equations to yield $7p + 7h = 105$. Dividing both sides of the equation by 7 yields $p + h = 15$.

9 C. A 10% commission could be expressed as $0.10p$, where p is equal to the total price of the items she sold, because $10\% = 0.10$.

10 C. This shows that the total number of cookies and brownies the class will sell will be less than or equal to 400, because the students can sell less than they have, but they can't sell more than they have. It also shows that the earnings from the bake sale will be more than or equal to $500.

11 B. The question is asking for the number of years after release when $b = f$. Solve for x in $6.5 + 0.5x = 4.5 + 0.7x$, which simplifies to $2 = 0.2x$. So $x = 10$.

12 B. This system of inequalities shows that the sum of the number of baseballs and tennis balls is greater than or equal to 50, meaning the teacher wants to buy at least 50 total, and the sum of the prices is less than or equal to the $150 budget. A, C, and D show the less than or greater than signs applying to the incorrect values.

13 C. Write a system of equations to solve. If c is equal to the number of boxes of cupcakes sold and m is equal to the number of boxes of muffins sold, then: $\begin{matrix} c + m = 30 \\ 6c + 4m = 140 \end{matrix}$. Solving for m will yield the number of boxes of muffins. We know from the first equation that $c = 30 - m$. This can be substituted in the second equation to find that $6(30 - m) + 4m = 140$, which simplifies to $m = 20$.

14 C. If she sold 26 shirts on Friday and twice as many on Thursday, then $x = (26)(2)$. Written another way, $\frac{x}{2} = 26$.

Student Produced Response – No Calculator

1 30. Determine the value of m when $a = b$, so $10 + 0.1m = 5.5 + 0.25m$. This simplifies to $4.5 = 0.15m$, so $m = 30$. Therefore, the Wi-Fi plans will cost the same at 30 minutes of use.

2 10. If c represents the number of pounds of chicken sold and p represents the number of pounds of pork sold, then: $\begin{matrix} c + p = 15 \\ 3c + 4p = 50 \end{matrix}$. Isolate a variable and substitute. $p = 15 - c$, so $3c + 4(15 - c) = 50$. This simplifies to $-c = -10$, so $c = 10$.

3 1.6 or 8/5. A change in depth of 500 meters results in a temperature change of 8°C. Dividing both values by 5 gives us a temperature change of 1.6°C for every 100 meters.

4 0.6 or 6/10 or 3/5. Determine the price of both when $t = l$: $2.6 - 0.5x = 1.8 - 0.3x$. This simplifies to $0.8 = 0.2x$, so $x = 4$. After 4 weeks, the price of tile is equal to $2.6 - 0.5(4) = 0.6$, or $0.60 in dollars.

5 16. The maximum number of yoga classes requires spending as much as possible on instructors. If instructors are paid $40 per hour for a 1.5-hour class, they earn $60 per class. Since we are looking for the maximum number of classes, we will assign that number a variable, x. The cost for all the classes will be $60x$. Since the total cost cannot exceed $1,000, we have the inequality $60x \leq 1{,}000$, in which case $x \leq 16.67$. Since Tamara cannot offer part of a yoga class, she can offer a maximum of 16 classes per week.

Multiple Choice – Calculator

1 C. It shows the rate of online subscriptions multiplied by the number of online subscriptions, added to the rate of print subscriptions multiplied by the number of print subscriptions.

2 C. Sam's bill can be represented by the equation $9.98 + 2.5x = 17.48$, where x is the number of on-demand programs he watched. Solving for x results in $x = 3$.

3 B. It's easiest to assign the variable to the smaller value. Let x = Rafael's hours. Then Joanna's hours will be $x + 12$, and their total hours can be represented as $x + x + 12 = 64$. Solving for x results in $x = 26$, meaning Rafael worked 26 hours. If Joanna worked 12 more hours than Rafael, then $26 + 12 = 38$.

4 C. If x is the number of adults and y is the number of students, then $x + y$ equals the total number of people who went to the theme park, and $12.5x + 6.75y$ represents the total admission fee of $2,118.

5 D. It shows the total books in January, plus 95 books per month. When this number is greater than or equal to 15,000, the library will be at or above capacity.

6 A. It shows that $x + y$ is equal to the total number of slots. We don't see 60 or 30 in the answer choices, so we should conclude that the numbers in the equations are not in seconds. Since most of the second equations have coefficients of 0.5 and 1, we can conclude that the numbers represent minutes. In an 18-hour period, there are 216 minutes of ads (6 minutes per 30-minute program, (6)(2)(18) equals the total number of minutes of ads), so the number of 1-minute ads plus the number of 0.5-minute ads is equal to the total number of minutes.

7 A. This system of equations shows that the total weight of the boxes is less than or equal to 96 ounces and the total number of boxes is less than or equal to 12.

8 B. Adding 2.5 to the cost of a six-inch sandwich yields the cost of a foot-long, and the total cost of 2 foot-long sandwiches and 4 six-inch sandwiches is \$37.70, so these equations can be used to find the value of a foot-long sandwich. A and D use multiplication instead of addition to find the price of a foot-long sandwich. C uses subtraction instead of addition to find the total cost.

9 A. If t is equal to the number of one-way trips the plane can take: $2{,}000t \leq 5{,}300$ and $190t \leq 400$. A round trip is equal to $2t$, since it is 2 one-way trips, so the pilot can only make 1 round trip, which will use 4,000 gallons of fuel and last 380 minutes. In both inequalities $2 \leq t \leq 3$, so the plane only has the fuel capacity and allotted flying time to make 1 round trip.

10 D. This system of equations shows that the total weight in a truck is less than or equal to 2,000 pounds and the total number of crates is less than or equal to 30.

11 B. Based on the information in the equation, if c is the price of one cup and p is the price of one plate, then: $\begin{matrix} p - c = 6.5 \\ 5c + 3p = 55.5 \end{matrix}$. Therefore, $p = 6.5 + c$, which you can substitute into the second equation: $5c + 3(6.5 + c) = 55.5$, which simplifies to $8c = 36$, so c is equal to 4.5.

12 A. To solve, find the value of p when $s = d$. So, $\frac{1}{2}p + 65 = 200 - p$. This simplifies to $1\frac{1}{2}p = 135$. So $p = \$90$.

13 C. This scenario can be represented by the equation $k - 5q = p$, where q is equal to the number of incorrectly answered questions and p is equal to the number of points the player has. Therefore, $k - 5(30) = 50$. So, $k = 200$.

14 A. The total distance is 6 miles, so $b + w = 6$. The other three choices are all incorrect because none of the second equations have $b + w = 6$. Additionally, Sharon bikes b miles at 9 miles per hour, which takes $\frac{b}{9}$ hours. Similarly, she walks $\frac{w}{3}$ hours. The sum of these is less than 1 hour. Therefore, $\frac{b}{9} + \frac{w}{3} < 1$.

15 B. These inequalities separate time and sheets of paper into two separate inequalities. The first inequality shows that the sum of each 90-minute writing assignment and each 20-minute math assignment is greater than 300 minutes (thereby eliminating choices A and D because they do not show less than or equal to 300 minutes) and that the number of sheets of paper used is less than or equal to the greatest amount of paper that can be used, which is 20 sheets (eliminating choice C for showing the amount of paper must be less than 20 sheets instead of less than or equal to 20).

16 D. If the total dinner sales at the restaurant were \$785, then the tips earned by all servers is shown by $t = 0.15\,(785) = 117.75$. So, during an 8 hour shift, the server earns

$$s = 5.5(8) + \frac{117.75}{6} = 44 + 19.625 = 63.625.$$

17 A. If x and y are the length and width of the playground, this system of inequalities shows that the difference between the length and width is greater than or equal to 20 (eliminate choices B and D). Since the perimeter cannot be greater than 2,000 feet, the sum of 2 lengths and 2 widths ($2x + 2y$) must be less than or equal to 2,000 (eliminate choice C).

18 C. If a is the cost of an adult ticket and c is the cost of a child ticket, the information in the problem can be represented as: $\begin{matrix} 4a+8c=116 \\ 6a+2c=89 \end{matrix}$. Isolate one of the variables and use substitution to solve. One way is to add both equations together to find that $10a + 10c = 205$. This yields $c = 20.5 - a$, which can be substitute back into one of the original equations to solve for a. The child ticket costs \$8.50 and the adult tickets costs \$12.00.

19 B. x student tickets at \$12 each can be represented as $12x$. 2 adult tickets at \$15 each is \$30. Since the total amount of money must be between \$90 and \$105, we can create the compound inequality $90 \le 15(2) + 12x \le 105$. Taking 30 away from all sides simplifies the equation to $60 \le 12x \le 75$, or $5 \le x \le 6.25$. The only integer values that make this inequality true are 5 and 6, so the only possible answer is B.

20 C. If s is equal to the number of superhero comics sold and g is equal to the number of graphic novels sold, then: $\begin{matrix} s-g=40 \\ 3.5s+12.5g=700 \end{matrix}$. Therefore, $g = s - 40$. Substitute this in the second equation: $3.5s + 12.5(s - 40) = 700$. This simplifies to $16s = 1{,}200$, or $s = 75$. So the comic book store sold 75 superhero comics.

21 C. If n is the number of copies made and there is a fee of \$0.10 per page, then the cost for the copies is $0.10n$. Adding the base fee of \$1.00 to use the machine results in a total cost of $t = 1.00 + 0.10n$.

22 C. The expression shows the number of miles traveled per day divided by the number of miles per gallon of gas to find the number of gallons of gas used per day. This amount is then multiplied by x days to find the total number of gallons of gas needed for the trip, which is then multiplied by 2.19 to find the total cost, in dollars.

23 C. If 4 times x plus 10 is equal to 6, then $4x + 10 = x$, and $x = -1$. In that case, 2 times x plus 5 feet can be written as $2(-1) + 5$, which equals 3.

24 C. For Store A, $y = 500 + (25 + 70)x$. For Store B, $y = 450 + (20 + 80)x$. These two equations simplify to $y = 500 + 95x$ and $y = 450 + 100x$. To find when Store B is less than or equal to Store A, solve the inequality $450 + 100x \le 500 + 95x$. Solving for x results in $x \le 10$.

25 C. The number of boxes of cereal in each carton (18) is irrelevant to the question asked. The machine fills c cartons in 60 minutes, and the question asks how many cartons the machine fills in 5 minutes, so $\frac{c}{60} = \frac{x}{5}$, where x is equal to the number of cartons the machine fills in 5 minutes. This can be simplified as $\frac{5c}{60}$.

26 C. Since we are looking for the number of players, we will assign that value a variable, x. In the first situation, each player gets 8 ounces, which is a total of $8x$ ounces, plus the 76 extra ounces. In the second situation, each player gets 10 ounces, or a total of $10x$ ounces, but they are 30 ounces short. The total amount of water can be represented as both $8x + 76$ and $10x - 30$, in which case $8x + 76 = 10x - 30$. Solving for x results in $x = 53$.

Student Produced Response – Calculator

1 15. To solve, write an equation. $36 = 2.4y$ where y is equal to the number of years. $y = 36 \div 2.4 = 15$ years.

2 45. If r represents the number of bouquets of roses and d represents the number of bouquets of daisies sold by the florist, then from the problem we know: $\begin{matrix} r-d=30 \\ 25r+15d=1{,}350 \end{matrix}$. You can use substitution to solve. $d = r - 30$, so $25r + 15(r - 30) = 1{,}350$. This simplifies to $40r = 1{,}800$, so $r = 45$. So the florist sold 45 bouquets of roses.

3 65. If b is equal to the cost of a bush and t is equal to the cost of a tree, then: $\begin{matrix} 13b+4t=485 \\ 6b+2t=230 \end{matrix}$. Multiplying both sides of the second equation by –2 yields $\begin{matrix} 13b+4t=485 \\ -12b-4t=-460 \end{matrix}$. Adding these equations yields $b = 25$. Plug this into one of the equations solve for t: $6(25) + 2t = 230$. This simplifies to $t = 40$. So $b + t = 25 + 40 = 65$.

4 1,925 (entered into bubble sheet as 1925). We assign the smaller value the variable, so Damon drove x hours, Geoff drove $x + 15$ hours, and they drove a total of $x + x + 15 = 55$, resulting in $x = 20$. Damon therefore drove 20 hours and Geoff drove 35 hours. At an average speed of 55 mph, Geoff drove $(35)(55) = 1{,}925$ miles.

5 38. Write a system of equations to solve. If p is equal to the price of the pants and b is equal to the price of the backpack, this system represents the problem: $\begin{matrix} p+b=78 \\ p+1.06b=80.40 \end{matrix}$. Subtracting the first equation from the second yields $(p + 1.06b) - (p + b) = 80.4 - 78$, which simplifies to $0.06b = 2.4$. This gives $b = 40$. So the backpack cost \$40 before tax. Substituting in the first equation yields $p + 40 = 78$ so $p = 38$. Thus, the cost of the pants is \$38.

Guided Practice – Linear Equations, Inequalities, & Systems on the Coordinate Grid

Multiple Choice – No Calculator

1 D. If the x-axis represents time t, and the y-axis represents the amount of money in the account, then at the beginning (time zero) when money is first deposited into the account, the y-value should be 500. Only choice D represents this. The equation of the line is $V = 150x + 500$, where 500 represents the initial deposit (the y-intercept) and 150 represents the amount of change over a particular period t.

2 C. If Jake starts each week with 95 computers and can repair 12 per day, then the number of computers to be repaired that week minus the number of days he worked multiplied by the number of computers he can fix in a day will leave the number of computers still left to repair.

3 C. If $3y - x = 12$, then $x = -12 + 3y$. Plugging into the first equation: $2(-12 + 3y) + 5y = -2$ and $-24 + 6y + 5y = -2$ and $11y = 22$, resulting in $y = 2$. Thus, $2x + 5(2) = -2$, so $2x = -12$ and $x = -6$.

4 A. A line parallel to the line of the given equation will have the same slope. Choice A can be rewritten as $y = 4x + 12$, where the coefficient of x is the slope of the line. This is the same as the slope in the given equation.

5 B. If the first equation has an x intercept at (1, 0), then we can plug those (x,y) values into the equation: $0 = a(1) + 1$, then $a = -1$. If a system of equations has no solutions, the linear equations must be parallel lines. Therefore, they both have the same coefficient of –1.

6 B. A line with the slope $\frac{1}{5}$ will have an equation $y = \frac{1}{5}x$. Therefore, the value of x will be 5 times the value of y. This is only true for choice B.

7 C. The equation is written in the form of $y = mx + b$, where (x,y) is a point on the line. In this case, (3,26) is a point on the line $c = 7h + b$, so $26 = 7(3) + b$, which simplifies to $b = 5$.

8 D. A line with slope m has a perpendicular line with slope $-\frac{1}{m}$. The given equation is written in the form $y = mx + b$, so the slope is –6. Therefore, the slope of the perpendicular line will be $\frac{1}{6}$. This is only true of choice D, which can be rewritten as $y = \frac{x-18}{6} = \frac{1}{6}x - 3$.

Student Produced Response – No Calculator

1 2. The equation is written in the form $y = mx + b$ and (8,26) is a point on the line of this equation. Therefore, $26 = a(8) + 10$, which simplifies to $a = 2$. The hourly rate is \$2 per hour.

2 5. Since the slope of the first line is 3, an equation of this line can be written in the form $y = 3x + c$, where c is the y-intercept of the line. Since the line contains the point (1,9), one can substitute 1 for x and 9 for y, which gives $9 = 3(1) + c$, so $c = 6$. Thus, an equation for the first line is $y = 3x + 6$. The slope of the second line is equal to $\frac{1-3}{4-2} = -1$. Thus, an equation of the second line can be written in the form of $y = -x + d$, where d is the y-intercept of the line. Substituting 2 for x and 3 for y gives $3 = -2 + d$, so $d = 5$, so an equation for the second line is $y = -x + 5$. Since a is the x-coordinate and b is the y coordinate of the intersection point of the two lines, one can substitute a for x and b for y in the two equations, giving the system $b = 3a + 6$ and $b = -a + 5$. Use substitution to find the values of a and b: $3a + 6 = -a + 5$, so $a = -\frac{1}{4}$ and $b = \frac{1}{4} + 5$. So the value of $a + b = 5$.

Multiple Choice – Calculator

1 C. This equation can be written as $y = mx$, where m is the slope of the line. In the context of the problem, the slope represents the daily rental fee, because multiplying the fee by the number of days, x, yields the total cost, y.

2 B. If the system has infinite solutions, then both lines overlap on the xy-plane and must have the exact same equation. Therefore, a must be equal to b.

3 B. The only points both graphs have in common are solutions to the system. If $y = 2x - 3$, then $2x - 3 = x - 3$, so $x = 0$. $y = 2(0) - 3 = -3$. The graphs intersect at (0,–3).

4 A. The second equation can be rewritten as $y = 3x - 5$. Both equations have the form $y = mx + b$, where m is equal to the slope and b is equal to the y-intercept. Since both equations have the same slope but different y-intercepts, the lines must be parallel. Only the points where both graphs have in common are solutions to the system. Since the lines are parallel, they will never intersect, so there are no solutions.

5 B. Since the inequalities show that y values are greater than or equal to (or greater than) a value, the solutions for both inequalities will be all possible values above the lines on the xy-plane. Both lines have negative slopes and intercept the y-axis at 1, so no solutions will be in Quadrant III.

6 C. A line with points Quadrants I, II, and III is a line with a positive slope that intercepts the y-axis at a point above the x-axis. It cannot have a slope of zero, because it would be a horizontal line, and so would only pass through two quadrants.

7 D. To find the point where any two straight lines intersect, find the solutions (x,y) for both. At this point, $2x - 4 = -3x + 6$, so $5x = 10$, so $x = 2$. Thus, $2(2) - 4 = y$, and $y = 0$. So, the two lines intersect at (2,0).

8 C. A line that is perpendicular to a line with slope m will have a slope equal to $\frac{-1}{m}$. The line above intercepts the y-axis at (0,3) and a point at (–1,1). You can find the slope by substituting into $y = mx + b$, where m is the slope and b is the y-intercept: $1 = m(-1) + 3$, so $m = 2$. The slope of the perpendicular line will be $-\frac{1}{2}$. That rules out choices A and B. Rewriting choice C in terms of y yields $y = -\frac{1}{2}x + 3$. Rewriting choice D in terms of y yields $y = \frac{1}{2}x + 2$. C is the perpendicular line.

9 A. B is the amount of water, so the B-intercept is the initial amount of water in the bucket.

10 C. The equation of a line can be expressed as $y = mx + b$, where m is the slope and b is the y-intercept. Since the line is decreasing, it has a negative slope, which rules out choice A and B. Since the line intercepts the B-axis at 10, the equation will be $B = -2d + 10$.

11 D. The graph shows the increase in the number of loaves of bread the bakery baked, so the L-intercept shows the number of loaves of bread the bakery had when they started tracking how many they baked.

12 B. The equation of a line can be expressed as $y = mx + b$, where m is the slope, or rate of change, and b is the y-intercept. Since the line is increasing, it has a positive slope, which rules out choice D. According to the graph, the bakery bakes 8 loaves per hour, so 8 is the slope. Since the line intercepts the L-axis at 3, the equation will be $L = 8h + 3$.

Student Produced Response – Calculator

1 4. If the system has one solution, then both lines intercept at the point (1,1). Therefore, $1 = -b(1) + 5$, so $b = 4$. Since b is a constant, this is also true for the second equation.

2 2. The cost, c, will be the same for both companies for the same number of hours at the point (h,c) where the graphs overlap. In this case, $30 + 12(3)h = 42 + 10(3)h$. This simplifies to $h = 2$. Plugging 2 in for h in the equation would give you the cost, but the question only asks for after how many hours the jobs are the same price, so the answer is 2.

Guided Practice – Absolute Value

Multiple Choice – No Calculator

1 A. Solve by isolating the absolute value part of the equation: $|x + 4| = 4$. This equation has two possible solutions: 8 – 4 = 4and 0 – 4 = –, so the value of x could be 8 or 0. Since 8 is not an option, 0 is the correct answer.

2 A. $|x - 2| - 2$ will equal 0 if $|x - 2| = 2$. So x could be equal to 4 or 0. The other three choices cannot be the correct answer because any value of x substituted into the absolute value part of the expression will yield a positive number, so adding 2 will yield a sum greater than 0.

3 D. When isolating $|5x + 4|$ on one side of the equation, you'll find that $|5x + 4| = -8$. An absolute value cannot be negative, so this equation has no solution.

4 A. $|2 - 4(-4)| = 18$, which is greater than 10. For choice B, $|2 - 4(-1)| = 6$. For choice C, $|2 - 4(2)| = 6$. For choice D, $|2 - 4(3)| = 10$. You can also solve the absolute value inequality by rewriting it as a compound inequality: $-10 > 2 - 4x > 10$. We isolate x by subtracting 2 from all three sides and then dividing by –4, which gives us $3 < x < -2$. (Don't forget that multiplying or dividing an inequality by a negative number makes the inequality change direction!) Therefore, x must be greater than 3 or less than –2. Choice A is the only number that works.

5 C. This states that the difference between dancers' heights and 70 must be less than or equal to 4.

6 D. The easiest way to answer this question is to try all four answer choices. $f(2) = |5 - 4(2)| = 3$. $f\left(\frac{1}{2}\right) = \left|5 - 4\left(\frac{1}{2}\right)\right| = 3$. None of the other choices are equal to 3.

7 A. If a is a negative number, then the absolute value of a is the opposite of a, or $-a$. So the absolute value of a plus the absolute value of b is equal to $-a + b$.

Student Produced Response – No Calculator

1 1. There are two possible solutions to x. First, isolate the part of the equation within the negative value signs: $|x - 3| = 2$, therefore $x - 3 = \pm 2$. Solving for x in both equations yields possible values of x of 5 and 1, respectively. Since 1 is the least value, it is the correct answer.

2 13. Solving the first equation results in $8 - k = 5$ and $8 - k = -5$, in which case k could be equal to 3 or 13. Solving the second equation results in $k - 2 = 11$ and $k - 2 = -11$, in which case k could be equal to 13 or –9. 13 is the only value that works for both equations.

3 2,000. If $t = 1$, $s = -2|1 - 20| + 40$, which simplifies to $s = -2(19) + 40 = -38 + 40 = 2$. Since s is the weekly sales in thousands, the maximum sales the band will have in one week is 2,000.

Multiple Choice – Calculator

1 D. If $|n - 2| + 2 = 0$, then $|n - 2| = -2$. However, the absolute value can never be a negative number. Therefore, there is no value for which $|n - 2| + 2 = 0$.

2 B. For each point, find the value of $|x| - |y|$. For point B, $|-5| - |-2| = 3$. For choice A, the equation will be 2 – 4 = –2. For choice C, 2 – 3 = –1. For choice D, 3 – 3 = 0.

3 B. Set up an absolute value inequality to solve. $|x-2.5| \le 0.05$, which is solved with the compound inequality $-0.05 \le x-2.5 \le 0.05$. This gives a possible range of values of $2.45 \le x \le 2.55$. The only choice in this range is B.

4 C. In the given inequality, $-y < y$, so y must be positive, and statement II is true. If $-y < x < y$, the value of x is either between y and 0 or $-y$ and 0, so statement III is true. It is possible for x to be either positive or negative, so statement I is not necessarily true. Therefore, only II and III must be true.

5 A. This scenario can be represented by the inequality $|x-12.25| \le 0.06$, in which the difference between the diameter of the gear x and 12.25 is less than 0.06, so the range of possible diameters can be represented by $2.19 \le x \le 12.31$.

6 B. This expression can be simplified as $-|-1|+|-11| = -1+11 = 10$.

7 C. To find the range, find the values for which $t-75 = \pm 145$. When $t = -70$, $t-75 = -145$. When $t = 220$, $t-75 = 145$. So the range is $-70 \le t \le 220$.

8 D. This expression shows that the low or high temperature subtracted from 34 will yield 7, which gives a range of $27 \le t \le 41$, which shows a high temperature of 41° and a low of 27°. You can check by putting these values back into the equation. $34-27 = 7$ and $34-41 = -7$.

9 A. If $|26x| = 13$, then $26x = \pm 13$. So $x = \pm\frac{13}{26}$ or $\pm\frac{1}{2}$. So x can only be equal to $\frac{1}{2}$ and $-\frac{1}{2}$.

10 B. This equation shows that the absolute value of the difference between the weight and 16 is 0.5, which means that the range of acceptable weights is $15.5 \le x \le 16.5$.

11 C. The difference between the actual temperature of the oven and 180° cannot exceed 5°. Answer choice B says the difference between the actual temperature and 5° cannot exceed 180°. Answer choice A says the difference between the actual temperature and 180° must be at least 180°. Answer choice D says the difference between the actual temperature and 5° must be at least 180°. Only answer choice C says the difference between the actual temperature and 180° cannot exceed 5°.

12 B. It takes Jane $30 \le m \le 40$ minutes to get to her friend's house, but the question asks for this in miles per hour. The distance is 25 miles, so to solve, determine Jane's speed to travel 25 miles per hour in 30 minutes and Jane's speed to travel 25 miles per hour in 40 minutes. This is equal to $37.5 \le m \le 50$. To find the correct inequality, find the midpoint (or average) of the two values, which is 43.75, then find the difference between that midpoint and the two values, which is ±6.25.

13 A. If $x < 0$, x is a negative number, so the absolute value will be positive; in other words, it will be $-x$. You can check by substituting a few negative values for x. For example, if $x = -2$, then $|-(-2)| = 2$.

Student Produced Response – Calculator

1 61. If $|x-72| = 10.8$, the low score will be $x-72 = -10.8$, so x will be equal to 61.2 or 61 rounded to the nearest whole number.

2 93.5. If $|x-110| \le 16.5$, there is a range of possible prices from $93.5 \le x \le 126.5$. So, the lowest possible price for a painting is $93.50.

3 2. If $|s-30| \le 5$, $30 \pm 5 = 7.5p + 3.75 + 2.25$, where p is equal to the number of people Tom buys tickets for. Since the question asks for the maximum number of people Tom can buy tickets for while staying within his preferred budget, you want to find how many tickets he can purchase within the $35 budget. So, $35 = 7.5p + 3.75 + 2.25$. Isolating the variable yields $29 = 7.5p$. So $p \approx 3.87$, but since Tom can't bring a partial person to the movies, he can bring himself and 2 friends with him to the movies.

Mixed Practice – Algebra

Multiple Choice – No Calculator

1 Solving Linear Equations & Inequalities: D. If $3x = 27$, then $x = 9$, so $6x + 5 = 6(9) + 5 = 59$.

2 Linear Equations, Inequalities, & Systems in Word Problems: C. If a peanut butter bar has 40 more calories than a fruit bar, then the fruit bar can have x calories and the peanut butter bar can have $x + 40$

calories, in which case 3 peanut butter bars and 4 fruit bars having a total of 750 calories can be represented as $3(x + 40) + 4x = 750$. Solving for x yields $x = 90$, to which we must add 40 to arrive at 130.

3 Absolute Value: C. Julio's score must be between 80 and 100. To create the correct inequality, take the midpoint (or average) of the high and low values, which is 90. Now, notice that he must score within 10 points of that midpoint. The difference between Julio's score and 90 must be no more than 10. Only choice C shows this relationship.

4 Linear Equations, Inequalities, & Systems on the Coordinate Grid: D. The slope of a line that passes through two points is $m = \frac{y_2 - y_1}{x_2 - x_1}$, so the slope of the line through the given points is $\frac{3-6}{-1-0} = 3$. The diagonals of a square are perpendicular, and so the line that passes through A and C must have a perpendicular slope. Perpendicular slopes are negative reciprocals of each other. If the given line has a slope of 3, the perpendicular line will have a slope of $-\frac{1}{3}$. Choice D is the only one with that slope.

5 Solving Systems of Equations: B. If we solve the top equation for x, we get $x = b + 7$. If we solve the bottom equation for y, we get $y = c + 7$. Since we're told that $b = c + \frac{1}{2}$ we can use substitution and get $x = c + \frac{1}{2} + 7$, or $x = c + 7 + \frac{1}{2}$. Now we can substitute y into the equation, getting $x = y + \frac{1}{2}$, which is answer choice B.

Student Produced Response – No Calculator

1 Linear Equations, Inequalities, & Systems on the Coordinate Grid: 4. Since we are solving for b, we only need to work with the second equation. If (1,–1) is a point in both lines, then $-1 = b(1) - 5$, so $b = 4$.

2 Solving Linear Equations & Inequalities: 11. If $3a - 5 \geq 1$, then $3a \geq 6$ and $a \geq 2$. Since a must be greater than or equal to 2, the least possible value of a is 2. Therefore, the least possible value of $3a + 5$ occurs when $a = 2$: $3(2) + 5 = 11$.

Multiple Choice – Calculator

1 Solving Linear Equations & Inequalities: C. The given information states that $9 + 3x = 3$, so $x = -2$. The question asks to find the value of $15 + 2x$, which is $15 + 2(-2) = 11$.

2 Linear Equations, Inequalities, & Systems in Word Problems: C. If we assign Lisa's dinner the variable x, then the price of Stacey's dinner is $x + 3$, and the total cost of the meal is $2x + 3$ dollars. If this is split evenly, Lisa and Stacey each paid $\frac{2x+3}{2} = x + 1.5$ dollars, plus an 18% tip. After adding the tip, each of them paid $(x + 1.5) + 0.18(x + 1.5) = 1.18(x + 1.5) = 1.18x + 0.27$ dollars.

3 Solving Linear Equations & Inequalities: D. Isolating the variable yields $b \leq -4$, meaning the greatest possible value of b is –4, so –3 cannot be a possible solution to the inequality.

4 Linear Equations, Inequalities, & Systems in Word Problems: A. Let s represent the number of songs and p represent the number of podcasts. Based on the data in the problem: $\begin{matrix} s + p = 25 \\ 3.5s + 20p \leq 300 \end{matrix}$. So, $s = 25 - p$ and $3.5(25 - p) + 20p \leq 300$. This simplifies to $16.5p \leq 212.5$. Dividing both sides of the inequality by 16.5 yields approximately $p \leq 12.878787$. Any value less than this is a possible answer for p. The other choices will either require more than 300 megabytes of data.

5 Absolute Value: D. In the first equation, m could be equal to 9 or –1. In the second equation, k could be equal to 5 or –15. Since both m and k are less than 0, $m = -1$ and $k = -15$. So, $m - k = -1 - (-15) = 14$.

Student Produced Response – Calculator

1 Linear Equations, Inequalities, & Systems in Word Problems: 38. Determine the price of both when $a = b$: $26 + 0.3x = 18 + 0.5x$. This simplifies to $8 = 0.2x$, so $x = 40$. This means that their houses had the same price per square foot after 40 weeks. At that time, the price of both of their houses is equal to $26 + 0.3(40) = 38$, or \$38.

Data Analysis

Guided Practice – Ratios & Proportions

Multiple Choice – Calculator

1 B. Since $y = kx$ and $y = 42$ when $x = 7$, then we can plug in 42 and 7, giving us $42 = k(7)$. Solving for k results in $k = 6$. Substitute 6 into the equation yields 6 times 9, which is 54.

2 B. To find the unit price of the cupcakes sold by the dozen, divide 21 by 12, which equals \$1.75. This is \$1.00 cheaper than the individual price of \$2.75.

3 B. If Rover started at 20 kg and now weighs 46 kg, his weight gain was 26 kg. Rover's weight gain was 30% more than Spot's, meaning Rover's weight gain was 1.3 times Spot's weight gain. If we call Spot's weight gain x, we can create an equation $1.3x = 26$. Solving for x results in $x = 20$. This is how much weight Spot gained; the question asks for Spot's current weight, which is $20 + 20 = 40$.

4 B. Though it may seem intuitive that the area would remain the same, the best way to tackle this problem is to calculate the new dimensions and area. A change of 10% to a side with length 20 meters would be a change of 2 meters (either increasing or decreasing, depending on the side).
$20 + 2 = 22$, and $20 - 2 = 18$; $18 \times 22 = 396$. Alternatively, $20 \times 0.9 = 18$ and $20 \times 1.1 = 22$ (a decrease and increase of 10 percent, respectively).

5 C. Calculate the old area and the new and use the formula $\frac{new-old}{old}$ to determine the percentage change.
The old area of the rectangle is equal to the length times the width: $50 \times 40 = 2{,}000$. To find the area of the new rectangle, find the length and width after the 20% decrease and increase, respectively. 20% = 0.2. A 20% decrease means 80% remains, and so can be determined by multiplying 0.8 by the length; in this case, $50 \times 0.8 = 40$. A 20% increase can be determined by multiplying 1.2 by the width; in this case, $40 \times 1.2 = 48$. The new area is $40 \times 48 = 1{,}920$. Substituting into the percentage change formula: $\frac{2{,}000 - 1{,}920}{1{,}920} = 0.04$ or 4%.

6 C. Since 1 hour = 60 minutes, there are $\frac{60}{15} = 4$ periods of 15 minutes per hour. The total number of hours the messenger worked is $5 + 4 = 9$ hours. $9 \cdot 4 = 36$ periods. At \$5 per delivery, the messenger made \$180 for the nine hours of work.

7 D. Let y be the amount of money earned for selling 180 items. Set up the proportion $\frac{40}{1500} = \frac{180}{y}$. Cross multiplying produces $40y = (180)(1{,}500)$. Solving for y results in $y = \$6{,}750$.

8 A. 67% of \$1,500 is \$1,005. Using that much for living expenses leaves \$495 for his savings account.

9 B. You must take note of any word or information written in italicized, bold, or underlined font. The question asks how many juniors were *not* wearing the school colors that day. If 48% are wearing the school colors, then 52% are *not* wearing the school colors. To find a percent of any value, multiply the percent (in decimal form) by the value: $0.52(650) = 338$.

10 C. To get the total number of hours for a 9.5-year trip, multiply 9.5 years × 365 days × 24 hours per day to arrive at 83,220 hours. To find the average miles per hour for the trip, divide the total miles 4,670,000,000 by 83,220 hours ≈ 56,116 miles per hour, which can be written in scientific notation as approximately 5.61×10^4.

11 B. 30% of 200 can be found by multiplying $(0.30)(200) = 60$. Subtracting 60 from 200 results in 140.

12 B. We are looking for the number of pages reviewed on Wednesday, which we will call x. The number of pages reviewed on Thursday was 125% of the pages on Wednesday, so Thursday was $1.25x$. Since we know she reviewed 40 pages on Thursday, $1.25x = 40$. Solving for x results in 32.

13 C. If a \$600 phone is on sale for 40% off, then the customer still has to pay 60% of the original price, so the sale price is $0.6(600) = \$360$. The question asks what the discount is, which is the same as asking for the percent change. We find percent change by dividing the difference of the two values ($360 - 252 = 108$) by the sale price (360). This gives us 0.3, which is 30%.

14 D. A formula is described in the first sentence: $D = \frac{m}{V}$. All we have to do is plug in the values and solve. If density is 2.7 and volume is 8, and we are looking for mass, then $2.7 = \frac{m}{8}$. Solving for m results in 21.6.

15 B. To solve, we set up the proportion $\frac{7}{200} = \frac{x}{35{,}000}$, where x is the number of widgets the analyst will inspect. Cross multiplying produces $200x = (7)(35{,}000)$, which results in $x = 1{,}225$.

16 B. 3 miles is equal to 15,840 feet, found by multiplying 5,280 by 3. 15,840 feet is equal to 24 furlongs, found by dividing 15,840 by 660.

17 C. 3.5 acres = 3.5(43,560) square feet, which is 152,460 square feet. If there are 9 square feet in 1 square yard, then we can find the number of square yards by dividing 152,460 by 9, which gives us 16,940.

18 A. 11.9 stone is equal to 166.6 pounds (found by multiplying 11.9 by 14), and 166.6 pounds is equal to 2,665.6 ounces (found by multiplying 166.6 by 16).

19 D. We are trying to find Ms. Martinez's original salary, so we will call that value x. If her salary increases by the constant amount of 5% of her starting salary, then 10 years later, her salary is $x + 0.05(10)x$, which is $1.5x$. Since her current salary is \$72,000, we can solve for x by setting up and solving $1.5x = 72{,}000$, which results in $x = 48{,}000$.

20 C. 0.1 pounds = 1.6 ounces (found by multiplying 0.1 by 16), which equals 45.36 grams (found by multiplying 1.6 by 28.35). If he is paying 20% less than \$40 per gram, he is paying \$32 per gram. 45.36 grams × \$32 = 1,451.

21 C. To find the total percentage, we set up a proportion using the total values from the two years: $\frac{x}{100} = \frac{240 + 144}{300 + 240}$, where x is the percentage. Cross multiplying results in $540x = 38{,}400$. Solving for x gives us $x \approx 71.11$, which is the percentage of students who passed over the last two years.

22 B. One way to solve this question is to set up a proportion. The ratio of clothing percentage to dollar amount is $\frac{3}{1{,}920}$. This should be equal to the ratio of rent percentage to dollar amount: $\frac{26}{x}$. Setting them equal to each other creates a proportion, $\frac{3}{1{,}920} = \frac{26}{x}$, we can solve by cross multiplying: $3x = 26(1{,}920)$. Solving for x results in $x = 16{,}640$. The question asks for the approximate amount in thousands. Rounding 16,640 to the nearest thousand results in 17 thousands.

23 B. Write the given rate as a ratio, $\frac{300{,}000{,}000 \text{ m}}{1 \text{ sec}}$, and multiply by identity fractions in order to cancel out unwanted units and replace with the units that the question is asking for: $\frac{300{,}000{,}000 \text{ m}}{1 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{1 \text{ km}}{1{,}000 \text{ m}} = \frac{18{,}000{,}000{,}000 \text{ km}}{1{,}000 \text{ m}}$. This simplifies to $\frac{18{,}000{,}000 \text{ km}}{\text{min}}$.

24 A. To convert units, start by writing the given rate as a ratio. Here, it would be $\frac{30 \text{ miles}}{1 \text{ hour}}$. Then multiply by identity fractions (fractions that are equal to 1) that will cancel out unwanted units and leave you with your desired units. Here, our desired unit ratio is feet per minute, or $\frac{\text{feet}}{\text{minute}}$. That means multiplying $\frac{30 \text{ miles}}{1 \text{ hour}} \cdot \frac{5{,}280 \text{ feet}}{1 \text{ mile}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}}$. The miles cancel each other out and the hours cancel each other out, leaving you with feet per minute.

25 C. If there are 3 boys for every 7 kids in the school, then there must be 4 girls for every 3 boys, so the ratio of boys to girls is $\frac{3}{4}$. If there are 1,200 girls in the school, then we can set up the proportion $\frac{3}{4} = \frac{x}{1{,}200}$. Cross multiplying gives us $4x = 3{,}600$, which simplifies to $x = 900$.

26 C. You can set up a proportion to solve. The ratio of aluminum to steel is 330:2400. This is proportional to the total number of pounds used in one full day. However, since we need the units to match, so we must turn 200 tons into pounds by multiplying by 2,000, which gives us 400,000 pounds. Therefore, the proportion is $\frac{330}{2{,}400} = \frac{400{,}000}{x}$. Cross multiplying gives us $330x = 960{,}000{,}000$, which simplifies to $x = 2{,}909{,}090$. This is closest to answer choice C.

27 C. You can set up a proportion to solve. The ratio of cars to minutes is 10:12. This is proportional to the number of cars made in 12 hours. However, we need the units to match, so we must turn 12 hours in minutes by multiplying by 60, which gives us 720 minutes. Therefore, the proportion is $\frac{10}{12} = \frac{x}{720}$. Cross multiplying gives us $12x = 7{,}200$, which simplifies to $x = 600$.

28 D. When a ratio is given as a single number, we can rewrite it was a fraction over 1. Here, the given ratio is $\frac{1.618}{1}$, which is equal to the ratio of the length to the height of the wall, which is $\frac{x}{20}$. We can create the proportion $\frac{1.618}{1} = \frac{x}{13}$. Cross multiplying gives us $x = 21.034$.

29 A. If one hour uses 16% of a tank of gas, we want to find out how many times 16% fits into a full tank, or 100%. Dividing 100 by 16 results in 6.25, which is the number of hours she'll be able to drive on a full tank. However, the question asks how many *more* hours will she be able to drive after having driven that first hour, which is 5.25.

30 A. To find percent decrease, divide the positive change in values by the original number. The percent decrease in carbon monoxide is $(197 - 89) \div 197$, which equals a 54.8% decrease. The percent decrease in sulfur dioxide is $(31 - 15) \div 31$, which equals a 51.6% decrease. Carbon monoxide had the greater percent decrease.

31 D. To find a percentage divide the part by the whole. However, we're not going to divide 1,312,960 by 4,594,539 because we are asked to find the percent *outside* the U.S., which means we must subtract 1,312,960 from 4,594,539 first, giving us 3,281,579. Dividing $3{,}281{,}579 \div 4{,}594{,}539 \approx 0.714$, which equals 71.4%.

Student Produced Response – Calculator

1 393. Write the given rate as a ratio, $\frac{2\text{m}}{1\text{sec}}$, and multiply by identity fractions in order to cancel out unwanted units and replace with the units that the question is asking for: $\frac{2\text{ m}}{1\text{ sec}} \cdot \frac{60\text{ sec}}{1\text{ min}} \cdot \frac{1\text{ ft}}{0.305\text{ m}} = \frac{120\text{ ft}}{0.305\text{ sec}}$. This simplifies to approximately $393.44\frac{\text{ft}}{\text{sec}}$, which rounds to 393.

2 60. First, we need to know how large the actual park is. The map shows that 1 in = 10 m. Since we are looking for square units, we should square both sides of that equation, giving us $1\text{ in}^2 = 100\text{ m}^2$. Since the park takes up 120 in^2 on the blueprint, the actual park is $100(120) = 12{,}000\text{ m}^2$. To find how many tons of concrete are needed, we divide the total area of $12{,}000\text{ m}^2$ by 200 m^2, which gives us 60.

3 89. To find the average of a set of numbers, we take the sum of all the numbers and divide that sum by the number of values in the set. Here, we need to find the total sum of all of Susan's test scores and divide that sum by 8. To find the total of the first three test scores, multiply the average by the number of tests, or $84 \times 3 = 252$. Do the same to find the total of the last five test scores, or $92 \times 5 = 460$. The total of all 8 scores is $252 + 460 = 712$. Divide that sum by 8 to get the final average, which is 89.

4 11.1. If 935.8 is equal to 8.4% of a total amount, we can call the total amount x, in which case $0.084x = 935.8$. Dividing both sides by 0.084 results in $x \approx 11{,}140$. However, that number is in millions, but the question asks for the number to be provided in billions, so we must divide that number by 1,000 (because there are 1,000 millions in a billion), giving us 11.14, which is rounded to 11.1.

Guided Practice – Linear & Exponential Growth

Multiple Choice – Calculator

1 C. The formula for compound interest is $A = P(1+\frac{r}{n})^{nt}$, where P is the principal, r is the annual interest rate expressed as a decimal, t is the number of years the deposit is held, and n is the number of times the interest is compounded per year. Substituting the provided information into the formula for compound interest yields $A = \$10{,}000\left(1+\frac{0.15}{1}\right)^{(1)(t)} = \$10{,}000(1+0.15)^t$.

2 D. Choice A is incorrect because the amount of interest earned depends on how much the interest from the previous year increases the principle. Choice B is incorrect because this also describes a situation where the principle increases as a result of the interest rate being added to it. Choice C is incorrect because it describes a situation where interest increases based on the interest earned the previous year. Choice D is correct because it describes a situation where a constant amount is added and is therefore linear growth, not exponential.

3 C. Choice A is incorrect because each succeeding term is found by adding $\frac{2}{2}$ to the term before it, a constant increase. Choice B is incorrect because each succeeding term is less than the term before it and is exponential decay. Choice D is incorrect because each succeeding term is found by adding 4 to each term before it. Choice C is exponential growth because each term is found by raising the common ratio $\frac{3}{2}$ to increasing integer exponents. $\left(\frac{3}{2}\right)^1 = \frac{3}{2}, \left(\frac{3}{2}\right)^2 = \frac{9}{4}, \left(\frac{3}{2}\right)^3 = \frac{27}{8}, \left(\frac{3}{2}\right)^4 = \frac{81}{16}, \left(\frac{3}{2}\right)^5 = \frac{243}{32}$. The sequence is exponential growth, not exponential decay because the common ratio is greater than 1.

4 B. The number of lizards observed decreases at a constant rate of 30 every three weeks. This is a linear decrease.

5 C. Since \$480/\$240 = 2, we are trying to find the number of years required to double the amount \$240. Following the rule, we divide 72 by the fixed annual rate of 6 to get 12.

6 C. The pattern of exponential growth can be found by dividing each number after the first by its preceding number: 150,000 ÷ 100,000 = 1.5; 225,000 ÷ 150,000 = 1.5. This means the values are growing by 50% at each interval. The next value should be 225,000 × 1.5 = 337,500. However, that is only at 3:30PM. We are looking for 4PM, so we need to add 50% one more time: 337,500 × 1.5 = 506,250.

7 B. This problem may look confusing, but it is nothing more than simple plug-in algebra. You are given a formula. You are given the values of every variable except one. Plug everything in and then solve for the last remaining variable. Here, you will plug in 10,000 for every instance of N_{NOW} and 50,000 for every instance of K, then solve for the variable N_{NOW+20}. $N_{NOW+20} = 10{,}000 + .07(10{,}000)\left(1-\frac{10{,}000}{50{,}000}\right) =$

$10{,}000 + 0.7(10{,}000)(0.8) = 10{,}000 + 5{,}600 = 15{,}600.$

8 A. Plug 14,200 in for N_{NOW+20} and 10,000 in for N_{NOW} then solve for K. $14{,}200 = 10{,}000 + 0.7(10{,}000)\left(1-\frac{10{,}000}{K}\right)$. First, subtract 10,000 from both sides: 4,200 =

$0.7(10{,}000)\left(1-\frac{10{,}000}{K}\right)$. Then divide both sides by 0.7(10,000): $0.6=1-\frac{10{,}000}{K}$. Subtracting 1 from both sides and multiplying by –1 yields $0.4=\frac{10{,}000}{K}$. K then equals 25,000.

9 A. When thinking about percent problems, it can be helpful to use 100 as your starting number. If an investor invests \$100 and earns 100% on that investment, then she earns \$100 and now has \$200, effectively doubling her money. If she reinvests her profit (meaning she now has \$200 invested) and again earns 100%, she then earns an additional \$200, for a total of \$400, again doubling her money. Choice B is incorrect because growing by 100% is not the same as multiplying by 100. Choice C is incorrect because her investment is growing, not remaining unchanged. Choice D is incorrect because the amount is not growing by a constant \$100 each year.

10 B. Answer A is incorrect because it can be simplified to $A = P(0.8)^t$. Since the expression in the parentheses is less than 1, this would lead to an exponential decay, not an exponential growth equation. The amount described in the problem would still be undergoing exponential growth, just at a lower rate. Answer C is incorrect because the amount in the parentheses reflects an increase in the rate to 20%. Answer D is incorrect because it is the simplified version of answer A. Answer B is correct because it results from the amount of interest added to the account at 9% minus the amount of interest added at 7%.

11 D. Percent increase is a form of percent change, which is found by (amount of change) ÷ (original amount). Trying each pair of consecutive numbers, we get $\frac{20{,}000-16{,}000}{16{,}000}=0.25$, $\frac{25{,}000-20{,}000}{20{,}000}=0.25$, and $\frac{31{,}250-25{,}000}{25{,}000}=0.25$. Each time, the change is 25%.

12 D. The formula for compound interest is $A=P\left(1+\frac{r}{n}\right)^{nt}$, where P is the principal, r is the annual interest rate, t is the number of years the deposit is held, and n is the number of times the interest is compounded per year. For annual compounding, $n = 1$, so the formula becomes $A = P(1 + r)^t$. For semiannual compounding, $n = 2$, so the formula becomes $A=P\left(1+\frac{r}{2}\right)^{2t}$. The difference between semiannual compounding and annual compounding is found by subtracting the two amounts and factoring out P.

13 D. The formula for interest is $A=P\left(1+\frac{r}{n}\right)^{nt}$, where r = rate, and n = number of times per year interest is compounded. If $r = n$, the formula can be rewritten as $A=P\left(1+\frac{n}{100n}\right)^{nt}$, (where $\frac{n}{100}$ represents the percentage interest rate) which can be rewritten as to $A=P\left(\frac{100n}{100n}+\frac{n}{100n}\right)^{nt}$, which equals $A=P\left(\frac{101n}{100n}\right)^{nt}$, and finally simplifies to $A = P(1.01)^{nt}$. Answer A is incorrect because it does not include division of the rate by 100 to account for r being a percent.

Student Produced Response – Calculator

1 $\frac{5}{3}$ or 1.66. Each successive term is found by raising the common ratio by the next integer power. The common ratio can be found by dividing any term in the sequence by the term before it.

$$\frac{125}{27}\div\frac{25}{9}=\frac{125}{27}\times\frac{9}{25}=\frac{5}{3}$$

2 7. This is essentially a simple interest problem where the number of perch increase exponentially based on the original 4,290 fish. The percent change in the number of perch can be found by subtracting the

observed number of perch in any year from the number of perch observed during the previous year and then dividing the difference by the number in that previous year. Percent change = $\frac{4{,}590 - 4{,}290}{4{,}290}(100) = 7\%$.

3 96. Percent problems like this can be solved by starting with 100%. If the panda's weight increases by 40% of the original weight, it is now 140%. If it increases by 40% again, we must find 40% of 140%, which is 56%, and add that to the total, giving us 196%. This represents an increase of 96% from the original weight.

4 3,125. Percent change can be found by subtracting any year's observed number of rabbits minus the number of rabbits from the previous year and dividing by the number of rabbits from the previous year: $\frac{2{,}000 - 1{,}600}{1{,}600} = 0.25$. This means there is 25% growth each year. Adding 25% to 2,500 results in 3,125.

5 8. The percentage rate can be found from: $\frac{\$43{,}200 - \$40{,}000}{\$40{,}000} = \frac{3{,}200}{40{,}000} = 0.08 = 8\%$.

6 10.5. We must figure out the number of grams of the two samples separately. The oxygen-exposed element will decrease by 0.4g per second for 60 seconds: $48 - 0.6(60) = 12$. Since there are five 12-second intervals in one minute, the hydrogen-exposed element will decrease by half five times: $48(0.5)^5 = 1.5$. The difference between the two is $12 - 1.5 = 10.5$.

7 3,795. We must find how many stars each astrophysicist found on the 10th day. Since the first astrophysicist doubles each day, we can use $10(2)^{10-1}$, which gives us 5,120. For the second astrophysicist, we can use $200 + 125(10 - 1)$, which gives us 1,325. The difference is 3,795.

Guided Practice – Interpreting Graphs & Tables

Multiple Choice – Calculator

1 C. On the graph, a line segment with a positive slope represents an interval over which the target heart rate is strictly increasing as time passes. A line segment with a negative slope represents an interval over which the target heart rate is strictly decreasing. The only interval in which the slope strictly increases and then strictly decreases is between 30 and 50 minutes.

2 B. A graph with a strong negative association between x and y would have the points on the graph closely aligned with a line that has a negative slope. Graph B most closely shows such a line.

3 B. If Jennifer writes 70 words per minute and the assignment is 7,500 words long, it will take her approximately 107.14 minutes to type the information, or 1.79 hours. She also needs to complete 3 hours of research. Therefore, she needs about 5 hours to complete the assignment. If she works 2 hours a day, it will take her 3 days to complete the assignment.

4 C. The question is simply asking us to find the point on the given line that aligns with 6,500 on the y-axis, and find what number on the x-axis it matches with. If a horizontal line is traced from 6,500 on the y-axis to the line of best fit, and then drop a line straight down from the point where they intersect, it will go to 2010.

5 B. There is a 30-minute period in which Joe did not move further from home, which shows the time when he stopped to take photos. The graph begins increasing again at 2:00 to show he is traveling further from home, so that is when his break ended. The rest of the choices indicate times when Joe resumed biking, but after a break other than 30 minutes in length.

6 A. There were 5 adult dramas put on at the theater, out of a total of 35 productions, so the fraction of adult drama productions is $\frac{5}{35} = \frac{1}{7}$.

7 A. To find the average decrease, first take note of the unit of measure in each axis. Each gridline represents 5 degrees on the vertical axis, and 4 days on the horizontal axis. The line does not pass through intersecting gridline points, and is in fact less steep than the 5/4 slope that a line passing through each intersecting gridline point would have. A slope of 3, 5, and 20 would be very steep on this graph.

8 B. The line of best fit shows a decrease in temperature.

9 C. If a line of best fit were drawn, one could tell that the most likely length of an eruption after an 80-minute interval would be 4 minutes.

10 A. About 60% of people in this age group voted for Candidate B, compared with 51% for the 30–44 age group, 40% for the 45–64 age group, and 55% for the 65 or over age group.

11 A. For a data point to be above the line $y = x$, the value of y must be greater than the value of x. Here, that means years during which homes built after 2000 have a higher value than homes built before 2000. That occurs 1 time in this bar graph.

12 C. The amount of energy consumed by most appliances has gone down since 2000, but the number of appliances per household has gone up. Therefore, the decrease in energy consumption leads to an increase in appliances.

13 A. Since the number of bacteria in the dish doubles each hour, it increases exponentially over time. Only graph A shows an exponential increase.

14 C. The line of best fit shows, from a data set, what expected values are. In this case, these values are the number of books sold at different price points. Thus, the line of best fit shows the expected number of books sold at any given price point. There are five data points that lie above the line of best fit, which mean that at these points (for a given price), more books were sold than expected.

15 D. The x-intercept represents a situation where the number of books sold is almost zero. The actual data shows that there were about 10 books sold at \$10. However, the question does not ask how many books were sold at a price point of \$10, but what the x-intercept of the line of best fit means. Since, where the line of best fit crosses the x-axis, the number of books sold is 0, and since the crossing happens around \$10, we can expect that there would be no books sold at that price point.

16 D. The x-axis represents the number of bathrooms, so if we follow the midpoint of 3 and 4 up to the line of best fit, we can trace it over to \$60. However, \$60 is the price per night per bathroom, so the approximate cost of the entire vacation home would be $60 \times 3.5 = \$210$. A better way to solve this question would be to plug 3.5 into the equation for the line of best fit: $y = 90 - 9(3.5) = 58.5$. If we multiply 58.5×3.5, we get 204.75, which rounds to 205.

17 B. The slope of the line is –9, so for every integer increase in bathroom, the average price per night per bathroom drops \$9.

18 C. Interest compounded annually will lead to exponential growth, so the amount in Tanisha's account after t years is equal to the initial deposit multiplied by the interest rate expressed as a decimal plus 1, to the power of the number of years.

19 B. Grace's top speed is 5 miles per hour, which occurs from the 25-minute mark to the 30-minute mark. We need only properly interpret the graph to know that we are looking for coordinates where the y value is highest.

20 D. The graph tells us commute time, but not distance. It is possible that the person who takes the longest to get to work lives very close to work, but takes long because he walks slowly while everyone else drives. There is no definite correlation between commute time and distance from work.

21 A. The ratios of each point are as follows: A: $\frac{30}{4}$, B: $\frac{15}{7}$, C: $\frac{45}{8}$, and D: $\frac{50}{10}$. Of these, A is the greatest.

Student Produced Response – Calculator

1 1/3 or 0.33. According to the graph, there were 400 phones sold in 2012 and 1,200 sold in 2015. The fraction of phones sold in 2012 is equal to $\frac{400}{1{,}200} = \frac{1}{3}$.

2 5/11 or 0.45. There were 500 shirts sold in January and 1,100 shirts sold in April, so the fraction of shirts sold in January to shirts sold in April is $\frac{5}{11}$.

3 1200. There were 1,600 shirts sold in May, the month with the highest number of shirts sold. The month with the fewest number of shirts sold was March, with 400 shirts having been sold. The difference is 1,200. Note the graph indicates that shirts sold is represented in hundreds.

Guided Practice – Additional Data Analysis & Statistics

Multiple Choice – Calculator

1 C. The graph shows the number of households in intervals of 1,000. In other words, if x is equal to the number of households that each unit on the y-axis represents, then $8x+6x+9x+7x+4x=34{,}000$, and $x = 1{,}000$.

2 A. The probability of the winner being over 18 is equal to the total number of men and women out of 100 that are over 18, so: $\frac{75}{100}$.

3 C. The world population does not increase by a fixed amount each year, as it would under a linear growth model. Instead, the increases between periods are greater and greater, indicating exponential growth.

4 A. The lowest price per egg is in the carton of 18. 5 cartons of 18 eggs hold a total of 90 eggs and cost $29.45. An additional carton of 6 costs $3.50, and then 4 additional eggs cost a dollar each. So the lowest possible cost for exactly 100 eggs is $36.95.

5 B. The sum of the two categories is approximately 6 + 22 = 28, which represents 28% of 100 commuters.

6 A. The confidence interval only applies to the average life of batteries manufactured that day, not to individual batteries or to all batteries the factory produces.

7 C. Illegal tickets will be detected for 50 customers, 12 of which were not illegal. So the probability that the customer did not use an illegal ticket is $\frac{12}{50} = 0.24$ or 24%.

8 D. The better results of these students may have been a result of being more motivated, as shown in their willingness to do extra work, and not in their attendance at the study session necessarily, so no conclusion can be drawn regarding a cause and effect relationship between the two events.

9 C. The total number of students surveyed is 260. 15% of 260 is 39, which is the number of male students taking chemistry.

10 C. The range changes from 78 – 59 = 19 to 70 – 59 = 11 . This is a greater change than will occur for either the mean or the median. The median will not change at all. The mean will change from 65.2 to 64.5, which is a change of only 0.7.

11 B. The budget for public protection increased by $0.31 million, from $7.32 million to $7.63 million, over the course of two years, for an average rate of change of $0.155 million, which is equal to $155,000.

12 A. The recreation & culture department budget can be represented as $\frac{0.65}{0.85} \approx 0.76$. The closest is the budget for health: $\frac{6.48}{8.56} \approx 0.76$.

13 D. Each point in the scatterplot represents a house in the town and tells how many square feet it is and what its price is. The line of best fit represents expected prices for houses of different sizes, but doesn't necessarily represent any particular house. The line intersects the y-axis at $71,000 when $x = 0$, meaning a house that is 0 square feet in size would still cost $71,000. However, this is an expected cost, as there are no data points there to represent an actual house. Choice A is incorrect because the 71 represents the total price of the house in thousands, not the price per square foot. Choice B is incorrect because we don't know how much the cheapest house in town costs. Choice C is incorrect because, even if there was a house that was 0 square feet, again, 71 does not represent price per square foot. Only Choice D makes sense.

14 A. The sample size is so small, it is difficult for the researcher to draw a conclusion that represents the entire population of the city.

15 D. Calculate the mean by adding the values together and dividing by the number of values. 32 + 27 + 54 + 43 + 35 + 48 + 32 + 29 = 300, which divided by 8 results in 37.5.

16 C. To solve, set up a proportion: $\frac{150}{159.6} = \frac{175}{x}$. Cross multiply to get $150x = 27{,}930$, so $x = 186.2$.

17 B. We can solve using a proportion: $\frac{150}{354.6} = \frac{100}{x}$, so $150x = 35{,}460$ and $x = 236.4$. We can also logically eliminate all the incorrect choices because we are looking for a planet where objects weigh more than twice what they do on Earth. Looking at the chart, the only planet that even comes close is Jupiter.

18 D. The sample for this study is well defined, so it is reasonable to conclude that, if conditions are the same – that is, if this model of car is given the new engines – that gas mileage would improve for this model of car produced by this factory. Choices B and C are not supported by the given data because other conditions were not tested. Choice A is too strong a conclusion and not supported by the given data.

19 C. Let x be the number of female students who do not play sports and y be the number of male students who do not play sports. Given the data in the problem, we know that $x + y$ must equal 45. We also know that $2x$ represents the number of female students who play sports and $3y$ represents the number of male students who play sports, giving us the system of equations:

$x + y = 45$

$2x + 3y = 110$

Solving this system yields $x = 25$ and $y = 20$, in which case 50 of the students who play sports are female. Therefore, the probability that a randomly selected student who plays sports will be female is $\frac{50}{110}$, or about 0.45.

20 A. The standard deviation is a measure of how far the data set values are from the mean. In the data set for City B, the vast majority of the data are in the middle three of the five possible values, which are the three values closest to the mean. In the data set for City A, the data are more spread out, with many values at the minimum and maximum values. Therefore, by observation, the data for City A have a larger standard deviation.

21 B. If the team scores an average of 11 points per game, their total for 4 games is 44 points. $44 - (7 + 18 + 6) = 13$, so the team must have scored 13 points at the 4th game.

22 A. In order to answer a probability question, we need to know the total. Here, the total of all students is 8,221 + 5,243 + 4,723 + 4,916 = 23,103. The probability of a student chosen at random from the entire population being a junior from the Midwest is equal to $\frac{1{,}212}{23{,}103} \approx 0.0525$.

23 C. The average of Wendy's times is 2.5 minutes. However, that is the average time it took her to run one lap at the track, which is 0.25 mile. Since there are 24 blocks of 2.5 minutes in every hour (60 minutes ÷ 2.5 minutes = 24), Wendy could on average run a total of 6 miles in that hour (24 × 0.25 miles). Thus, Wendy's average speed is 6 miles per hour.

24 C. If Amar's average is 134 points, then the sum of all 3 rounds was 134 × = 402. Subtract the known scores to find the missing one: 402 – (116 + 148) = 138.

25 B. The range will remain the same (a change of 0) and the mean only changes slightly. Putting the numbers in order – 5, 8, 13, 36, 43 – we see that the median of the original data set is 13, but with the addition of a chapter with 25 recipes, the new list becomes 5, 8, 13, 25, 36, 43, in which case the median number of recipes per category becomes 19.

26 C. Statement I is true because adding 5 to every score will increase the mean by 5. Statement II is also true, because increasing every score by 5 will also increase the median by 5. But statement III is false, because increasing every score by the same amount will result in the same spread of scores as the original, so the standard deviation will remain the same.

27 C. The vertical axis of the chart shows us the number of values in the data set (for example, there are 5 people who work between 10-20 hours). The best way to question of median is to approximate the vertical values: 5, 12, 23, 8, and 2 would be reasonable. There are, then, approximately 50 values in the data set. The median will occur in the middle of that data set, so approximately 25 values in from either the lowest

or highest ends of the data set. Starting from either side, we can see that the median must occur in the bucket of 23 employees working from 30-40 hours. This means either choices B or C are correct. The mean, in this case, can be visually determined to be in the same bucket, since there are so many values in the data set corresponding to 30-40, which happens to be in the middle of the data set.

28 C. If 12 people are taking both classes, then there are 42 people taking only martial arts, 11 people taking only yoga, and 12 people taking both, or a total of 65 people enrolled in these classes. $121 - 65 = 56$.

Student Produced Response – Calculator

1 64. The sum of the temperatures for the first days is 396. Adding 52 yields a sum of 448. Divided by 7, the new average is 64°F.

2 7.5. To find the range of a data set, subtract the greatest value from the lowest value. In this case, 9.5 is the greatest and 8.3 is the lowest. To find the least possible value, subtract the range of 2 from the highest number, meaning x is equal to 7.5.

3 633. Find the mean by adding all of the home runs together: 7,059. Then divide by the number of players (11): 641.72. The average age, rounded to the nearest whole number, is 642.

4 5. If the mean is 7, the sum of the first 5 ratings is 35. In order for the mean to be 8 for the first 10 ratings, the sum of the first 10 ratings should be $8 \times 10 = 80$. So, the sum of the second ten ratings is equal to $80 - 35 = 45$. The maximum rating is 10, so the maximum possible value of the sum for the 7th through the 10th rating is $4 \times 10 = 40$. Therefore, for the product to be able to have an average of at least 8 for the first 10 ratings, the least possible value for the 6th rating is $45 - 40 = 5$.

5 81.4. If the average for 18 students is 82, then the sum of their scores is $18 \times 82 = 1{,}476$. The sum of the scores of the additional students is $2 \times 76 = 152$. Therefore, the sum of the scores of all 20 students is equal to 1,628. Dividing by 20 yields an average score of 81.4.

6 0.13. If the winner is chosen from the entire pool of customers, then the odds of the winner being someone who purchased a CD after the show would be $\frac{44}{342} \approx 0.1286$, so the probability rounded to the nearest hundredth is 0.13.

7 99. If 20 students scored an average of 80, then the sum of the scores is 1,600. If 19 students averaged 79 points, then the sum of their scores was 1,501. The 20th score is the difference between these two sums: 99 points.

Mixed Practice – Data Analysis

Multiple Choice –Calculator

1 Additional Data Analysis & Statistics: A. One way to solve this is to try each answer choice. Looking at choice A, if there were 2 athletes who each scored 40 points, then there were 15 athletes in all, making the eighth number from the left (i.e. from lowest to highest) the median. The eighth number is 20, so that works. Looking at choice B, if 3 athletes each scored 40 points, then there were 16 athletes, making the average of the 8th and 9th numbers the median. The 8th and 9th numbers are 20 and 30, making the median 25, which does not work. That means that C and D won't work either.

2 Linear & Exponential Growth: B. The acceleration of gravity is given at 9.8m/s^2, which acts on an object continuously as it falls. This means that the speed of the object (for example, in meters per second) that falls does not stay constant. Instead, the object's speed is constantly increasing, or accelerating.

3 Ratios & Proportions: C. This problem asks about percent change, which is found by dividing the difference of the two values by the original value. Tom's percent change is $\frac{|8-9|}{9}$, which equals a 11.1% change. Everett's percent change is $\frac{|10-12|}{12}$, which equals a 16.7% change.

4 Ratios & Proportions: C. An efficient way to solve this problem is just to find the price per square foot of each house, which we can find simply by dividing the price of each house by its square footage. The Howell Boulevard house results in the lowest cost per square foot, of approximately $321 per square foot.

5 Interpreting Graphs & Tables: D. Since age is on the vertical axis, the oldest sprinter is represented by the data point that is highest in the graph. Here, if we follow that point down to the time axis, it lands just past 9.95. Choice D is the only choice that satisfies that value, as all others are below 9.95.

6 Additional Data Analysis & Statistics: C. The histogram shows that there are 23 parks in total, meaning the median is the 12th park from the left. That places the 12th park in the 600-699 bar, and only 602 falls within that range.

Student Produced Response – Calculator

1 Linear & Exponential Growth: 2,500. This problem may look confusing, but it is nothing more than simple plug-in algebra. You are given a formula. You are given the values of every variable except one. Plug everything in and then solve for the last remaining variable. Here, you will plug in 580 for "$N_{\text{next month}}$" and 500 for "$N_{\text{this month}}$", then solve for the variable K: $580 = 500 + (0.2)(500)\left(1 - \frac{1}{K} \cdot 500\right) = 500 + (100)\left(1 - \frac{500}{K}\right)$. Then subtract 500 from both sides and divide by 100, giving you $0.8 = 1 - \frac{500}{K}$. Solving for K results in $K = 2{,}500$.

2 Interpreting Graphs & Tables: 3. If Jason can get 36 miles per gallon and his gas tank can hold 18 gallons of gas, he can travel 648 miles on each tank of gas. The total mileage (2,522 miles) divided by 648 means he will need approximately 3.89 tanks of gas to make the trip. Since he starts with a full tank, he will need to refill 3 times.

3 Ratios & Proportions: 450. If 60% of the penguins are female, then 40% are male. The 300 male penguins therefore represent 40% of the total. We can solve most percent problems using a proportion: $\frac{40}{100} = \frac{300}{x}$, where x is the total number of penguins. Cross multiplying results in $40x = 30{,}000$, in which case $x = 750$. That is the total number of penguins, so there must be 450 female penguins.

Advanced Math

Guided Practice – Working with Polynomials

Multiple Choice – No Calculator

1 B. Add together like terms. $7x^2 + 3x^2 = 10x^2$ and $(-4x) + (-x) = -5x$ and $8 + 7 = 15$.

2 C. Multiply by FOIL method. $(4x+3)(5x-10) = 20x^2 - 40x + 15x - 30 = 20x^2 - 25x - 30$.

3 D. Multiply by FOIL method, then distribute the 5. $5(3x+1)(2x+1) = 5(6x^2 + 3x + 2x + 1)$. This simplifies to $30x^2 + 25x + 5$.

4 B. Multiply by FOIL method, then distribute the $\frac{1}{2}$.

$$\frac{1}{2}(2x+4)(x+1) = \frac{1}{2}(2x^2 + 2x + 4x + 4) = \frac{1}{2}(2x^2 + 6x + 4) = x^2 + 3x + 2$$

5 C. Since both $7x$ and 3 are multiplied with $(2x+1)$, the given expression is equivalent to $(7x+3)(2x+1)$. By expanding this expression out, you get $14x^2 + 13x + 3$. Therefore, both I and II are equivalent to the given expression.

6 C. Since $\frac{20}{ab} = 10$, it follows that $ab = 2$, so $a^2b^2 = 2^2 = 4$. Plugging this into the second expression, we get $\frac{32}{a^2b^2} = \frac{32}{4} = 8$.

7 B. Since $\frac{x}{y} = 3$, it follows that $\frac{y}{x} = \frac{1}{3}$, so $\frac{6y}{x} = 6 \cdot \frac{1}{3} = 2$.

8 D. $\dfrac{\dfrac{1}{x+4}+1}{\dfrac{2}{x-2}} = \dfrac{\dfrac{1}{x+4}+\dfrac{x+4}{x+4}}{\dfrac{2}{x-2}} = \dfrac{\dfrac{x+5}{x+4}}{\dfrac{2}{x-2}} = \dfrac{x+5}{x+4}\cdot\dfrac{x-2}{2} = \dfrac{x^2+2x-10}{2x+8}$

9 A. $\dfrac{\dfrac{x+5}{x+4}}{\dfrac{x-7}{2x+8}} = \dfrac{\dfrac{x+5}{x+4}}{\dfrac{x-7}{2(x+4)}}$ By multiplying the numerator and denominator by $2(x+4)$, we can make cancellations and get $\dfrac{2(x+5)}{x-7}$ which equals $\dfrac{2x+10}{x-7}$.

10 D. Multiplying with the FOIL method, choice D gives us $(2a^2+5b^2)(2a^2+5b^2) = 4a^4+10a^2b^2+10a^2b^2+25b^4$ which simplifies to $4a^4+20a^2b^2+25b^4$.

11 C. $\dfrac{a+3}{x}+\dfrac{b-5}{2x} = \dfrac{2(a+3)}{2x}+\dfrac{b-5}{2x} = \dfrac{2a+6+b-5}{2x} = \dfrac{2a+b+1}{2x}$

12 C. $(3x-6)^3 = (3x-6)^2(3x-6)$ and becomes $(9x^2-36x+36)(3x-6)$. This equals $27x^3-54x^2-108x^2+216x+108x-216$, which simplifies to $27x^3-162x^2+324x-216$.

13 D. The expression in the question will have different values depending on the value of x. Therefore, choice A and B must be incorrect because they are constant. Simplifying choice D into a single fraction, we get $6-\dfrac{25}{x+4} = \dfrac{6(x+4)}{x+4}-\dfrac{25}{x+4} = \dfrac{6x+24-25}{x+4} = \dfrac{6x-1}{x+4}$, so D is the correct answer.

14 A. $\dfrac{3}{x+2}-\dfrac{2}{x^2+4x+4} = \dfrac{3}{x+2}-\dfrac{2}{(x+2)^2} = \dfrac{3(x+2)}{(x+2)^2}-\dfrac{2}{(x+2)^2} = \dfrac{3x+6-2}{(x+2)^2} = \dfrac{3x+4}{x^2+4x+4}$

15 B. All the answer choices start with $\dfrac{y}{x}=$, which means that we have to solve for $\dfrac{y}{x}$. Breaking the fraction on the left side of the equation into two fractions, we get $\dfrac{y}{x}+\dfrac{1}{x} = \dfrac{6}{7}$. Subtracting $\dfrac{1}{x}$ to both sides, we get $\dfrac{y}{x} = \dfrac{6}{7}-\dfrac{1}{x}$. Combining the fractions on the right, we get $\dfrac{y}{x} = \dfrac{6x}{7x}-\dfrac{7}{7x} = \dfrac{6x-7}{7x}$.

16 D. Expanding out the original function and converting it to standard form, we get $f(x) = x^2-10x+24$. However, this does not give us the roots, so we much convert it again into factored form, which is $f(x) = (x-4)(x-6)$. This indicate that both roots are positive (4 and 6). Negating each factor, we get the equivalent function $f(x) = (4-x)(6-x)$ which has constants 4 and 6.

Student Produced Response – No Calculator

1 32. We get this by manipulating $4x(2x+3)+5(4x+1) = 8x^2+12x+20x+5$, which simplifies to $8x^2+32x+5$, so $b = 32$.

2 3. The left side of the equation is a square, so we get $(x^2-9)^2 = 0$. Factoring this further, we get $[(x+3)(x-3)]^2 = 0$, so $x = \pm 3$. However, the question asks for a positive answer, so it is 3.

3 7. Factoring the equation by grouping, we get $x^2(x-7)+3(x-7) = 0$ and then $(x^2+3)(x-7) = 0$, so the only real solution to the equation is $x = 7$.

4 1. Dividing both sides of the equation by 6 we get $x^4(x^4-2) = -1$. Expanding out the left side we get $x^8-2x^4 = -1$ or equivalently $x^8-2x^4+1 = 0$ which is a square polynomial, $(x^4-1) = 0$. This has real solutions $x = \pm 1$, and the problem specifies to take the positive answer.

Multiple Choice – Calculator

1 D. The given expression is a difference of squares. It can be rewritten as $(8w^3)^2 - (v^3)^2$. Using the difference of squares formula, we get choice D.

2 B. Expanding the function using FOIL, we get $f(x) = x^2 - 4x - 21$. The minimum will occur at the vertex when $x = -\frac{b}{2a} = -\frac{-4}{2(1)} = 2$. By completing the square, we can rewrite the function as $f(x) = (x-2)^2 - 25$ which shows that when $x = 2$, the value of the function is –25. B is the correct choice because it includes the minimum value (–25) and it is equivalent to the original function.

3 C. $f(x)$ is an upturned parabola with vertex at the origin, so its minimum value is zero. $g(x)$ has its minimum at its vertex, where $x = -\frac{b}{2a} = -\frac{-6}{2(1)} = 3$. Plugging 3 in for x, we get $g(3) = 3^2 - 6(3) + 19 = 10$ so the minimum value of $g(x)$ is 10. Thus, the difference in the minimum values is $10 - 0 = 10$.

Student Produced Response – Calculator

1 17. Distribute the –5 to get $7x^2 + 2x - 4 - 5x^2 + 15x - 10$. Then combine like terms to get $2x^2 + 17x - 14$. The value of b is 17.

2 25. The first part of the expression factors as $3x(y-5)$. Since the first part of the expression is divisible by $y - 5$, the second part of the expression, $y^2 - c$, must also be divisible by $y - 5$ in order for the entire expression to be divisible by $y - 5$. The expression $y^2 - c$ resembles a difference of squares. Using the factor $y - 5$, we get $(y-5)(y+5) = y^2 - 25$. Therefore, $c = 25$. Another way to solve this problem is to plug in 5 for y, and see what value of c will make the expression equal to 0.

3 6. Apply the distributive property to get $x^3 = 48x - 2x^2$, which can equivalently be written as $x^3 + 2x^2 - 48x = 0$. Factoring this, we get $x(x + 8)(x - 6) = 0$ so the solutions are $x = -8, 0, 6$. The question is asking for the greatest of the solutions, which is 6.

Guided Practice – Working with Polynomial Factors in Expressions & Equations

Multiple Choice – No Calculator

1 D. The value of t cannot be 3 because that would make the fraction undefined. Multiply both sides of the equation by $t - 3$ and we get $t + 3 = 6t - 18$. Subtract t from both sides and add 18 to both sides. It becomes $21 = 5t$, so $t = \frac{21}{5}$.

2 B. The value of the function is 0 when $x = 1$. Therefore, 1 is a root of the polynomial, so $x - 1$ must be a factor of the polynomial.

3 D. The value of the function is 0 when $x - 3$. Therefore, –3 is a root of the polynomial, so $x - (-3)$ or, equivalently, $x + 3$ must be a factor of the polynomial. $x - 1$ is also a factor, though it is not an answer choice.

4 A. The value of the function is 0 when $x = 6$. Therefore, 6 is a root of the polynomial, so $x - 6$ must be a factor of the polynomial.

5 B. If we were to use either long division or synthetic division, we would find that the quotient is $4x + 2$ with a remainder of 11. However, the SAT will NOT require students to perform long division of polynomials. Instead, this question tests students on their knowledge of the Polynomial Remainder Theorem, which states that any polynomial $p(x)$ is equal to the product of its quotient times its divisor $(x - a)$, plus its remainder. Here, that means $4x^2 - 2x + 9 = (4x + 2)(x - 1) + 11$. The trick to remember is that when you plug a (from the divisor) into $p(x)$, the resulting value is the remainder. Here, we see that plugging 1 into the polynomial results in 11.

6 D. The Polynomial Remainder Theorem (see question #5 above) states that when any polynomial $p(x)$ is divided by $(x - a)$, the remainder is equal to $p(a)$. Here, the value of a is –3. Plugging that into the original polynomial, we get $9(-3)2 + 5(-3) - 2$, which equals 64.

7 C. Remembering the Polynomial Remainder Theorem, we know that for any divisor $(x - a)$, when you plug a into the polynomial dividend, the result will be the remainder. Here, we are told that when a is –3 (meaning the divisor is $x + 3$) the remainder is 0, and when a is 2 (meaning the divisor is $x - 2$) the remainder is 4.

8 B. Cross-multiply the original equation. It becomes $2(x + 24) = 10x$, which multiplies out to $2x + 48 = 10x$. Subtract $2x$ from both sides to get $48 = 8x$. Then divide both sides by 8 to get $6 = x$. The question asks for the value of $\frac{x}{2}$ which is $\frac{6}{2}$ and simplifies to 3.

9 D. Cross-multiply the original equation. It becomes $4x = 1(x + 6)$ or simply $4x = x + 6$. Subtract x from both sides to get $3x = 6$. Then divide both sides by 3 to get $x = 2$. The question asks for the value of $\frac{1}{x+6}$, which is $\frac{1}{2+6}$ and simplifies to $\frac{1}{8}$.

Student Produced Response – No Calculator

1 28. Working backwards from the given information, we can see that the result, multiplied by $4x - 6$, will give us the original polynomial. As an equation, this becomes $8x^2 - 20x + 40 = (4x - 6)\left(2x - 2 + \frac{R}{4x-6}\right)$. The right side can equivalently be written as $(4x - 6)(2x - 2) + R$, which expands to $8x^2 - 20x + 12 + R$. So our new equation becomes $8x^2 - 20x + 40 = 8x^2 - 20x + 12 + R$. Cancelling terms and solving for R gives us 28.

2 6. We can clear the denominators in this equation by multiplying through by $(t + 2)(t + 10)(4)$. The new equation becomes $1(4)(t + 10) = -2(4)(t + 2) + 1(t + 2)(t + 10)$. This becomes $4t + 40 = (-8 - 16) + (t^2 + 12t + 20)$. If we collect all of these terms on the right side of the equation, we get $0 = t^2 - 36$. This factors as $(t + 6)(t - 6)$, so $t = \pm 6$. Because the problem states that t must be positive, the answer is 6.

Multiple Choice – Calculator

1 B. Multiply both sides of the equation by $t - 5$ and we get $t - 3 = 7t - 35$. Subtract t from both sides and add 35 to both sides. It becomes $32 = 6t$, so $t = \frac{32}{6}$. This simplifies to $\frac{16}{3}$.

2 C. Cross-multiply and we get $(3t)(10) = (t + 6)(1)$, which becomes $30t = t + 6$. Subtract t from both sides. It becomes $29t = 6$, so $t = \frac{6}{29}$.

3 A. The expression can be factored as $(x - 3)(x - 4)$, which means that $(x - 3)$ is a factor of $x^2 - 7x + 12$. When we divided by a factor, the remainder will be zero.

4 D. Remembering the Polynomial Remainder Theorem, we know that for any divisor $(x - a)$, when you plug a into the polynomial dividend, the result will be the remainder. Here, since we are plugging in 4, the divisor must be $(x - 4)$ and the resulting value of –1 is the remainder. This matches choice D.

5 C. Since $y \neq 0$ we can cancel y from the numerator and denominator of the right side. Next, cross-multiply the equation. It becomes $5(x + 2) = 2(x + 11)$, which multiplies out to $5x + 10 = 2x + 22$. Subtract $2x$ and 10 from both sides to get $3x = 12$. Then divide both sides by 3 to get $x = 4$. The question asks for the value of $x + 11$ which is $4 + 11$and simplifies to 15.

6 A. To eliminate the denominator, multiply both sides by $ax - 6$. The equation becomes $30x^2 - 26x - 19 = (6x + 2)(ax - 6) - 7$. The coefficient of x^2 on the left side is 30 and on the right side it will be $6a$. Therefore, $30 = 6a$, so the value of a is 5.

7 B. To eliminate the denominators, multiply through by $2x + b$. The equation becomes $8x^2 - 22x - 9 = (4x + 3)(2x + b) + 12$. The constant term on the left side of the equation is –9 and on the

right side it will be $3b + 12$. Therefore, $-9 = 3b + 12$. Subtract 12 from both sides and then divide by 3. We get -7 as the solution for b.

Student Produced Response – Calculator

1 4. We can clear denominators in this equation by multiplying through by $(6)(t-1)(t+2)$. The new equation becomes $2(6)(t+2)+1(t-1)(t+2)=5(6)(t-1)$. This simplifies to $(12t+24)+(t^2+t-2)=30t-30$. If we collect all of these terms on the left side of the equation, we get $t^2-17t+52=0$. This factors as $(t-13)(t-4)=0$ so the solutions are 13 and 4. Since the problem states that $t<12$, the answer is 4.

2 112. Working backwards from the given information, we can see that the result, multiplied by $7x+4$, will give us the original polynomial. As an equation this becomes $7x^2-38x+88=(7x+4)\left(x-6\frac{R}{7x+4}\right)$. The right side can equivalently be written as $(7x+4)(x-6)+R$, which expands to $7x^2-38x-24+R$. So our new equation becomes $7x^2-38x+88=7x^2-38x-24+R$. Cancelling terms and solving for R gives us 112.

3 74. Multiplying both sides by $5x + 8$, we get $10x^2-34x-6=(5x+8)(2x-10)+R$. The right side can equivalently be written as $10x^2-34x-80+R$. So our new equation becomes $10x^2-34x-6=10x^2-34x-80+R$. Cancelling terms and solving for R gives us 74.

Guided Practice – Quadratic Functions & Equations

Multiple Choice – No Calculator

1 B. All of the answer choices have the factors $(x-4)$ and $(x+5)$, except for choice B which has the factors $(x+4)$ and $(x-5)$. This means that the solutions to equation B are -4 and 5, whereas all the other equations have the solutions 4 and -5.

2 A. This equation factors as $(x+7)(x-5)=0$ so x is -7 or 5.

3 D. This equation factors as $(x+1)(2x+1)=0$ so x is -1 or $-\frac{1}{2}$.

4 B. The quadratic formula says $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. Plugging in $a = 1$, $b = 5$, $c = 2$, we arrive at $x=\frac{-5\pm\sqrt{25-8}}{2}$ which simplifies to answer choice B.

5 B. Before using the quadratic formula, we must determine the equation in standard form by adding $4x$ to both sides. This gives $x^2+4x+1=0$. Using the quadratic formula, we arrive at $x=\frac{-4\pm\sqrt{16-4}}{2}$, which simplifies to answer choice B.

6 B. The equation factors as $(x-4)(x-4)=0$. Since both factors are the same, the equation has only one real solution, $x = 4$.

7 D. Before using the quadratic formula, we must represent the equation in standard form by using the distributive property on the left side and then subtracting 1 from both sides. This gives $3x^2+6x-1=0$. Using the quadratic formula, we arrive at $x=\frac{-6\pm\sqrt{36+12}}{6}$ which simplifies to answer choice D.

8 D. From the original equation, we see that $ab=6$. Since $a+b=5$, the values of a and b must be 2 and 3, but we don't know which is which. Trying both possibilities, we arrive at $(2x+4)(3x+5)=6x^2+22x+20$ or $(3x+4)(2x+5)=6x^2+23x+20$, so c is either 22 or 23.

9 C. From the last term in the equation, we see that $ab = -7$, so the values of a and b are either –1 and 7, or –7 and 1. Trying both possibilities, we arrive at $(x-1)(x+7) = x^2+6x-7$ or $(x+1)(x-7) = x^2-6x-7$, so c is either 6 or –6.

10 C. The value of ab will be whatever we arrive at for the constant term on the right side of the equation when we put it in standard form. $(x+3)^2+11 = (x^2+6x+9)+11 = x^2+6x+20$. Therefore, the value of ab is 20.

11 D. To determine the value of a, plug in 0 for $f(x)$. $0 = x^2-10x+25 = (x-5)(x-5)$, so $x = 5$ when $f(x) = 0$. This tells us that the function passes through the point (5, 0), so the value of a is 5. To determine the value of b, plug in 0 for x. You arrive at $f(x) = 0^2-10\cdot 0+25 = 25$, so $f(x) = 25$ when $x = 0$. This means that the function passes through the point (0, 25), so the value of b is 25. Therefore, $a + b = 5 + 25 = 30$.

12 A. This question is asking when the function has a value of 0, so you have to solve the equation $25x^2-9 = 0$. This factors as $(5x+3)(5x-3) = 0$, so the solutions are $\pm\frac{3}{5}$ which yields $k = \frac{3}{5}$.

13 D. Notice that the right side of the equation has no x term. This means that the factor $(ax+b)$ must be $(3x-2)$ or some multiple thereof, because that is the only way to determine the x term to cancel out when we multiply $(ax+b)$ with $(3x+2)$. With any multiple of $(3x-2)$, the two constants a and b must have different signs (one positive, one negative). Using basic rules of positive and negative integers, we know their product ab must be negative. The example $ax + b = 3x - 2$ shows that choices A and C are false. The example $ax + b = -3x + 2$ shows that choice B is false.

14 B. To find the intersection points of two functions, you must find when they are equal. Setting these two functions equal to each other, we arrive at $-x^2+8x-7 = x^2-8x+17$. Bringing all terms to the same side of the equation, we arrive at $2x - 16x + 24 = 0$. This factors as $2(x - 6)(x - 2) = 0$ so the solutions are $x = 6$ and $x = 2$. These numbers give us the x-coordinates of the two intersection points. By plugging either of these values into either of the two functions, you arrive at 5 as the y-coordinate of both intersection points, so $k = 5$.

15 C. To find the intersection points of two functions, you must find when they are equal. Setting these two functions equal to each other, we arrive at $16 - x^2 = x^2$. Bringing the x^2 terms to the same side of the equation, we arrive at $2x^2 = 16$ and dividing both sides by 2 we arrive at $x^2 = 8$, so $x = \pm\sqrt{8} = \pm 2\sqrt{2}$. These are the x-coordinates of the two intersection points, so $a = 2\sqrt{2}$.

16 A. Notice that $g(x)$ will be undefined when its denominator is equal to zero. However, the equation $x^2-4x+8 = 0$ has no real solutions, because when you perform the quadratic formula, you get a negative number under the square root: $\sqrt{b^2-4ac} = \sqrt{(-4)^2-4\cdot 1\cdot 8} = \sqrt{-12}$. This means there are no values of x for which the function is undefined.

17 C. *Volume* = *Depth* × *Width* × *Length* = 60. If x represents the width, then the length is $(2x + 4)$. This gives us $3x(2x + 4) = 60$. Using the distributive property, we arrive at $6x^2 + 12x = 60$. Then, subtracting 60 from both sides and dividing both sides by 6, we arrive at $x^2 + 2x - 10 = 0$.

Student Produced Response – No Calculator

1 15. First, put the equation in standard form by expanding the left side to get $x^2 + 8x + 16 = 1$ and then subtracting 1 from both sides to arrive at $x^2 + 8x + 15 = 0$. This factors as $(x+3)(x+5) = 0$ so the solutions are –3 and –5, which have a product of 15. Another way to figure this out is by using the following rule: For any equation $ax^2 + bx + c = 0$, the product of the solutions is $\frac{c}{a}$. In this case, $\frac{c}{a} = \frac{15}{1} = 15$.

2 7. Notice that $f(x)$ will be undefined when its denominator is equal to zero. The expression $x^2 - 5x - 14$ factors as $(x - 7)(x + 2)$ and this is equal to zero when x is 7 or –2. Since the problem only asks for positive values, the answer is 7.

Multiple Choice – Calculator

1 C. You can figure out how many solutions a quadratic will have by calculating the number under the square root in the quadratic formula. This number, $b^2 - 4ac$, is called the determinant. If the determinant is positive, the equation will have two real solutions. If the determinant is 0, there will be one solution, and if the determinant is negative, there will be no real solution. The determinants of equations I, II, and III are 61, 0, and –16 respectively, so equation III is the only one with no real solutions. (Since this is a calculator problem, you could also graph the three equations and see which one does not touch the x-axis).

2 A. The equation factors as $3(r + 3)(r + 4) = 0$, so the possible values of r are –3 and –4, which have a sum of –7. Another way to figure this out is by using the following rule: For any equation $ax^2 + bx + c = 0$, the sum of the solutions is $-\frac{b}{a}$. In this case, $-\frac{b}{a} = \frac{21}{3} = -7$.

3 B. A function is undefined when its denominator is zero. You can calculate the zeros for each denominator, but in this case, the question only asks about $x = 2$. Therefore, you can just plug 2 into the denominator of each function. For the denominator of $f(x)$, you get $(2 - 2)^2 + (2 - 4)^2 = 4$. For the denominator of $g(x)$, you get $2^2 - 6 \times 8 = 0$. For the denominator of $h(x)$, you get $2^2 + 10 \times 2 + 16 = 40$. Since $g(x)$ is the only one with 0 in the denominator, the answer is g only.

4 D. Since the problem gives us the total distance, we can make an equation involving distance. Remember that *distance = rate * time*. For the first part of the hike, the hiker's rate is $\frac{x}{2}$ and his time is $x + 2$, so his distance is $\frac{x}{2}(x + 2)$. For the second part of the hike, his rate is the half of the original rate, which gives us $\frac{1}{2} \cdot \frac{x}{2} = \frac{x}{4}$. His time for the second part is 3, so his distance is $\frac{x}{4} \cdot 3$. The problem tells us that the total distance is 15. By adding up the distances from the two parts of the hike we arrive at $\frac{x}{2}(x + 2) + \frac{x}{4} \cdot 3 = 15$. If we multiply both sides of the equation by 4, we arrive at $2x(x + 2) + 3x = 60$. Rearranging this into standard form gives us choice D.

5 C. Since the problem gives us the final distance for her trip, we can make an equation involving distance. Remember that *distance = rate * time*. For the first part of the trip, the driver's rate is $\frac{2x}{3}$ and her time is $3x$, so her distance is $\frac{2x}{3}(3x)$. For the second part of the drive, her rate is still $\frac{2x}{3}$ and her time is 30 minutes, but we need to convert this to hours, so the time is $\frac{1}{2}$ an hour. Therefore, the distance for the second part of her trip is $\frac{2x}{3} \cdot \frac{1}{2}$. After both parts of the trip, she was 50 miles from home. Since she reversed direction, we must subtract the distances so we arrive at $\frac{2x}{3}(3x) - \frac{2x}{3} \cdot \frac{1}{2} = 50$. If we multiply both sides of the equation by 3, we arrive at $2x(3x) - 2x \cdot \frac{1}{2} = 150$. Rearranging this into standard form gives us choice C.

Student Produced Response – Calculator

1 1/3 or 0.33. This equation factors as $(3x - 1)(x + 5) = 0$, so x is $\frac{1}{3}$ or -5. However, the question asks for a positive solution, so the answer is $\frac{1}{3}$, or its decimal equivalent, 0.33.

2 17/2 or 8.5. First put the equation in standard form: $2x^2 - 17x + 35 = 0$. The equation factors as $(2x - 7)(x - 5) = 0$, so the possible values of x are $\frac{7}{2}$ and 5, which have a sum of $\frac{17}{2}$. Another way to figure this out is by using the following rule: For any equation $ax^2 + bx + c = 0$, the sum of the solutions is $-\frac{b}{a}$. In this case, $-\frac{b}{a} = \frac{-17}{2} = \frac{17}{2}$.

3 6. Notice that $h(x)$ will be undefined when its denominator is equal to zero. First, put the denominator in standard form. $(x-7)^2 + 2(x-7) + 1 = (x^2 - 14x + 49) + (2x - 14) + 1 = x^2 - 12x + 36$. This expression factors as $(x - 6)(x - 6)$, so the denominator is equal to zero when $x = 6$.

Guided Practice – Exponents & Radicals

Multiple Choice – No Calculator

1 D. By definition, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$. It follows, therefore, that $a^{\frac{4}{5}} = \sqrt[5]{a^4}$.

2 B. $4 = 2^2$, so 4^x can be written as 2^{2x}. Since both the numerator and denominator have a common base, $\frac{4^x}{2^y} = 2^{2x-y}$. Since we know that $2x - y = 18$, the value of the original expression is 2^{18}.

3 C. If $\sqrt{3k^2 + 9} - x = 0$ and $x = 6$, then $\sqrt{3k^2 + 9} = 6$. Square both sides to get $3k^2 + 9 = 36$. This yields $k^2 = 9$, or $k = 3$.

4 A. If there are 4 million subscribers at the start of 2014, and that number increases by 3% each year, then after 1 year, the website has 1.03 times more subscribers. It has 1.03 times more subscribers in each subsequent year, so multiplying 4 by 1.03 to the power of the number of years will yield the number of subscribers after t years.

5 A. $\frac{x^a}{x^b} = x$ can be rewritten as $x^{(a-b)} = x^1$. Therefore, $a - b = 1$. We now know that a and b must be consecutive whole numbers, so $b - a$ is equal to -1.

6 B. Since $10^3 = 1{,}000$, if $b = 1$, $a = 3$. None of the other responses are mathematically possible.

7 C. This function shows the initial amount multiplied by the percent of the substance that remains to the power of the time, in days.

8 D. Squaring each side yields $(x-6)^2 = (\sqrt{x+42})^2$ or $x^2 - 12x + 36 = x - 6$. To find the solutions, subtract the right side from the left side of the equation: $x^2 - 13x + 42 = 0$. This quadratic equation can be factored: $(x-7)(x-6)$. So $x = 7$ and $x = 6$.

9 C. Keep in mind that $k^{-1} = \frac{1}{k}$. Simplifying $m^2k^{-1} = 10m$ yields $10k = m$, so $\frac{1}{m} = \frac{1}{10k}$.

Student Produced Response – No Calculator

1 20. If $5x^2 = 20$, $x^2 = 4$. If $4y = 20$, $y = 5$. So, $x^2y = 20$.

2 9. Since the numerator and denominator have a common base, the expression can be rewritten using the law of exponents as $x^{a^2-b^2}$. Since this expression is also equal to x^{36}, $a^2 - b^2 = 36$. This can be factored as $(a + b)(a - b) = 36$. Since it is given that $a + b = 4$, substituting gives $4(a - b) = 36$. Dividing both sides by 4 yields $a - b = 9$.

3 36. If $a = 3\sqrt{3}$, $2a = 6\sqrt{3}$. Therefore, $6\sqrt{3} = \sqrt{3x}$. Squaring each side yields $(6^2)(\sqrt{3})^2 = (\sqrt{3x})^2$, which simplifies to $108 = 3x$, so $x = 36$.

Multiple Choice – Calculator

1 B. When $t = 2$, $P(t) = 4{,}000$. When $t = 4$, $P(t) = 8{,}000$, so the population increased by 4,000 organisms.

2 B. $64 = 2^6$, so $a + b + c = 6$. Since they are different integers, 1, 2, and 3 are the only possible values. $2^1 + 2^2 + 2^3 = 2 + 4 + 8 = 14$.

3 C. Squaring both sides yields $x + 16 = x^2 - 8x + 16$. Subtracting 16 from both sides yields $x = x^2 - 8x$.

4 C. One fast and simple way to solve this is to just try each answer choice by plugging them in for n in the equation. $20{,}000(0.8)^3$ is the only value that gives 10,240. If you are comfortable working with logarithms, you could also solve algebraically with the equation $10{,}240 = 20{,}000(0.8)^n$. Dividing both sides by 20,000 gives you $0.512 = 0.8^n$. You can solve for n with $n = log_{0.8}0.512$, which equals 3.

5 C. Substituting 4 or 5 into the expression do not yield integers, but 20 does: $\sqrt{\frac{5 \times 20}{4}} = \sqrt{\frac{100}{4}} = \sqrt{25} = 5$.

6 B. The perimeter of the triangle is 6 + 10 + 12 = 28, so s is 14. Therefore, the area of the triangle can be represented by the formula as $A = \sqrt{14(14-6)(14-10)(14-12)} = \sqrt{14(8)(4)(2)} = \sqrt{896}$. If you're calculator does not put radicals into simplest form, just try each answer choice. $\sqrt{896} \approx 29.93$ so we're looking for the answer choice that equals the same value, which is choice B.

7 A. Squaring both sides of the equation yields $4x^2 + 1 = 4x^2 + 4x + 1$. Subtracting $4x^2$ and 1 from both sides leaves you with $0 = 4x$, so x must equal 0.

8 D. A decrease of 10% means the population will be 90% of the initial amount after 4 years. Since this decrease happens every 4 years, the decrease must be calculated for every 4-year period of time that elapses. Since t represents a number of years, dividing by 4 will yield the correct model.

9 C. The problem can be represented by the expression $500{,}000(0.95)^t$. So, after ten years, the bee population will be $500{,}000(0.95)^{10} \approx 299{,}368.47$. So 300,000 is the closest approximation of the possible answers.

Student Produced Response – Calculator

1 2. In this equation, $r = 0.76$, because the value of the stock will be 76% of its total value if it decreases by 24%. If the stock decreases by \$114, its value is \$156. So, $156 = 270(0.76)^t$. The easiest way to solve might be to substitute possible values for t, since this model only works for 3 weeks. If $t = 1$, $V = 270 \times 0.76$ which is not true because $156 \neq 205.2$. If $t = 2$, $V = 270 \times 0.76^2$, so $V = 155.95$, which rounds to \$156.

2 7. To solve, substitute a few values for t. Since, after 3 weeks, $V \approx 118.52$, less than half its original value, it is reasonable to expect that it would only take a few more weeks for the value to be less than \$1. Plugging the numbers into the original equation, when $t = 4$, $V = 90.08$. When $t = 5$, $V = 68.46$, when $t = 6$, $V = 52.03$, and when $t = 7$, $V = 39.54$. So the value for the stock will be < \$50 after 7 weeks.

3 5/4 or 1.25. We want to solve for $\frac{x}{t}$. First, squaring both sides yields $x^2 - t^2 = 4t^2 - 4tx + x^2$. Subtracting x^2 from both sides and adding t^2 to both sides gives us $0 = 5t^2 - 4tx$. Dividing everything by t leaves us with $0 = 5t - 4x$, or $5t = 4x$. The last steps are to divide both sides by t and then divide both sides by 4, finally leaving us with $\frac{x}{t} = \frac{5}{4}$, or the decimal equivalent 1.25.

Guided Practice – Systems of Equations

Multiple Choice – No Calculator

1 D. We have two expressions for y, which we can set equal to each other. We get the equation $2x - 3 = x^2 + 6x$. In standard form, this is $x^2 + 4x + 3 = 0$, which has the solutions $x = -1$ or $x = -3$. Since

none of the answer choices have an x value of –3, we can plug –1 back into either equation and get $y = -5$, so the answer is D. Another method would be to simply test every answer choice. Choice A satisfies the first equation, but D is the only ordered pair that satisfies both equations.

2 D. The second equation represents a vertical line. The first equation represents a function. Since a vertical line can only intersect a function once, the system has only one solution. Another way of solving this problem is to replace x with –3 in the first equation. You get $y = 9 - 3a + b$. No matter what constants you choose for a and b, there is only one possible value for y (and –3 is the only possible value for x). Therefore, the system has only one solution.

3 B. The first equation is a circle centered at the origin. The second equation is an upward opening V shape with its vertex at the origin. This is true for any non-zero value of r. Therefore, the two graphs will intersect in quadrants I and II.

4 C. In answer choices A and B, we have the equations of two lines, which will only intersect at one point. These cannot be the answer, because the problem says there are exactly two solutions. Answer choice C has a circle crossed by a horizontal line, so there will be two solutions with the same y-coordinate (this is what the problem asks for). Answer choice D has a circle crossed by a vertical line, so there will be two solutions, but they will have the same x-coordinate and different y-coordinates, so D is incorrect. Another method would be to solve each system. Choice A has solution (5,0). Choice B has solution (0,5). Choice C has solutions (3,0) and (–3,0). Choice D has solutions (0,3) and (0,–3).

5 C. Take the expression for x that is given in the second equation, and substitute it for x in the first equation. The new equation is $y = 2(y^2 - 2y) - 10$ which simplifies to $y = 2y^2 - 4y - 10$. In standard form, this becomes $0 = 2y^2 - 5y - 10$. The discriminant is $b^2 - 4ac = (-5)^2 - 4(2)(-10) = 105$. Because the discriminant is positive, there are two real solutions for y (which would yield two distinct ordered pairs if we plugged y back in to find x). If you didn't remember the discriminant rule, you could also use the quadratic formula to see that there are two real solutions for y.

6 A. A system of quadratic equations has no solution when the parabolas never intersect. One way to solve is to draw a rough sketch of the first equation $y = x^2 + 4$. The second equation must either fit inside it (meaning have a higher y-intercept and steeper slope) or it must fit outside of it (meaning a lower y-intercept and a wider slope). A is the correct choice because the y-intercept is 2 and the slope is 1/2.

7 C. Take the expressions for y in each equation and set them equal to each other. The new equation is $x^3 + 15x = cx^2$, which is $x^3 + 15x - cx^2 = 0$ in standard form. This factors as $x(x^2 - cx + 15) = 0$. This equation has $x = 0$ as a solution, so the system will have three solutions if the equation $x^2 - cx + 15 = 0$ has two solutions. That will occur for $c = 8$. It will not occur for $c = -5$ or $c = 3$, because in both of those cases, the equation $x^2 - cx + 15 = 0$ would have no real solutions, so the entire system would have only one solution.

8 A. Solve the first equation for x. You get $x = -\frac{4}{3}y$. Substitute this for x in the second equation. You get $y = \frac{9}{4}\left(-\frac{4}{3}y\right)^2 - 6\left(-\frac{4}{3}y\right) + 3$. This simplifies to $y = 4y^2 + 8y + 3$. In standard form, this becomes $4y^2 + 7y + 3 = 0$, which factors as $(4x + 3)(x + 1) = 0$ so $x = -\frac{3}{4}$ or $x = -1$. The only solution that appears in the answer choices is A.

Student Produced Response – No Calculator

1 7. Replace y with x in the first equation. It becomes $x = x^2 - x - 35$. In standard form, this becomes $x^2 - 2x - 35 = 0$, which factors as $(x - 7)(x + 5) = 0$, so the solutions are 7 and –5. The problem states that $x > 0$, so 7 is the correct answer.

2 202. The problem tells us that x is negative, which means that $|x - 2| = -(x - 2) = -x + 2$, so we get $y + 4 = -x + 2$. Solve this equation for x and it becomes $x = -y - 2$. Substitute this for x in the second equation and we get $(-y - 2) + 2y = 200$. Finally, we can solve this equation for y and we get $y = 202$.

Multiple Choice – Calculator

1 B. This choice does not match with the others. The solution to the system is (1,4), so choice B has the x and y coordinate reversed. The other three choices all represent the same system of equations.

2 C. Take the expression that is given for y in the first equation, and substitute it for y in the second equation. The new equation is $4x + (x-3)(x-7) = 13$. This simplifies to $4x + x^2 - 10x + 21 = 13$. In standard form this becomes $x^2 - 6x + 8 = 0$ which has the solutions $x = 2$ or $x = 4$. None of the answer choices have an x value of 4, so we can plug in 2 for x in either equation and get $y = 5$. This problem can also be solved by testing each answer choice.

3 D. If we put both equations in standard form, we can see that they are both equivalent to $y = x^2 + 10x - 24$. Since the equations are equivalent, there are infinitely many solutions.

4 A. Set the two y expressions equal to each other. The new equation is $5x^2 + x + 8 = 3x^2 + 4$. In standard form, this becomes $2x^2 + x + 4 = 0$. The discriminant is $b^2 - 4ac = (1)^2 - 4(2)(4) = -15$. Because the discriminant is negative, there is no real solution. If you didn't remember the discriminant rule, you could also use the quadratic formula to see that there is no real solution to the equation $2x^2 + x + 4 = 0$.

5 B. Take the expression for y in the second equation and substitute it into the first equation. The new equation is $4x + mx^2 + n = 0$, which we write in standard form as $mx^2 + 4x + n = 0$. This equation will have exactly one real solution when its discriminant is equal to zero. The discriminant is $b^2 - 4ac = (4)^2 - 4(m)(n) = 16 - 4mn$. If we test each answer choice by plugging in the values of a and b, the only choice that yields a negative discriminant is B.

Student Produced Response – Calculator

1 236. Replace x with -4 in the second equation. We get $y = (-4)^3 + 300 = 236$.

2 3/2 or 7/2 or 1.5 or 3.5. Set the two expressions for y equal to each other. The new equation is $5(x^2 - 4x + 4) = x^2 - 1$. Distribute the 5 to get $5x^2 - 20x + 20 = x^2 - 1$. When we collect all terms on the left side, this becomes $4x^2 - 20x + 21$. This factors as $(2x-7)(2x-3) = 0$, so $x = \frac{7}{2}$ or $x = \frac{3}{2}$. Since a calculator is allowed for this problem, you could also graph both functions on the calculator and find the x-coordinates of their intersection points.

Guided Practice – Function Notation

Multiple Choice – No Calculator

1 D. Calculate $g(4)$ and plug it into the $f(x)$ equation. $f(4) = 5g(4) = 5[2(4)+1] = 5[9] = 45$.

2 C. We know that $g(6) = 7$. Therefore, $f(g(6)) = f(7) = 6$.

3 A. From the table, $f(10) = 10{,}000$ and $f(10{,}000) = 1$. It follows that $f(f(10)) = 1$.

4 B. Check the answers by substituting $2x+7$ for x in each of the answer choices. For answer choice B, you get $\sqrt{(2x+7)-4}$ which simplifies to $\sqrt{2x+3}$ so this is the correct answer.

5 C. Plug 2 into the equation for x. We get $g(2) = a(2)^2 - 2a(2) + 6 = 4a - 4a + 6$. The terms $4a$ and $-4a$ cancel out, and we are left with $g(2) = 6$.

6 A. We know that $f(a) = b$. Therefore, $g(f(a)) = g(b) = a$.

7 C. Replace the variable x in the $f(x)$ equation with the function $g(x)$. We get $f(g(x)) = (x+5)^2 - 9$. This simplifies to $x^2 + 10x + 25 - 9$, which becomes $x^2 + 10x + 16$.

8 B. This is the only option that makes all of the possible values of n true for $f(n)$.

9 C. This is the only option that makes all of the possible values of n true for $f(n)$.

10 C. Test each answer choice by replacing x in the original equation with the expression given by the answer choice. Choice C becomes $f(2x+2) = 2(2x+2) + 4 = 4x + 4 + 4 = 4x + 8$. None of the other answer choices result in $4x + 8$.

11 D. To calculate $f(g(x))$, replace x in the $f(x)$ equation with $5x$. We get $f(g(x)) = (5x)^2 + 2 = 25x^2 + 2$. To calculate $g(f(x))$, replace x in the $g(x)$ equation with $x^2 + 2$. We get $g(f(x)) = 5(x^2 + 2) = 5x^2 + 10$. Finally, we subtract these two expressions and get $f(g(x)) - g(f(x)) = (25x^2 + 2) - (5x^2 + 10) = 20x^2 - 8$.

12 C. Substitute $\sqrt{x^2 - 6}$ for x in the original equation. You get $\sqrt{\sqrt{x^2 - 6}^2 - 6}$. This simplifies to $\sqrt{x^2 - 6 - 6}$ which equals $\sqrt{x^2 - 12}$.

13 B. When any point or function is reflected over the y-axis, its x-values negate and its y-values stay the same. In function notation, the x inside the parentheses should negate and the value of the function, or g, should stay the same.

14 D. Based on the transformations described, we can write the equations $q(x) = -p(x) - 4$ and $r(x) = -p(x) + 4$. It follows that $q(x) = r(x) - 8$.

Student Produced Response – No Calculator

1 23. Calculate $g(3)$ and plug it into the $f(x)$ equation. $f(3) = g(3) + 10 = [3(3) + 4] + 10 = 13 + 10 = 23$.

2 13. The value of the function at 0 is $f(0) = (0)^2 - 13(0) + 40 = 40$. To find the value of a, we must find the other point at which the function has a value of 40. This means solving the equation $40 = x^2 - 13x + 40$ which simplifies to $x^2 - 13x = 0$. The solutions to this quadratic equation are 0 and 13. Since the problem says that $a > 0$, the answer must be 13.

Multiple Choice – Calculator

1 A. Since we know that $g(2) = 6$, we can plug in 2 for x and 6 for $g(x)$. This gives the equation $6 = c(2)^2 - 6$. Solving this equation, we get $c = 3$, so the function becomes $g(x) = 3x^2 - 6$. When we plug in -2 for x, we get $g(-2) = 6$. There is also a shortcut. Notice that the variable x only appears as x^2. Since the value of 2^2 is the same as the value of -2^2, the values of $g(2)$ and $g(-2)$ will also be equal to each other, so $g(-2) = 6$.

2 A. Since we know that $f(2) = 16$, we can plug in 2 for x and 16 for $f(x)$. This gives the equation $16 = a(2) - 4$. Solving this equation, we get $a = 10$, so the function becomes $f(x) = 10x - 4$. When we plug in 3 for x, we get $f(3) = 26$.

3 B. Choice B is true because $f(-x) = (-x)^2 - 3 = x^2 - 3 = f(x)$ for all values of x. All of the other choices will be false for most values of x. For example, if you plug in 2 for x, the equations in choices A, C, and D will all be false.

4 B. First calculate the value of $g(3)$, the inner function. $g(3) = 2(3) + 2 = 8$. Next, plug this into the outer function. $f(g(3)) = f(8) = 3(8) - 4 = 20$.

5 A. The graph of $y = 2f(-x)$ is equivalent to the graph of $y = f(x)$ reflected across the y-axis and then stretched vertically by a factor of 2. This is shown in answer choice A. Choice B is incorrect because the graph is translate up two units, not stretched. Choices C and D are incorrect because they show the original function reflected over the x-axis instead of the y-axis.

6 B. Replace x in the original equation with $-5x$. We get $f(-5x) = 4(-5x) + 6 = -20x + 6$.

Student Produced Response – Calculator

1 108. Calculate $f(20)$ and plug it into the $g(x)$ equation. $g(20) = 3f(20) = 3[2(20) - 4] = 3[36] = 108$.

2 22. Calculate $f(2)$ and $g(2)$ and add them together. $h(2) = f(2) + g(2) = [2(2) + 5] + [3(2) + 7] = 9 + 13 = 22$.

3 9. The graph has a y-value of 1 when the x-value is 3. This means that $f((3) + 6) = 1$. It follows that $f(9) = 1$, so the answer is 9. Another approach is to recognize that the graph $y = f(x + 6)$ is equivalent to $y = f(x)$ translated 6 units left. If we translate it 6 units back to the right, the point (3,1) maps to (9,1) which means that $f(9) = 1$.

Guided Practice – Graphs of Functions

Multiple Choice – No Calculator

1 A. The x-intercepts of the graph of a function correspond to the zeroes of the function. If a function has x-intercepts at –3 and 2, then the values of the function at –3 and 2 are each 0. The function in choice A is in factored form and shows that $f(x) = 0$ if and only if $x + 3 = 0$ or $x - 2 = 0$, so this function has x-intercepts at –3 and 2.

2 C. Choices A, B, and D all intercept the y-axis at –1, so there is at least one value less than 0 in each of these graphs. If we expand $y = (x - 1)^2$ we get $y = x^2 - 2x + 1$, which is a parabola that opens upward and has a minimum at (1,0), so every point on this function will have a y-value of at least 0.

3 A. The line at $y = 4$ intercepts with the parabola when $(x - 5)2 = 4$. Taking the square root of both sides, we get $x - 5 = 2$ or $x - 5 = -2$, so x could be equal to 7 or 3. The line intersects the parabola at (3,4) and (7,4). The length of AB is therefore equal to the difference between 7 and 3, which is 4.

4 C. Since a negative square root yields an imaginary number, $2x - 8 \geq 0$. This can be simplified to $x \geq 4$.

5 C. The range of a function is the set of all possible y values. Since x^2 is positive, we know the graph of the function will be a parabola that opens upward. You can either graph the function on a graphing calculator or find the vertex of the parabola, which will show the lowest value of y, by solving $x = -b/2a$, which equals –1, and then plugging that into the function, which gives you $f(-1) = (-1)^2 + 2(-1) + 2 = 1$. The vertex of the parabola is located at (–1,1). Since the domain of this function is all real numbers, and substituting any real number for x would yield a y-value greater than 1, the range of this function is $y \geq 1$.

6 D. The equation $f(x) = k$ will have 4 solutions if and only if the graph of the horizontal line with equation $y = k$ intersects the graph of f at 4 points. The graph shows that of the given choices, only for -1 will the graph of $y = -1$ intersect the graph of f at 4 points.

7 D. This parabola intersects the x-axis at 2 and –6. The x-coordinate of the vertex of the parabola is halfway between these points (because a parabola is symmetrical): $\frac{-6+2}{2} = -2$, which is the value of c. To find the y coordinate of the vertex, substitute –2 for x in the equation: $y = a(-2 - 2)(-2 + 6) = -16a$.

8 C. One can find the intersection points of the two graphs by setting the functions $f(x)$ and $g(x)$ equal to one another and then solving for x. Thus, $3x^2 - 3 = -3x^2 + 3$. This can be simplified to $6x^2 = 6$, so $x^2 = 1$ and $x = \pm 1$. Choice C is the only value that matches.

9 C. Since the graph shows two straight lines that form a "V", it is an absolute value function. The absolute value by itself would show all values greater than 0, but since this graph has values for $-2 \leq x \leq 0$, this graph shows a transformation. For a graph of a function f, a change in the form of $f(x) - 2$ will result in a vertical shift of 2 units down, and a change in the form of $f(x - 2)$ will result in a horizontal shift of 2 units, so $f(x) = |x - 2| - 2$ is the only possible answer.

Student Produced Response – No Calculator

1 3. In this function $f(1) = 5$, so $5 = b(1) + 2$. So $b = 3$.

2 10. The line at $y = 25$ intercepts with the parabola when $(x - 4)^2 = 25$. Taking the square root of both sides, we get $x - 4 = 5$ or $x - 4 = -5$, so x could be equal to 9 or –1. The line intersects the parabola at (–1,25) and (9,25). Therefore, the length of AB is equal to the difference between –1 and 9, which is 10.

3 2. If the vertex is located at (1,–8), it is a point on the graph of the function. Therefore, $-8 = a(1 + 1)(1 - 3)$, so $-8 = -4a$ and $a = 2$.

Student Produced Response – Calculator

1 6. If $f(x) = 2x^2 - bx + 2$, then $2 = 2(3)^2 - b(3) + 2$, so $2 = 20 - 3b$ and $3b = 18$. So $b = 6$.

2 2. Substituting 1 for x and 0 for y yields: $0 = (1)^3 + 3(1)^2 + k(1) - 6 = 1 + 3 + 1k - 6 = k - 2$. So, $k = 2$.

3 26. The equation for $g(x)$ is $g(x) = (x + 2)^2 + 1$. So, $g(3) = (3 + 2)^2 + 1 = 26$.

Multiple Choice – Calculator

1 D. The x-intercepts of the parabola are the values of x for which y is equal to 0. The factored form of the equation shows that y equals 0 if and only if $x = 3$ or $x = 4$. Thus, the factored form shows the x-intercepts of the parabola as the constants 3 and 4.

2 D. The graph of a function f on the xy-plane is the set of all points $(x, f(x))$. In this function, the value of $f(a)$ is 1 if and only if the unique point on the graph of f with the x-coordinate a has a y-coordinate equal to 1. This is true for the x-coordinates –3, 1, and 2. Therefore, I, II, and III are all equal to 1.

3 D. To find the vertex, we can plug into the formula $x = \frac{-b}{2a}$ to find the axis of symmetry, which results in 4, and then plug that value back into the original function to find the y-value, which gives us –4. The vertex is therefore (4,–4), which are values found only in choice D.
because, when $y = 0$, the possible values of x, and thus, the x intercepts, are 2 and 6.

4 C. Since $f(-x) = f(x)$, $f(-1) = f(1) = 4$, so point (–1,4) will also be on the graph of $f(x)$.

5 C. If (–1,0) is a point on the function, we can substitute –1 for x and 0 for y:
$0 = 2(-1)^3 + (-1)^2 + k(-1) + 4$, which simplifies to $0 = 1 - 2 - k + 4$, so $k = 3$.

6 D. The x-intercepts of the parabola are the values of x for which y is equal to 0. The factored form of the equation shows that y equals 0 if and only if $x = -3$. Thus, the factored form shows the x-intercept of the parabola as the constant –3. (The vertex of the parabola is located at (–3,0).)

7 A. In a rational function, the asymptote is equal to the value of x that makes the denominator equal to zero. In this case a is equal to the value of x when $x + 2 = 0$. So a is equal to –2.

8 C. The smallest value of the function would be equal to $f(x)$ when $|x - 4|$ is equal to 0, because the absolute value indicates all other values will be positive numbers greater than 0. In this case, when $|x - 4| = 0$, $f(x) = 2$, so 2 is the smallest value in the range. (In the graph of this function, the vertex is at (4,2), making 2 the smallest value of y.)

9 D. We can see that the vertex is at point (2,–16). Choice D is the only one that has these two numbers. (It is in the form of $f(x) = a(x - h)^2 + k$ where a, h, and k are constants and (h, k) is the vertex of the parabola.)

10 C. All of the given choices are polynomials. If the graph of a polynomial function f on the xy-plane has an x-intercept at b, then $(x - b)$ must be a factor of $f(x)$. Since –2, –1, and 2 are each x-intercepts of the graph of f, it follows that $(x + 2)$, $(x + 1)$, and $(x - 2)$ must each be a factor of $f(x)$.

11 B. When the graph of a quadratic function is translated, we use $f(x) = a(x - h)^2 + k$, where h represents the translation along the x-axis and k represents the translation along the y-axis. The equation for $g(x)$ is therefore $g(x) = -(x + 3)^2 + 1$. We then plug –2 into the function: $g(-2) = -(-2 + 3)^2 + 1 = 0$.

12 D. The graph only has one x-intercept, so it has 1 real solution. Therefore, II is not true. The graph has two turning points, the second of which is at (1,6). Therefore, the graph is increasing for $x \geq 1$ and $f(x) \geq 6$ for all $x \geq 0$, so I and III are true.

13 B. Note that the question asks for the range, so we need to solve in terms of y. First, solve for x in terms of y: $y = \frac{1}{x} - 2$, so $y + 2 = \frac{1}{x}$. Therefore, $x = \frac{1}{y+2}$, meaning that there is an x value for all y values except –2 (since there cannot be a 0 in the denominator). You can also check this by graphing the equation; there is a horizontal asymptote at $y = -2$.

14 B. The graph of this function has vertical asymptotes when the denominator of the fraction is equal to 0. Factoring the denominator yields $0 = (x + 5)(x - 2)$, so –5 and 2 cannot be possible values of x.

15 B. For any value of x, say $x = a$, the point $(a, f(a))$ lies on the graph of f and $(a, g(a))$ lies on the graph of g. Thus, for any value of x the values of $f(a) + g(a)$ is equal to the sum of the y coordinates of the points on the graph with an x coordinate equal to a. Therefore, the value of x for which $f(x) + g(x) = 0$ will occur when the y-coordinates of the points representing $f(x)$ and $g(x)$ at the same value of x are equidistant from the x-axis and are on opposite sides of the x-axis. On this graph, this occurs at $x = -1$. The point (–1,3) lies on $f(x)$ and (–1,–3) on $g(x)$. Thus, at $x = -1$, $f(x) + g(x) = 3 + -3 = 0$.

16 B. One way to solve is to isolate y so that the equation is in $y = mx + b$ format. First, we distribute k: $y + x = kx - ky$. Then we move ky to the left side of the equation and x to the left side of the equation so that both y's and both x's are together: $ky + y = kx - x$. We can factor out the common terms:
$y(k + 1) = x(k - 1)$. Finally, we isolate y by dividing both sides by $k + 1$:

$y = \left(\frac{1-k}{1+k}\right)x$. Since this equation does not have a y-intercept, the graph must be a line that passes through the origin.

Guided Practice – More Word Problems

Multiple Choice – No Calculator

1 B. Solve the equation for m. First, subtract 40 from both sides to get $b - 40 = 0.45m$. Then divide both sides by 0.45 to get $m = \frac{b-40}{0.45}$.

2 A. Solve the equation for y. First, add 24 to both sides. We get $c + 24 = 12y$. Next, divide both sides by 12. We get $\frac{c+24}{12} = y$ or, equivalently, $\frac{c}{12} + 2 = y$.

3 C. Solve for a. First, subtract b from both sides and add $0.5y^2$ to both sides. The result is $w - b + 0.5y^2 = ay$ Next, divide both sides by y to get $\frac{w-b+0.5y^2}{y} = a$ or, equivalently, $\frac{w-b}{y} + 0.5y = a$.

4 C. The rate of change for the function $L(t)$ is the coefficient of t, which is 2. The rate of change is positive, so the temperature will increase at a rate of 2 degrees per minute. Therefore, in ten minutes, the temperature will increase 20 degrees. 33 represents the starting temperature.

5 B. The function $L(t)$ increases at a rate of 2 degrees per minute. The problem tells us that the temperature of the lemonade increased by 10 degrees, which means that 5 minutes passed. Now examine the equation for $C(t)$. The coefficient of t is -1, which means that the coffee temperature is decreasing at a rate of 1 degree per minute. Therefore, in a period of 5 minutes, the temperature of the coffee with decrease by 5 degrees.

6 D. Begin with the equation $R = \frac{\sqrt{P}}{2}$. To solve for P, the first step is to multiply both sides by 2. The equation becomes $2R = \sqrt{P}$. The next step is to square both sides. We get $4R^2 = P$.

7 A. The simplest way to solve this problem is to pick numbers for Anna and Brenda. We know that Anna completed 9 times as many projects as Brenda. Let's say that Anna completed 9 projects and Brenda completed 1. Then Anna's rating is $\frac{\sqrt{9}}{2} = \frac{3}{2}$ and Brenda's rating is $\frac{\sqrt{1}}{2} = \frac{1}{2}$. Brenda's rating is $\frac{1}{3}$ of Anna's rating because $\frac{\text{Anna Rating}}{\text{Brenda Rating}} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$.

8 A. The rate of change in this function is the coefficient of m, which is $\frac{1}{6}$. This means that the animal grows $\frac{1}{6}$ inch per month. Therefore, in one year (12 months) the animal will grow $\frac{1}{6} \cdot 12 = 2$ inches.

9 B. Begin with the equation $h = \frac{w}{5r^2}$. Get r^2 out of the denominator by taking the reciprocal of both sides. The new equation is $\frac{1}{h} = \frac{5r^2}{w}$. To get r^2 by itself, we must multiply both sides by w and divide both sides by 5. The final equation is $\frac{w}{5h} = r^2$.

10 A. Let h represent the amount of time that it takes for Field A to absorb the water. Then Field B absorbs the water in the time $4h$. Using the rearranged formula from the previous problem we see that for Field A,

$r^2 = \frac{w}{5h}$, so $r = \sqrt{\frac{w}{5h}}$. Similarly, for Field B, $r = \sqrt{\frac{w}{5(4h)}} = \sqrt{\frac{1}{4} \cdot \frac{w}{5h}} = \frac{1}{2}\sqrt{\frac{w}{5h}}$. This demonstrates that the radius of Field B is half of the radius of Field A. Another approach to this problem would be to plug in numbers (e.g. Let $w = 5$ for both fields and let h be equal to 1 for Field A but 4 for Field B).

11 C. The old mass was $6ds^2$. The new mass is $6(2d)\left(\frac{s}{2}\right)^2 = 6(2d)\left(\frac{s^2}{4}\right) = 3ds^2$. This is half of the original mass because 3 is half of 6 and ds^2 remains the same.

12 A. When the rock hits the bottom of the lake, the height will be 0, so the question is really asking for the time when $h = 0$, so we solve the equation $-3t^2 - 3t + 60 = 0$. If we divide by -3, we get $t^2 + t - 20 = 0$, which factors as $(t + 5)(t - 4) = 0$. The solutions are -5 and 4, but time cannot be negative, so the only acceptable answer is 4.

13 B. The straightforward way to figure out the answer to this question is to look at the units associated with each variable. The cost, c, is measured in dollars, the distance, d, is measured in miles and the price, p, is measured in dollars per gallon or $\frac{dollars}{gallons}$. If we replace each variable with its unit, we get $\frac{dollars}{1} = \frac{miles}{1} \cdot \frac{dollars}{gallons} \cdot k$. In order to get the proper cancelation of units on the right side of the equation, the unit for k must be $\frac{gallons}{miles}$, which is shown in choice B.

Student Produced Response – No Calculator

1 15. Based on the information described in the problem, the left side of the equation has been multiplied by 2 (because V was doubled) and the right side of the equation has been multiplied by $1.5 \times 20 = 30$ (because T was multiplied by 1.5, and n was multiplied by 20). In order for the two sides to balance, we need the left side to be multiplied by 15, so the pressure must be multiplied by 15.

2 $\frac{4}{9}$ or 0.44. Let l_A represent the length of the thicker rod and l_B represent the length of the thinner rod. We know their masses are equal, so $m = 12\left(\frac{3}{2}t\right)^2 l_A$ for the thicker rod and $m = 12t^2 l_B$ for the thinner rod. We can put these equations together to get $12\left(\frac{3}{2}t\right)^2 l_A = 12t^2 l_B$. We want to find the ratio of the length of the thicker rod to the length of the thinner rod, so we should solve for $\frac{l_A}{l_B}$. We get

$$\frac{l_A}{l_B} = \frac{12t^2}{12\left(\frac{3}{2}t\right)^2} = \frac{1}{\left(\frac{3}{2}\right)^2} = \frac{4}{9}, \text{ or } 0.44.$$

Multiple Choice – Calculator

1 C. Changing the material with which the bricks are made would change the weight per brick. It would not change the number of bricks (n), number of boxes (b), or weight of an empty box (w), so it must be K by process of elimination. Another way to see this is that the expression $n(bK + w)$ represents the total weight and n represents the number of boxes, so the expression $(bK + w)$ must represent the weight per box. Since w is the weight of an empty box, bK must represent the weight of all the bricks in each box. Since b is the number of bricks in each box, K must be the weight of each brick. That is the quantity that changes in this scenario.

2 B. We want to find the time when the ball hits the ground, which occurs when the height is 0. Therefore,

we want to solve the equation $-16t^2 + 65t = 0$. This factors as $t(-16t + 65) = 0$, so the solutions are $t = 0$ and $-16t + 65 = 0$. The first solution represents when the ball is launched and the second represents when it lands. Solving for the second solution, we get $t = \frac{65}{16} \approx 4.0$.

3 A. Whenever a problem asks for an equation that shows the minimum/maximum and where it occurs, you should put the equation in vertex form. This means it should look like $y = a(x - h)^2 + k$. The only equation in this form is answer choice A. To get the equation in this form, you need to complete the square. $t(s) = s^2 - 16s + 75 = s^2 - 16s + 64 - 64 + 75 = s^2 - 16s + 64 + 11 = (s - 8)^2 + 11$. Another approach would be to calculate the coordinates of the vertex and see which equation contains those numbers. To find the coordinates of the vertex, use the equation $x = -\frac{b}{2a} = -\frac{(-16)}{2(1)} = 8$. If we plug 8 into the original equation for s, we get $t(8) = 11$. The only answer choice with 8 and 11 in the equation is Choice A.

4 D. Solve for w. First, square both sides to get $\frac{\pi^2 d^2}{16} = \frac{w}{p}$. Then multiply both sides by p to get $\frac{\pi^2 d^2 p}{16} = w$.

5 C. One option is to test the answer choices. By looking at the equation $D = \frac{2r}{2r + b}$, we can see that D is not equal to P or $2P$, so we only need to test the last two answer choices. For answer choice C, we get $\frac{2P}{P+1} = \frac{2\left(\frac{r}{r+b}\right)}{\frac{r}{r+b}+1}$, which simplifies to $\frac{2r}{2r+b}$. However, with answer choice D, we get $\frac{2P}{2P+1} = \frac{\frac{2r}{r+b}}{\frac{2r}{r+b}+1}$ which simplifies to $\frac{2r}{3r+b}$. Alternatively, the quantity D results when r is doubled, so $D = \frac{2r}{2r+b}$. We want to rearrange this expression to get it in terms of P. A good first step is to divide the numerator and denominator by $r + b$ so that they look more like P. We get $\frac{2r}{2r+b} = \frac{\frac{2r}{r+b}}{\frac{2r+b}{r+b}} = \frac{2\left(\frac{r}{r+b}\right)}{\frac{r}{r+b}+\frac{r+b}{r+b}} = \frac{2P}{P+1}$.

6 D. Whenever a problem asks for an equation that shows the minimum/maximum and where it occurs, you should put the equation in vertex form. This means it should look like $y = a(x - h)^2 + k$. The only equation in this form is answer choice D. To get the equation in this form, start by factoring out the leading coefficient. The new equation becomes $m(v) = -\frac{1}{10}(v - 40v)$. Next, complete the square. This gives you $m(v) = -\frac{1}{10}(v^2 - 40v + 400 - 400) = -\frac{1}{10}(v^2 - 40v + 400) + 40 = -\frac{1}{10}(v - 20)^2 + 40$. Another way to solve this problem is to calculate the coordinates of the vertex and see which equation contains those numbers.

7 D. If we replace r with $2r$ we get $c = ke^{2rt} = ke^{rt+rt} = ke^{rt}e^{rt}$. Therefore, the original value of c is multiplied by e^{rt}.

Student Produced Response – Calculator

1 0.88. The value of r is the number by which V will be multiplied each day. Since the volume is decreasing, we must subtract 12% from 100%, so $r = 1 - 0.12 = 0.88$.

2 184. To calculate this, we must plug in 0.88 for r and 3 for t. The answer is $270(0.88)^3 = 183.99744$, which rounds to 184.

3 0.81. The taller person's body mass index is $\frac{m}{h^2}$. The shorter person's body mass index is $\frac{m}{(0.9h)^2}$.

Therefore, the ratio is $\frac{m}{h^2} \div \frac{m}{(0.9h)^2} = \frac{m}{h^2} \cdot \frac{(0.9h)^2}{m} = (0.9)^2 = 0.81$.

Mixed Practice – Advanced Math

Multiple Choice – No Calculator

1 Exponents & Radicals: C. Since three of the answer choices start with $x^3 =$, we can start by isolating x^3. First, subtract 3 from both sides, giving you $9\sqrt{x^3} = x - 9$. Then divide both sides by 9, giving you $\sqrt{x^3} = \frac{x-9}{9}$. Finally, square both sides, which gives you $x^3 = \left(\frac{x-9}{9}\right)^2 = \frac{x^2 - 18x + 81}{81}$. Answer choice B is incorrect because you cannot cancel out the 81's from the top and bottom of the fraction. Answer choice D is incorrect because solving for x requires finding the cube root of both sides ($\sqrt[3]{\ }$), not the square root.

2 More Word Problems: D. Solve for k. First, subtract $20x + b$ from both sides to get $c - 20x - b = kx^2$ or, equivalently, $c - b - 20x = kx^2$. Next, divide both sides by x^2 to get $\frac{c-b-20x}{x^2} = k$, or equivalently, $\frac{c-b}{x^2} - \frac{20}{x} = k$.

3 Systems of Equations: D. Take the expressions that are equal to y in each equation and set them equal to each other. The new equation is $3x^2 - 8x + 36 = 2x^2 + 2x + 11$. In standard form, this becomes $x^2 - 10x + 25 = 0$ which factors as $(x - 5)(x - 5) = 0$, so the only solution is $x = 5$. Plug this back into either equation to find the value of y. We get $y = 71$. This means that $x + y = 5 + 71 = 76$.

4 More Word Problems: D. The original energy was $E = kdr^3$. The new energy is $k(2d)(2r)^3 = k(2d)(8r^3) = 16kdr^3$. This is 16 times the original energy.

5 Function Notation: D. We know that $f(0) = -1$. Therefore, $g(f(0)) = g(-1) = 2$.

6 Working with Polynomials: B. Equation I factors as $(x - 3)(x - 3) = 0$ so it only has the solution $x = 3$. Equation II factors as $x(x^2 + 5) = 0$ where $x^2 + 5$ is not factorable, so the only real solution is $x = 0$. Equation III factors as $(x + 2)(x - 2) = 0$, so it has two real solutions, $x = 2$ and $x = -2$. Thus, I & II have exactly one real solution.

7 Graphs of Functions: B. If the graph of a polynomial function f on the xy-plane has an x-intercept at b, then $(x - b)$ must be a factor of $f(x)$. In this case, the vertex of the parabola is at (0,–2), so this is the only x-intercept.

Multiple Choice – Calculator

1 Working with Polynomial Factors in Expressions & Equations: D. To add fractions, we need them all to have the same denominator. Factoring $t^2 - 6t + 8$ gives us $(t - 2)(t - 4)$. All three fractions will have the same denominator if we multiply the first fraction by $\frac{t-4}{t-4}$ and the second fraction by $\frac{t-2}{t-2}$. This gives us $\frac{t^2 - 4t}{(t-2)(t-4)} + \frac{t-2}{(t-2)(t-4)} = \frac{2}{(t-2)(t-4)}$. Therefore, $t^2 - 4t + t - 2 = 2$, or $t^2 - 3t - 4 = 0$, which we can solve by factoring: $(t - 4)(t + 1) = 0$. Finally, t can equal –1 or 4. However, 4 cannot be a solution because plugging 4 into the original equation results in a denominator of 0, so the only solution is –1.

2 Working with Polynomial Factors in Expressions & Equations: C. Since this equation shows two fractions equal to each other, we can solve by cross multiplying. $4(x + 8) = 7(x - 1)$, which gives us $4x + 32 = 7x - 7$. This produces $3x = 39$, or $x = 13$.

3 Function Notation: A. Replace x in the original equation with $(x + 4)$. We get $f(x + 4) = 2(x + 4) - 7 = 2x + 8 - 7 = 2x + 1$.

4 Quadratic Functions & Equations: A. The problem talks about two sets of books. The first set of books had x books costing 10 dollars each for a total cost of $10x$. The second set of books was the rest of the books, which is $30 - x$ and they cost $\frac{x}{2}$ dollars each, so the cost for the second set of books was $\frac{x}{2}(30 - x)$. Adding up the cost of the first and second set of books, we arrive at $10x + \frac{x}{2}(30 - x) = 200$. If we multiply both sides by 2 we arrive at $20x + x(30 - x) = 400$. When you rearrange that into standard form you arrive at choice A.

5 Systems of Equations: B. The second equation can be rearranged to get $y = 3 - 2x$. If we substitute this expression for y in the first equation, we get $(3 - 2x) - 6 = -(x + 2)^2$ which simplifies to $-3 - 2x = -x^2 - 4x - 4$. In standard form, this becomes $x^2 + 2x + 1 = 0$. This equation has only one solution, $x = -1$, and if we plugged that solution into either equation, we would get only one possible value for y. Therefore, only one ordered pair satisfies the equations.

6 Working with Polynomials: B. Answer choice A factors as $(x^4 + 17)(x - 8)$. Answer choice C factors as $x(x - 5)(x - 8)$. Answer choice D factors as $y(x + 8)(x - 8)$, so all of those are divisible by $x - 8$. In choice B, $x - 8$ is a factor in first part of the expression, but there is a 6 at the end, which is not divisible by $x - 8$.

Student Produced Response – Calculator

1 Exponents & Radicals: 1,200. The amount of money in Joe's account can be represented by the equation $A = d(r)^t$, where d is the initial deposit, r is the interest rate plus 1, and t is the time in years. We know the current balance (A) but not the value of d. Substitute into the equation: $1{,}325 = d(1.02)^5$. Solving for d yields approximately \$1,200.

2 Graphs of Functions: 127. The formula for a parabola is $y = (x - h)^2 + k$, where h is the horizontal shift and k is the vertical shift. For this function, shifting 3 units to the left and 2 units up gives us $g(x) = (x + 3)^3 + 2$. We can then plug in 2 for x: $g(2) = (2 + 3)^3 + 2 = 127$.

Additional Math Topics

Guided Practice – Geometry

Multiple Choice – No Calculator

1 B. Since AB and DE are parallel, $\angle EDC$ and $\angle DBA$ are congruent because they are alternate interior angles. $\angle BAC$ and $\angle DEC$ are congruent for the same reason. Finally, $\angle DCE$ and $\angle ACB$ are congruent because they are vertical angles. Thus, the triangles are similar. AB corresponds to ED in the ratio 3:1, meaning the top triangle is three times the size of the bottom triangle. DC is 5 meters long, so BC must be 15 meters long.

2 B. The formula for a cylinder is given on the reference sheet at the beginning of each math section: $V = \pi r^2 h$. Since the diameter is 20 m, the radius is 10 m. We simply plug 10 in for r and 8 in for h: $V = \pi(10)^2(8) = 800\pi$.

3 B. Since lines j, k, and l are parallel, $\angle 1$ is congruent to the acute angle between line l and line n. Since lines m and n are parallel, $\angle 1$ and $\angle 2$ are congruent because they are formed by the intersection of parallel transversals to parallel lines. Basically, all the acute angles are congruent and all the obtuse angles are congruent.

4 A. $\angle ADC$ is congruent to $\angle 1$ because they are vertical angles so it must be 78°. If we focus on the angles we know the measures of, we see that we have two angles in triangle ADC. Therefore, $\angle ACB$ must equal $180° - (33° + 78°) = 69°$. Note that lines m and p are unnecessary to solving this question.

5 C. Since $\angle ACB$ is supplemental to $\angle DCA$, their measures must add up to 180°. Therefore, $\angle ACB = 180° - 105° = 75°$. Since the internal angles in triangle ABC must add up to 180°, then $\angle ABC = 180° - (75° + 35°) = 70°$.

6 C. The diagonal of a square effectively creates a 45°– 45°– 90° triangle. Using the reference sheet at the beginning of each math section, we can figure out that each side of the square should have a length of 4, meaning each side of the hexagon is 4. If we connect all of the opposite vertices, we can see that a hexagon is composed of 6 congruent equilateral triangles, each of which can be split into two 30°– 60°–

90° triangles with base 4 and height = $2\sqrt{3}$. This leads to the area of each equilateral triangle: $A = \frac{1}{2}bh = \frac{1}{2}(4)(2\sqrt{3}) = 4\sqrt{3}$. Since the area of the regular hexagon is 6 times the area of the triangle, $4\sqrt{3} \cdot 6 = 24\sqrt{3}$.

7 C. An important rule to know is that the central angle subtended by two points on a circle is twice the inscribed angle subtended by those points. Here, that means angle AOC is twice the size of angle ABC. Since angle $AOC = 60°$, angle ABC must equal 30°.

8 B. If angle $AOC = 56°$, then angle $AOB = 180° - 56° = 124°$ because the two angles form a straight line. Triangle AOB is isosceles because two of its sides are radii of the circle. Thus, angle $OAB = \frac{1}{2}(180° - 124°) = 28°$.

9 C. An important rule to know is that the angle formed by the intersection of two tangents (in this case, angle M) equals half the central angle formed by the radii to the tangent points. Therefore, the central angle here is 60°. Since $60° = \frac{1}{6}(360°)$, and the entire circumference is 720, then the major arc is $\frac{5}{6}(720) = 600$.

10 D. I is true because $B + C + D$ and $A + F + B$ both add up to 180° because both sums are half-circles. II is true because $A + B = C + D$ and $A = D$ because they are vertical angles leaving $B = C$ and $E = B$ because they are vertical angles. Thus, $E = C$. III is true because A and D are vertical angles. Thus, all three statements must be true.

11 B. From the hash marks on the sides, we can see that both triangles are isosceles. That means $w = 180° - 2v = 180° - 2(40°) = 100°$. Since $w = 2x$, x must be 50°, and so the other two angles in that triangle = (180° – 50°)/2= 65°. Finally, $y = 180° - 65° = 115°$.

12 B. Since the diameter of the circle = 20, the area of the semicircle = $\frac{\pi \cdot 10^2}{2} = 50\pi$. Since the triangle is inscribed in a semicircle, it must be a right triangle and angle $BAC = 30°$ because it subtends an arc of 60° from a point on the circle. Therefore, the triangle is a 30°– 60°– 90° triangle and has an area = $\frac{1}{2}(base)(height) = \frac{1}{2}(10)(10\sqrt{3}) = 50\sqrt{3}$. The area of the shaded region is the difference between the areas of the semicircle and the triangle. This yields $50\pi - 50\sqrt{3}$.

13 C. One way to solve this is to plug in values for each choice. The volumes of a right circular cylinder and a right circular cone are $\pi r^2 h$ and $\frac{1}{3}\pi r^2 h$, respectively. The only difference in the formulas is the $\frac{1}{3}$. That means that, if the radius and the height are the same, the volume of the cone will be one-third the volume of the cylinder. Statement I is wrong. The radius gets squared, so if the cone's base radius is one third the cylinder's, the volume will be multiplied by $\frac{1}{9}$, making the final volume $\frac{1}{27}$ that of the cylinder. The height of the cone would have to be 27 times that of the cylinder to balance out the volumes, not nine times. Since answer choices A, B, and D all have Statement I, they must all be eliminated, leaving only choice C. Statement II is correct because doubling the radius of the cylinder would result in a base area 4 times the base area of the cone and to compensate, the cone would have to have height 3 × 4 = 12 times as high as the cylinder. Statement III is correct because if the cone and cylinder have the same base areas, then the cylinder needs to have one third the height of the cone for their volumes to be equal.

Student Produced Response – No Calculator

1. 86. In a circle, the degree measure of an inscribed angle is half the measure of the arc it intercepts. If angle BAC is 43°, then arc BC must be 86°. The degree measure of an arc is equal to the measure of the central angle that intercepts the arc. Therefore, if arc BC is 86°, then central angle BOC must also be 86°.
2. 50. Segment BD is divided into parts in the same proportions as AB. The three parts of segment $AB = x$, $3x$, and $5x$, which add to $9x$. $BF = \frac{1}{9}$ of the length of $BD = 10$. Therefore, $DE = 5(10) = 50$.
3. 16. Since the circumference is 8π, the length of AC (the diameter) must be 8. Since the triangle is inscribed in a semicircle, it must be a right triangle, and since it is isosceles, it must be a 45°– 45°– 90° triangle. From here, we can solve in one of two ways: We either use the reference information at the beginning of each math section to figure out that sides AB and BC must each be $\frac{8}{\sqrt{2}} = 4\sqrt{2}$ and then plug into the formula for area of a triangle: $\frac{1}{2}(base)(altitude) = \frac{1}{2}(4\sqrt{2})(4\sqrt{2}) = 16$. Or, we can draw a line from point O to point B, splitting the triangle into two smaller right triangles, each with base 4 and altitude 4 (since OA, OB, and OC are all radii), giving us: $2\left(\frac{1}{2}(base)(altitude)\right) = 2\left(\frac{1}{2}(4)(4)\right) = 16$.

Multiple Choice – Calculator

1. D. The slant heights in the structure proportional to the vertical heights, so $4x + 3x + x = 8x = 32$ feet. Therefore, x must equal 4. The height of the first floor is $4x$, which is 4(4) = 16 feet.
2. B. Any problem involving the lengths of shadows of multiple objects in the same vicinity can usually be solved using a simple proportion because the sun's rays form the same angle with both objects, creating similar triangles. Here, the ratio of the bush's shadow to the tree's shadow is $\frac{2}{15}$, so we can set up and solve the following proportion: $\frac{2}{15} = \frac{0.7}{x}$. Cross multiplying gives us $2x = 10.5$, which means x must equal 5.25.
3. D. If angle x is 8°, it represents $\frac{8}{360}$, or $\frac{1}{45}$, of the circle, which means minor arc AB represents $\frac{1}{45}$ the circumference of the circle. The circumference must be $1.58 \times 45 = 71.1$. Since circumference $= 2\pi r$, we get $71.1 = 2\pi r$. Solving for r gets us approximately 11.32.
4. C. We are given that segment AD has a length of 3. Using the reference information at the beginning of each math section, we can figure out that side $BD = \frac{3}{\sqrt{3}}$. Side BC is then $\frac{3}{\sqrt{3}} \cdot \sqrt{2}$, which we can then simplify: $\frac{3\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{6}}{3} = \sqrt{6}$.
5. B. If the radius is 12, then the circumference is 24π, or approximately 75.4. If the arc length is between 23 and 24, then it represents a fractional part of the circumference that is between $\frac{23}{75.4}$ and $\frac{24}{75.4}$, or 0.305 and 0.318. Angle x represents the same fractional part of the total number of degrees in a circle (360), which would be greater than 0.305(360) = 109.8 and less than 0.318(360) = 114.48. Therefore, the integer values could be 110, 111, 112, 113, or 114.
6. C. This problem is describing a right triangle. It has given the lengths of two sides and is asking you to find the third side. Using the reference information at the beginning of each math section, we can use the formula $a^2 + b^2 = c^2$, where c is the hypotenuse, to find the missing side: $a^2 + 25^2 = 57^2$. Solving for a results in approximately 51.225, which is nearest to choice C.

7 D. The altitude, radius, and slant height form a 30°– 60°– 90° triangle with hypotenuse 12. Using the reference information at the beginning of each math section, we can figure out that the radius is 6 and the height is $6\sqrt{3}$. We then plug values for r and h into the formula:

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(6)^2(6\sqrt{3}) = 72\sqrt{3}\pi \approx 391.8$ cubic centimeters.

8 C. The reference information at the beginning of each section gives the volume of rectangular solids and rectangular pyramids. The volume of a right pyramid with square base $= \frac{1}{3}lwh$, and the volume a rectangular solid = lwh. Try each answer choice. Choice A has a pyramid with volume 48 and rectangular solid with volume 54. Choice B has a pyramid with volume 12 and rectangular solid with volume 18. Choice D has a pyramid with volume 96 and rectangular solid with volume 72. Choice C is correct: a pyramid and rectangular solid, both with volume 81.

9 A. As given in the reference information at the beginning of each section, the volume of the sphere is $V = \frac{4}{3}\pi r^3$. Plugging in 4 for r results in $V = \frac{4}{3}\pi(4)^3 = \frac{85}{3}\pi$. Since the question is asking for the closest of four values, we can estimate to 268.08. The volume of the cylinder: $V = \pi r^2 h = \pi(12)^2 h = 144\pi h \approx 452.39h$. The only variable left is h for height, which is what we're trying to find. Set the two equations equal to each other and solve: $268.08 = 452.39h$, which results in $h = 0.593$.

Student Produced Response – Calculator

1 320. Since $\angle ABC$ has its vertex on the circle, the angle of subtends an arc twice as large, so arc AC must be 40°. For the same reason, $\angle BAC$ subtends an arc of 160°. The remaining arc $AB = 360° - (40° + 160°) = 160°$. The ratio of the arc length to the circumference of the circle = the ratio of the arc angles 160° to 360°. $\frac{160}{360} = \frac{AB}{720}$. Therefore, arc AB must be 320.

2 12. As given in the reference information at the beginning of each math section, the volume of a sphere is $V = \frac{4}{3}\pi r^3$. Plugging in 288π for V gives us $288\pi = \frac{4}{3}\pi r^3$. The only variable left is r, which we can solve for by dividing both sides by $\frac{4}{3}\pi$, giving us $r^3 = 216$, and then finding the cube root of both sides, giving us $r = 6$. Since the radius is 6, the diameter must be 12.

Guided Practice – Equations of Circles

Multiple Choice – Calculator

1 A. Remember that the equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h,k) is the coordinate center of the circle and r is the radius. This means that the center of the circle must be at (6,–8) and the radius must be the square root of 16, or 4.

2 D. To answer this question, you must remember the equation of a circle, which is $(x - h)^2 + (y - k)^2 = r^2$, where (h,k) is the coordinate center of the circle and r is the radius. Note that the equation negates both h and k. Since the coordinates of the center are (5,–4), the equation must say $(x - 5)^2$ and $(y + 4)^2$. Squaring the radius gives an r^2 value of 64.

3 C. The equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h,k) is the coordinate center of the circle and r is the radius. Since the equation negates both h and k, the equation should say $(x + 3.6)^2 + (y + 13.4)^2$ which, when squaring the radius, should equal 8.

4 B. The equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h,k) is the coordinate center of the circle and r is the radius. Since the equation negates both h and k, the equation should say $(x - \frac{1}{6})^2 + (y - \frac{5}{7})^2$ which, when squaring the radius, should equal $\frac{4}{9}$.

5 D. The equation of a circle in standard form is $(x - h)^2 + (y - k)^2 = r^2$ where the center of the circle is located at (h,k) and r is the radius. Since this circle has its center at (4,2) and a radius of 2, the equation

would be $(x-4)^2+(x-3)^2=4$, which is choice D. Choices A and B are incorrect because the equations equal 2, which is the radius, instead of 4, which is the radius squared. Choice C is incorrect because the coordinates of the center are supposed to be negated.

6 C. We know that the equation of a circle is $(x-h)^2+(y-k)^2=r^2$, where (h,k) is the coordinate center of the circle and r is the radius. Since the two given points have the same y-value, they must lie on a horizontal line and their distance from each other is exactly 12, giving an r^2 value of 144. The equation negates h and k, so the equation must be $(x+8)^2+(y-11)^2=144$.

7 C. The original circle has its center at (0,–3) and radius of 4. If the center of the circle is translated 1 unit up and the radius is increased by 1, then the new center will be at (0,–2) and the radius will be 5, giving us the equation $x^2+(y+2)^2=25$.

8 C. The graph of this equation is a circle with the center (0,0) and a radius 5. If the center is translated 2 units up, the new center of the circle will be (0,2). If the radius is decreased by 1, the radius of the new circle will be 4. Therefore, the equation is $x^2+(y-2)^2=36$.

9 A. The equation for a circle is $(x-h)^2+(y-k)^2=r^2$ in which the center of the circle is at the point (h,k). In this circle, since the x coordinate in the radius is the same as the x coordinate of the endpoint, this is a vertical line, so the radius can be calculated by subtracting the y coordinates: $5-2=3$, so the radius of the circle is 3. Substitute within the equation to get $(x-2)^2+(y-5)^2=9$.

10 A. The equation for a circle is $(x-h)^2+(y-k)^2=r^2$ in which the center of the circle is at the point (h,k). In this circle, the diameter is a horizontal line with a length of 12, meaning the radius has a length of 6. r^2 must equal 36, allowing us to eliminate choices C and D. The center of the circle is halfway between –4 and 8, so the coordinates of the center are (2,–2). Thus the equation: $(x-2)^2+(y+2)^2=36$.

11 D. The equation of a circle in standard form is $(x-h)^2+(y-k)^2=r^2$ where the center of the circle is located at (h,k) and r is the radius. This circle would have a center at (4,5) and a radius of 6. This is only true for choice D.

12 C. The standard form of an equation of a circle is $(x-h)^2+(y-k)^2=r^2$ where the center of the circle is located at (h,k) and r is the radius. The center of this circle is located at (–4,2) and the radius is equal to 7, so the equation is $(x+4)^2+(y-2)^2=49$.

13 D. The graph of this equation is a circle with the center (0, 7) and a radius 3. If the center is translated 2 units to the left, the new center of the circle will be (–2, 7). If the radius is increased by 7, the radius of the new circle will be 10. Therefore, the equation is $(x+2)^2+(y-7)^2=100$.

14 A. The equation for a circle is $(x-h)^2+(y-k)^2=r^2$ in which the center of the circle is at the point (h,k). Since the length of the diameter is 14, the radius must be 7, and r^2 must be 49. We can thus eliminate choices C and D. Choice A has a center at (–1,–6) and choice B has a center of (–6,1). Only choice A has a center that is 7 units away from the given endpoint.

15 A. The given circle has a center at (3,–2) with radius 5. The radius was increased by 3; thus, the radius was 2 prior to the changes, and $r^2=4$, allowing us to eliminate choices B and D. To undo a translation of 3 units to the right, we decrease the value of the x-coordinate by 3. Thus the center was at (0,–2).

Student Produced Response – Calculator

1 5. The radius will be a diagonal line. To find the length of any diagonal line on a coordinate grid, use the distance formula: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(4-7)^2+(0-4)^2}=\sqrt{(-3)^2+(-4)^2}=\sqrt{9+16}=5$.

2 18. The given equation is not in the standard form for a circle, $(x-h)^2+(y-k)^2=r^2$. You can put it in standard form by completing the square. Since the coefficient of x is 4 and the coefficient of y is –12, write the equation in terms of x and y by first adding 4 and 36 to both sides of the equation: $(x^2+4x+4)+(y^2-12y+36)=41+4+36$. Next, rewrite each trinomial as a square, and combine the constants on the other side: $(x+2)^2+(y-6)^2=81$. 81 is the same as 9^2, so the radius of the circle is 9. The diameter of a circle is $d=2r$, so the diameter is 18.

3 20. The given equation is not in the standard form for a circle. You can put it into standard form by completing the square. Since the coefficient of x is –6 and the coefficient of y is –16, you can write the equation in terms of x and y by first adding 9 and 64 to both sides of the equation:

$(x^2-6x+9)+(y^2-16y+64)=27+9+64$. Next, rewrite each trinomial as a square, and combine the constants on the other side: $(x-3)^2+(y-8)^2=100$. 100 is the same as 10^2, so the radius of the circle is 10. The diameter of a circle is $d=2r$, so the diameter is 20.

Guided Practice – Trigonometry

Multiple Choice – No Calculator

1 D. The two acute angles in a right triangle are always complementary (add up to 90 degrees). Therefore, the sine of one is equal to the cosine of the other. Therefore, we get $\sin(m)=\cos(n)=0.8$.

2 A. The two acute angles in a right triangle are always complementary (add up to 90 degrees). Therefore, the sine of one is equal to the cosine of the other. The 75 degree angle is R, so we get $\sin(R)=\cos(T)$

3 C. Whenever we have two acute angles and the sine of one is equal to the cosine of the other, it means that the angles are complementary (add to 90 degrees). Therefore, $a+b=90$ which means that $(k+6)+(3k+4)=90$. Combining like terms, we get $4k+10=90$ and if we solve this for k, the answer is 20.

4 A. The question asks for the length of AD, which is a leg of triangle ADB, so we need to use either angle BAD or angle ABD. The measure of angle BAD is 20 degrees, so the measure of angle ABD must be $90-20=70$ degrees. If we try to find side AD using angle ABD, we must use the identity $\tan(70°)=\frac{AD}{10}$. If we solve this for AD, we get $AD=10\tan(70°)$. (We could try to use angle BAD to find the length of AD, but we would end up with $AD=\frac{10}{\tan(20°)}$ which is not one of the answer choices).

5 B. First look at the answer choices. They all involve either AD or DC, which are legs of right triangles, and we are trying to find BD, which is also a leg. Therefore, we should be using the tangent function (eliminate choice C). Since all the answer choices all involve $x°$, we should be looking at either angle DCB or angle DBA, since those are the only angles measuring $x°$. Using this second angle, we see that $\tan(x°)=\frac{AD}{BD}$. If we solve for line segment BD, we get $BD=\frac{1}{\tan(x°)}AD$. (You can try using angle DCB, but you would end up with $BD=\tan(x°)DC$, which is not an answer choice.)

Student Produced Response – No Calculator

1 30. Based on the given numbers, ABD is a 30-60-90 right triangle with $\angle ACB$ having measure 30 degrees. Because the figure is a rectangle, we know the two triangles are congruent, so and $\angle CAD$ is also 30 degrees.

Multiple Choice – Calculator

1 C. Because the triangles are similar, angles A and D are congruent, so they have the same sine.

2 D. The question gives us the length of CB and asks for the length of AC. In relation to $\angle BCA$, the segments CB and AC are the adjacent leg and hypotenuse, respectively. This suggests that we should use cosine. We get $\cos(65°)=\frac{3}{AC}$, which we can solve for AC to get $AC=\frac{3}{\cos(65°)}$. If we plug in the given cosine value, we get 7.14.

3 C. This is a 45-45-90 right triangle (45-45-90 right triangles have legs of length x and a hypotenuse with length $x\sqrt{2}$). Therefore, $\angle PRQ$ has measure 45°. The drawing is clearly not of a 45-45-90 triangle, which is why it is important to read the note that the triangle is not drawn to scale. If you didn't remember the rule for 45-45-90 right triangles, you could use the formula $\cos(PRQ)=\frac{3}{3\sqrt{2}}$. By taking the inverse cosine of the right side of the equation, you would get the measure of $\angle PRQ$.

4 B. The sine of x is equal to the sine of angle BCA, which is equal to $\frac{3}{AC}$. To calculate the length of AC, use the Pythagorean Theorem on triangle ABC. $3^2 + 4^2 = AC^2$. It follows that $AC = 5$. Therefore, the sine is $\frac{3}{5}$, which is between 0.5 and 0.7.

Student Produced Response – Calculator

1 4/5 or 0.8. The cosine of x is $\frac{20}{25}$, which simplifies to $\frac{4}{5}$ or 0.8.

2 $\frac{5}{13}$ or 0.38. Since the opposite side equals 12 m., the adjacent side equals 5 m.

$\text{Cos } A = \frac{adjacent}{hypotenuse} = \frac{5}{13}$.

3 38. The length of EG is equal to $EH + HG$. First figure out EH: Triangle EHF is a 30-60-90 right triangle, which means that $EH = \sqrt{3}FH \approx 1.73 \cdot 14 = 24.22$. Next, figure out HG: Triangle FHG is a 45-45-90 right triangle, so $HG = FH = 14$. Finally, we add together $EH + HG = 24.22 + 14 = 38.22$, which is just 38 when rounded to the nearest inch.

4 12/5 or 2.4. The tangent of x is equal to $\frac{YZ}{XY}$. We already know YZ and we can calculate XY using the Pythagorean Theorem. $XY^2 + 12^2 = 13^2$. Solving this for XY, we get $XY = 5$. Therefore, the tangent of x is $\frac{YZ}{XY} = \frac{12}{5}$.

Guided Practice – Radians

Multiple Choice – No Calculator

1 B. Because the x and y coordinates of point A are the same, the segment AO is halfway between the x-axis and y-axis. Since B is on the y-axis, this creates a 45 degree angle, which is $\frac{\pi}{4}$ radians (to convert from degrees to radians, multiply by $\frac{\pi}{180}$).

2 A. The sine and cosine of an angle are equal when the angle is 45 degrees or $\frac{\pi}{4}$ (to convert from degrees to radians, multiply by $\frac{\pi}{180}$). To see why it is 45 degrees, think about a right triangle. If the sine and cosine of an angle are equal, that means that the opposite side and adjacent side are equal. If the two legs of a right triangle are equal, then its angles are 45-45-90.

3 C. When you have two complementary angles, the sine of one is equal to the cosine of the other. This statement can be expressed in all of the following equivalent ways:

$\sin(x) = \cos(90 - x), \cos(x) = \sin(90 - x), \sin(x) = \cos\left(\frac{\pi}{2} - x\right), \cos(x) = \sin\left(\frac{\pi}{2} - x\right)$.

4 A. This is a 30-60-90 right triangle because the hypotenuse is twice the shorter leg. It is worth remembering (and it also is provided in the reference information at the beginning of each math section) that a 30-60-90 right triangle has sides with lengths $x, \sqrt{3}x, 2x$ for some number x. Since $\angle A$ is the smallest angle, it is 30 degrees or $\frac{\pi}{6}$ radians (to convert from degrees to radians, multiply by $\frac{\pi}{180}$).

5 C. Points A and B are each equidistant between the x-axis and y-axis, so they each make and angle of 45 degrees with the x-axis. Adding these together, we get 90 degrees or $\frac{\pi}{2}$ radians (to convert from degrees to radians, multiply by $\frac{\pi}{180}$).

6 D. The formula for converting degrees into radians is *degree measure* $\frac{\pi}{180}$. Therefore, $210° \cdot \frac{\pi}{180°} = \frac{210\pi}{180} = \frac{7\pi}{6}$. You can also remember that one full circle, or 360°, is equivalent to 2π radians and that 180° is therefore equal to π radians. A 210° measure is slightly larger than 180°, so the radian measure would be slightly larger than π. Choice D is the only answer with a value larger than π.

Student Produced Response – No Calculator

1 4. Because the x and y coordinates of point B are the same, the segment BO is halfway between the x-axis and y-axis. Since A is on the x-axis, this creates a 45 degree angle, which is $\frac{\pi}{4}$ radians(to convert from degrees to radians, multiply by $\frac{\pi}{180}$).

2 7/16 or 0.44. The question asks, "the area of the sector… is what fraction of the area of the circle?" However, we would get the same ratio if we asked, "the central angle is what fraction of the full circle?" The central angle is $\frac{7\pi}{8}$ out of a total of 2π. This becomes the fraction $\frac{\frac{7\pi}{8}}{2\pi} = \frac{7}{16}$.

Multiple Choice – Calculator

1 B. The angles shown in the figure are supplementary, so they must add up to 180 degrees or π radians. In other words, $\angle BOA + \angle COB = \pi$. This becomes $\frac{\pi}{6} + COB = \pi$. Solving this, we get $\angle COB = \frac{5\pi}{6}$.

2 B. The coordinates of B could represent the legs of a 30-60-90 right triangle whose hypotenuse is OB. This means that the angle between OB and the negative x-axis is 60 degrees or $\frac{\pi}{3}$ radians (to convert from degrees to radians, multiply by $\frac{\pi}{180}$). That angle is supplementary to $\angle AOB$, so we must subtract it from π. We get $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

3 C. The coordinates of A could represent the legs of a 30-60-90 right triangle whose hypotenuse is OA. This means that the angle between OA and the y-axis is 30 degrees or $\frac{\pi}{6}$ radians (to convert from degrees to radians, multiply by $\frac{\pi}{180}$). Similarly, OB makes an angle of $\frac{\pi}{6}$ radians to the other side of the y-axis. Together, they make $\frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$.

4 D. The sector for pizza is 45% of the circle so that is $\frac{45}{100} \cdot 2\pi$ which simplifies to $\frac{9\pi}{10}$. If we remember that a full circle is 2π and half a circle is π, then we can see that we are looking at an angle that is a little less than π. Choices A, B, and C are all far too small.

5 A. We can represent the measures of angles A and B as $2x$ and x respectively. Since they are supplementary, they must add up to 180 degrees or π radians. We get the equation $2x + x = \pi$, which simplifies to $3x = \pi$. Dividing by 3, we get $x = \frac{\pi}{3}$. This is the measure of angle B.

Student Produced Response – Calculator

1 3/10 or 0.3. The question asks, "the length of the arc… is what fraction of the circumference of the circle?" However, we would get the same ratio if we asked, "the central angle is what fraction of the full circle?" The central angle is $\frac{3\pi}{5}$ out of a total of 2π. This becomes the fraction $\frac{\frac{3\pi}{5}}{2\pi} = \frac{3}{10}$.

Guided Practice – Imaginary & Complex Numbers

Multiple Choice – No Calculator

1 D. Since $i = \sqrt{-1}$, we can determine that $i^2 = \sqrt{-1}^2 = -1$. Therefore, $i^2 + i^2 = (-1) + (-1) = -2$.

2 B. Complex numbers are added by combining the real parts and the imaginary parts separately. Essentially, just combine like terms: $(3 + 4i) + (7 + 6i) = (3 + 7) + (4i + 6i) = 10 + 10i$.

3 C. Complex numbers are subtracted by combining the real parts and the imaginary parts separately and distributing the negative through the second set of parentheses.
$(4 + 2i) - (7 - 2i) = (4 - 7) + (2i - (-2i)) = -3 + 4i$.

4 B. Multiply by distributing: $-6i(3 - 2i) = -18i + 12i^2$. Since $i^2 = -1$, we can determine that $-18i + 12i^2 = -18i + 12(-1) = -18i - 12$, or in complex number form, $-12 - 18i$.

5 B. Multiply by distributing and recognizing that $i^2 = -1$. $12i(4 + 3i) = 48i + 36i^2 = -36 + 48i$.

6 C. Dividing a complex number by a real number is done by dividing each part of the complex number by the real number: $\frac{12 + 8i}{2} = \frac{12}{2} + \frac{8i}{2} = 6 + 4i$.

7 D. Since $i = \sqrt{-1}$, we can determine that $i^2 = \sqrt{-1}^2 = -1$, and that $i^4 = (i^2)(i^2) = 1$.
Therefore, $5i^4 - 5i^2 - 5 = 5(1) - 5(-1) - 5 = 5$.

8 A. Complex numbers are subtracted by combining the real parts and the imaginary parts separately and distributing the negative through the second set of parentheses.
Here, $(-5i^2) - (-3 - 7i^2) = -5i^2 - (-7i^2) - (-3) = -5(-1) - (7) + 3 = 1$.

9 D. Multiply by distributing: $5i^3(4 - 2i^2) = 5i^3(4) - 5i^3(2i^2) = 20i^3 - 10i^5 = -20i - 10i = -30i$.

10 B. Distributing $8i$ through the parentheses yields $56i - 24i^2 = 24 + 56i$ after substituting $i^2 = -1$.

11 A. Multiply by distributing: $(4 - i)(4 - i) = 16 - 8i + i^2$. Since $i^2 = -1$, the equation simplifies to $15 - 8i$.

12 A. To multiply two binomials, distribute (i.e. FOIL): $(5 - 5i)(4 + 4i) = 20 + 20i - 20i - 20i^2 = 20 - 20i^2$. Since $i^2 = -1$, we arrive at $20 - 20(-1) = 20 + 20 = 40$.

13 C. Since $i^1 = i$, we can determine that $i^2 = -1$, $i^3 = -i$, and $i^4 = -i^2 = -(-1) = 1$ Every fourth power of i is equal to 1, so $i^{32} = (i^4)^8 = 1$.

14 B. Since every fourth power of $i = 1$, we can determine that $i^{47} = i^{44} \times i^3 = 1 \times i^3$. Since $i^3 = -i$, $1 \times i^3 = -i$.

15 D. Multiplication of binomials is done by distributing: $(4 + 2i)(3 - 7i) = 12 - 28i + 6i - 14i^2$. This equals $12 - 22i - 14i^2$. Since $i^2 = -1$, we arrive at $12 - 22i - 14(-1) = 26 - 22i$.

16 D. Whenever we FOIL a binomial by its conjugate, the two middle terms cancel each other out and we're left with the difference of squares. When we multiply a complex number by its complex conjugate, we always arrive at a real number answer: $(8 - 3i)(8 + 3i) = 64 + 24i - 24i - 9i^2 = 64 - 9i^2 = 64 + 9 = 73$.

17 D. Multiplying the two binomials written in reverse order yields $(i + 1)(i - 1) = i^2 - i + i - 1 = -1 - 1 = -2$.

18 D. Denominators must always be rational numbers, so answers may not be left with an imaginary number in the denominator. Multiply both numerator and denominator by i: $\left(\frac{7}{2i}\right)\left(\frac{i}{i}\right)=\left(\frac{7i}{2i^2}\right)=\frac{7i}{2(-1)}=\frac{-7i}{2}$.

19 B. Denominators must always be rational numbers, so answers may not be left with an imaginary number in the denominator. Simplify by multiplying the denominator and numerator by the complex conjugate of the denominator: $\left(\frac{9i}{1+8i}\right)\left(\frac{1-8i}{1-8i}\right)=\frac{9i-72i^2}{1-64i^2}=\frac{9i-72(-1)}{1-64(-1)}=\frac{72+9i}{65}$.

20 A. Denominators must always be rational numbers, so answers may not be left with an imaginary number in the denominator. Multiply both the numerator and denominator by i:

$\left(\frac{9-4i}{-5i}\right)\left(\frac{i}{i}\right)=\frac{9i-4i^2}{-5i^2}=\frac{9i-4(-1)}{-5(-1)}=\frac{9i+4}{5}$.

21 A. Since $i=\sqrt{-1}$, we can determine that $i^2=\sqrt{-1}^2=-1$, that $i^3=(i^2)(i)=-i$, and that $i^4=(i^2)(i^2)=1$. Therefore, $i+i^2+i^3+i^4=i+(-1)+(-i)+1=0$.

22 B. Simplify by multiplying the denominator and numerator by the complex conjugate of the denominator: $\left(\frac{4}{2+3i}\right)\left(\frac{2-3i}{2-3i}\right)=\frac{8-12i}{4-9i^2}=\frac{8-12i}{4-9(-1)}=\frac{8-12i}{13}$.

23 A. To rewrite in $a+bi$ form, we must rationalize the denominator. Multiply the numerator and denominator by the complex conjugate of the denominator: $\frac{(8-3i)}{(2+i)}\cdot\frac{(2-i)}{(2-i)}=\frac{16-8i-6i+3i^2}{4-i^2}=$

$\frac{16-8i-6i+3(-1)}{4-(-1)}=\frac{13-14i}{5}=\frac{13}{5}-\frac{14}{5}i$. Therefore, $a=\frac{13}{5}$.

24 A. To add fractions, we need a common denominator, so we'll multiply the numerator and denominator of each fraction by the denominator of the other fraction:

$\frac{5}{5+i}+\frac{i}{5-i}=\left(\frac{5}{5+i}\right)\left(\frac{5-i}{5-i}\right)+\left(\frac{i}{5-i}\right)\left(\frac{5+i}{5+i}\right)=\frac{25-5i}{25-5i+5i-i^2}+\frac{5i+i^2}{25-5i+5i-i^2}$. Since the denominators are now the same, we can add the fractions: $\frac{(25-5i)+(5i+i^2)}{25-5i+5i-i^2}=\frac{25+i^2}{25-i^2}$. Since we know that $i^2=-1$, we can arrive at $\frac{25+(-1)}{25-(-1)}=\frac{24}{26}=\frac{12}{13}$.

Student Produced Response – No Calculator

1 9. Complex numbers are added by combining the real parts and the imaginary parts separately. $(5+4)+(6-6)i=9+0i=9$.

2 15. Since $i=\sqrt{-1}$, we can determine that $i^2=-1$ and $i^4=1$. Therefore, $i^4-6i^2+9=1-6(-1)+9=1+6+9=15$.

3 41. Expand the expression using FOIL: $(4+5i)(4-5i)=16-20i+20i-25i^2=16-25i^2$. Since we know that $i^2=-1$, we can determine that $16-25i^2=16-25(-1)=41$.

4 1/5 or 0.2. Denominators must always be rational numbers, so answers may not be left with an imaginary number in the denominator. Multiply both the numerator and denominator by i:

$\left(\frac{i^3}{-5i}\right)\left(\frac{i}{i}\right)=\frac{i^4}{-5i^2}=\frac{1}{-5(-1)}=\frac{1}{5}$ or the decimal equivalent of 0.2. Alternatively, $\frac{i^3}{-5i}=\frac{i^2}{-5}=\frac{-1}{-5}=\frac{1}{5}$.

5 68. Expand the expression using FOIL: $(-2-8i)(-2+8i)=4-16i+16i-64i^2=4-64i^2$. Since we know that $i^2=-1$, we can determine that $4-64i^2=4-64(-1)=4+64=68$.

6 1. One way to solve this is to use rules of exponents: $\frac{i^7}{-i}=-i^{7-1}=-i^6$. Since $i^6 = i^2 = -1$, we can determine that $i^6 = -(-1) = 1$. Another way to solve is to divide by multiplying both the denominator and numerator by the complex conjugate of the denominator: $\left(\frac{i^7}{-i}\right)\left(\frac{i}{i}\right)=\frac{i^8}{-i^2}=\frac{(i^4)^2}{-(-1)}=\frac{1}{1}=1$.

7 6/5 or 1.2. To add fractions, we need a common denominator, so we'll multiply the numerator and denominator of each fraction by the denominator of the other fraction:
$\frac{2-i}{2+i}+\frac{2+i}{2-i}=\left(\frac{2-i}{2+i}\right)\left(\frac{2-i}{2-i}\right)+\left(\frac{2+i}{2-i}\right)\left(\frac{2+i}{2+i}\right)=\frac{4-2i-2i+i^2}{4-2i+2i-i^2}+\frac{4+2i+2i+i^2}{4+2i-2i-i^2}$. Since the denominators are now the same, we can add the fractions: $\frac{(4-2i-2i+i^2)+(4+2i+2i+i^2)}{4-2i+2i-i^2}=\frac{8+2i^2}{4-i^2}$. Since we know that $i^2=-1$, we can arrive at $\frac{8+2(-1)}{4-(-1)}=\frac{8-2}{4+1}=\frac{6}{5}$ or the decimal equivalent of 1.2.

Mixed Practice – Additional Math Topics

Multiple Choice – No Calculator

1 Trigonometry: B Whenever we have two acute angles and the sine of one is equal to the cosine of the other, it means that the angles are complementary (add to 90 degrees). If angles A and C add to 90 degrees, then the remaining angle, B, must also be 90 degrees in order for the triangle to have the required angle sum of 180.

2 Imaginary & Complex Numbers: D. Expand the expression as multiplication of two binomials: $(12+2i)^2 = (12+2i)(12+2i)=144+24i+24i+4i^2=144+48i-4=140+48i$.

3 Equations of Circles: C. The equation for a circle is $(x-h)^2+(y-k)^2=r^2$ in which the center of the circle is at the point (h,k). You can use the distance formula to find the radius, or the distance between (2,-4) and (8,–4): $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(8-2)^2+(-4-4)^2}=\sqrt{6^2+(-8)^2}=\sqrt{100}=10$. Therefore, the radius is 10. Plugging these back into the formula yields $(x-2)^2+(y-4)^2=100$.

4 Radians: D. The coordinates of point B, (–2,2), indicate that segment OB bisects the 4th quadrant, forming an angle of 45 degrees or $\frac{\pi}{4}$ with the positive y-axis and with the negative x-axis (to convert from degrees to radians, multiply by $\frac{\pi}{180}$). If we start at A, on the positive x-axis, we must travel counterclockwise by $\frac{\pi}{2}$ to get from the x-axis to the y-axis and then an additional $\frac{\pi}{4}$ to get from the y-axis to point B.

5 Imaginary & Complex Numbers: B. To rewrite in $a + bi$ form, we must rationalize the denominator. Multiply the numerator and denominator by the complex conjugate of the denominator:
$\frac{(10+40i)}{(-3+5i)}\cdot\frac{(-3-5i)}{(-3-5i)}=\frac{-30-50i-120i-200i^2}{9-25i^2}=\frac{-30-170i-200(-1)}{9-25(-1)}=\frac{170-170i}{34}=5-5i.$
Therefore, $b = -5$.

6 Equations of Circles: D. The equation of a circle in standard form is $(x-h)^2+(y-k)^2=r^2$ where the center of the circle is located at (h,k) and r is the radius. So this circle would have a center at (4,2) and a radius of 7. This is only true for choice D.

part of banks to pay back deposits. The other answer choices support incorrect explanations or pertain to actions after the crisis, not leading up to it.

39 *Word Choice.* D. Roosevelt's choice words – calling the closure of banks a "bank holiday" – not only makes it clear what is happening (i.e. the banks are closed), but also makes it clear that this closure is just temporary (as holidays typically are) and not permanent. The words do not ridicule the seriousness of the crisis, nor do they imply that the public should go on holiday. Roosevelt praises Congress for their speedy action, not their lack of it.

40 *Close Reading.* D. Roosevelt mentions that Congress worked with him to pass legislation and grant him necessary authority to act (lines 62-76). The passage does not suggest that Congress acted slowly or refused to act altogether.

41 *Relationships.* A. Roosevelt mentions that "legislation promptly" passed by Congress "gave authority to develop a program of rehabilitation." The passage does not mention a decree, vote, or referendum.

42 *Arguments.* D. Will Hunting's achievements surprise the math community; so too do Zhang's. The author does not show how both fiction and reality differ from the other, but how, in this case, the two are similar.

43 *Summarizing.* A. The author provides mathematical background knowledge in these paragraphs to ensure that the reader can understand later paragraphs discussing Zhang's accomplishments. Such concepts include the notion of primes, prime pairs, prime gaps, and twin primes. The author builds on these concepts and raises the question of prime gaps. The anecdote about Zhang's discovery is told in context of the concepts and question, but not until later in the passage. The author does not make comparisons with other concepts or discus answers (instead saying that the question "remains unanswered").

44 *Words/Phrases in Context.* B. The author suggests that without a rule or proof, it will be impossible to know how many twin primes there are. The key here is that mathematics seek to know, not simply to guess or approximate. Thus, "estimate" is inappropriate. The author does not suggest that solving the twin prime conjecture is difficult to understand. Instead, the very act of solving it – of ascertaining the solution – is difficult.

45 *Text Structure.* D. The author uses the first four paragraphs to describe information necessary to the main point of the story that follows: that there are real life examples of "miraculous events"; that in real life, the example centers around a difficult question in math. The story that follows, about Zhang, is really only understandable in context of the first four paragraphs. Otherwise, the author's point would be lost.

46 *Word Choice.* A. Zhang's story is interesting, like Hunting's, because of the otherwise humble origins of the protagonist. Part of the reason why both stories are impressive and interesting is because a significant accomplishment came from an unexpected place. By highlight the fact that Zhang worked as an accountant and in fast food, the author indirectly builds up Zhang's accomplishments, which are described later in the passage.

47 *Purpose.* C. The GPY paper intrigued Zhang. The paper mentions that Zhang modified a process described in the GPY paper, which ultimately led to Zhang's discovery (lines 59-63). The passage does not suggest that the GPY disproved Zhang's theory, or that it was useless to Zhang.

48 *Relationships.* B. The author writes that it was on a "break from his [Zhang's] work" that an "idea suddenly hit him." The passage does not suggest that Zhang's discovery was spurred on by a conversation or another unknown paper. In fact, the passage suggests that Zhang worked alone. The GPY paper was the basis for Zhang's work, but until his time away from work, he was unable to use the GPY methodology to solve the problem.

49 *Textual Evidence.* C. From the previous question, we know that Zhang's breakthrough was catalyzed by his vacation, or break from work. This is encapsulated in choice C. The other answer choices suggest that incorrect answers to the previous question are, in fact, correct.

50 *Close Reading.* C. The author indicates that after *Annals of Mathematics* published his work, Harvard reached out to him for a speaking engagement. The passage does not state that the speaking engagement was organized while the work was still in draft, or that it was first tested by the entire math community. Zhang earned his doctorate many years before the events of his discovery.

51 *Point of View.* B. The passage repeated refers to how the mathematics community was shocked by Zhang's accomplishments, how there was "great fanfare" after his publication, and how his work would "shake the theoretical mathematics community to its core." These reactions are positive; "esteem" is the only word with a positive connotation.

52 *Quantitative Information.* D. The graph shows prime gaps and the number of pairs forming those gaps for prime numbers less than 100. Zhang's theory deals with infinity, and does not suggest that analyzing prime numbers less than 100 will yield a proof or theory that can be applied to the infinite.

Writing & Language

1 *Style/Tone.* C. The preceding sentence begins with an independent clause containing the quality that answers the first sentence's question. That clause is followed by a dependent clause, which describes more about the independent

clause (i.e. intelligence is demonstrated by capacity to learn new things). The only choice that matches this style is C, which introduces an independent clause containing the quality answering the first sentence's question (curiosity) and is followed by a dependent clause explaining why this is the case (i.e. many scientists' lives).

2 *Frequently Confused Words.* B. The sentence compares two groups of people: scientists and gamblers. However, the best choice is to use the word "to" as a preposition, which identifies the thing being compared (gambler with scientist). "Too" is an adverb and would be incorrect to use as a preposition.

3 *Conventional Expression.* D. The sentence describes how scientists sometimes make discoveries by accident and offers radioactivity as an unknown concept that Becquerel unwittingly discovered. The best expression to use here is "after all," which means here "besides"; how could Becquerel have had an intention of discovering something that was unknown?

4 *Focus.* A. The paragraph – and passage in general – is concerned with explaining how some scientific discoveries are accidents. The facts that Becquerel's discovery was very important and that the unit of measurement of radiation is named after Becquerel are interesting, but blur the focus of the paragraph and do not support the paragraph's main idea.

5 *Precision.* D. From context, we can infer that the sentence describes Becquerel's early experiences in science – the things that intrigued Becquerel and the work that he was involved with (light and phosphorescence). A "foray" describes an attempt to become involved in something new, but can also refer to an attack or incursion into enemy territory. The latter does not make sense in context, nor does it make sense to use "excursion" or "expedition," which refer more to actual, physical trips.

6 *Shifting Tenses, Mood, Voice, and Number.* B. The entire paragraph is written in the simple past tense, describing Becquerel's work. It is inappropriate to shift the tense to the simple present (finds), future present (will find), and past perfect progressive (had been finding).

7 *Logical Sequence.* D. The paragraph describes the steps Becquerel took to conduct his experiment. Sentence 1 describes the repeating of these steps, but refers to things like "paper" and "phosphorescent substance" and "impression on the film." All of these elements are described only after sentence 5, so it would not be logical to include this sentence before these elements are explained.

8 *Syntax.* B. Because "days" is the topic of discussion in both sentences, it is possible to combine both sentences together. By doing so, the combined sentence would read "very cloudy days that did not provide…" This is more fluid and less choppy than separating the sentences with a period or semicolon, as the second sentence describes the impact of the days on Becquerel's experiment – the subject of the first sentence. Using "which" would only be appropriate if preceded by a comma.

9 *Modifier Placement.* C. The sentence describes Becquerel's surprise when he expected to find a weak image on the film but instead found a clear image on the film. The adverb "clearly" should be used to modify the appearance of the image. As written, the modifier is squinting and ambiguous. The other choices emphasize the wrong part of the sentence, changing the meaning entirely.

10 *Quantitative Information.* B. The graph shows that despite changing intensity of ambient light, the intensity of the image on photographic film remained relatively constant. This does not show any correlation or dependency of the image on the ambient light levels.

11 *Unnecessary Punctuation.* A. No commas are necessary in this sentence. The sentence is a single, independent clause and does not contain clauses, phrases, or appositives that would require the use of clarifying commas.

12 *Coordination.* B. The first two sentences are related in that they refer to the reasons why choosing a career can be confusing. However, because they are related, a coordinating conjunction that logically suggests the opposite – such as although – should not be used. Such conjunctions contradict the logic of the two sentences, especially the "after all" at the end of the second sentence, which provides a clue that the two sentences are logically related.

13 *Pronoun Clarity.* B. As written, "them" is unclear (a job that is too easy for whom?) and also disagrees in number to the noun "job" (which is singular). The only correct choice to ensure that the pronoun is both clear and agrees in number with the noun "job" is "a person." "Him," though singular, refers to an unknown entity, and "people" is plural.

14 *Precision.* D. Though the patients are technically the customers of RNs, the paragraph here discusses how RNs care for other people – patients and their families. The RNs "provide emotional support" to those that they care for – their wards. They are custodians of other people, not the other way around. The passage does not suggest that the RNs work with their friends.

15 *Support.* B. The sentence develops the point that RNs can also work in uncommon places in addition to places one would expect nurses to work. The only example here that supports this point is an insurance company. The other

answer choices provide examples of health-related locations where nurses can reasonably be expected to be employed.

16 *Logical Comparison.* B. As written, the sentence compares the "high rate of growth" of the RN occupation directly with "other occupations." Instead, the sentence should compare the rate of growth of the RN occupation with the rate of growth of other occupations. The only choice that accomplishes this goal is choice B.

17 *Possessive Nouns & Pronouns.* C. The sentence describes how the world population will continue to grow. In this sentence, the population is "belonging" to the world, not multiple worlds. Without an apostrophe, there is no possession indicated. "Worlds' population" suggests that the population of multiple worlds are being discussed. "World population's" suggests that the entire population of the world possesses something.

18 *Concision.* C. The sentence suggests that a growing population will result in higher demand for RNs, which is a different way of saying that people will need more RNs. To say that the demand is going to increase along with the number of people who need RNs is redundant, since they describe the same thing.

19 *Syntax.* A. The sentence is made up of two independent clauses separated by a semicolon because the two clauses discuss related topics: the RN satisfaction. The first clause makes a claim, and the second clause supports it. Using the word "however" would signal that the two clauses make opposing statements about the related topic, which is not the case here. Using a comma results in a run-on sentence. Using "and" is incorrect in this context because the two clauses remain independent.

20 *Organization.* A. The paragraph describes the requirements to becoming an RN and the steps one would need to take to do so. Preceding paragraphs discuss why people might consider becoming an RN and who might be interested in becoming an RN. The passage does not describe when it would be appropriate to become an RN.

21 *Items in a Series.* C. The two items in the series denoted by the preceding colon relate to two options to become an RN. Because there are only two options, and because the use of commas does not confuse the sentence, the comma is the most appropriate punctuation. Using a period or semicolon results in a fragment of the second option (diploma from nursing program), and the use of a dash is not appropriate. A comma (and the use of the conjunction "or") sufficiently separates the two options.

22 *Proposition.* D. The passage is primarily concerned with explaining both the pros and cons of becoming an RN. The final paragraph discusses the steps needed to become an RN, which involves education and licensure. The author does not seek to dissuade people from becoming RNs; rather, the author seeks to have the reader make up his or her own mind by evaluating information presented in the passage.

23 *Possessive Determiners.* D. Without an apostrophe, the word "its" is possessive. With the apostrophe, the word "it's" is a contraction for "it is." There is no such word as "It'" or "its'."

24 *Focus.* C. The paragraph raises a number of questions in order to demonstrate the wide breadth and depth of the field of linguistics, which is the subject of the entire passage. The question about a dog is relevant to the paragraph and passage, as it relates to the topic of morphology discussed in the fourth paragraph. The sentence does not make contradictions or make opposing arguments.

25 *Subject-verb Agreement.* C. The subject in this sentence is "question" which is a plural noun. Thus, the verb "are" – the plural form of "be/is" – is most appropriate.

26 *End-of-Sentence Punctuation.* A. When punctuating the end of a quotation, ending punctuation generally goes inside the quotation mark. However, placement of question marks and exclamation points depend on what is being quoted. In neither case is "double punctuation" used (as in choices C and D). The difference between choices A and B is subtle; the former implies that the question itself is being quoted, where the latter implies that a quoted statement is being questioned.

27 *Concision.* B. While the sentence as written and choice B are grammatically correct, "nuanced" is a more economical way of saying "subtly different" without changing the meaning of the sentence. Using words precisely and economically is preferable to using more words that accomplish the same goal. Similarly, choice C means something similar, but uses many words – some of which are confusing since they are repeated later in the sentence.

28 *Precision.* B. The sentence uses the word "though" to contrast the field of linguistics with the fact that it can be thought of more simply. Thus, the best word here is to use "broad," since a topic that is broad is one that covers many things and can be difficult to think of. It would not be appropriate to contrast a "narrow" or "inexact" field with the fact that it is possible to think of that field in two ways. Similarly, the author does not suggest that the two ways of thinking about linguistics are duplicative – in fact, the author argues that they are different.

29 *Parallel Structure.* C. The order of words here is important, though many choices are technically grammatically correct. Stylistically, the best way to ensure parallelism in writing is to ensure consistent structure. Only the second choice mirrors the earlier part of this series: "the first has to do with" and "the second has to do with."

30 *Style/Tone.* A. The second sentence in the paragraph and the last sentence in the paragraph are constructed in such a way that the different fields of study (morphology, syntax, phonology, etc) are introduced first, defined, then explained in more detail. Only choice D follows this pattern, with the other choices offering a different (but not grammatically incorrect) sequence.

31 *Proposition.* D. The paragraph enumerates different parts of the linguistics field, and how those fields relate to language itself. By first enumerating these different parts, then ending the paragraph by saying "the study of language is divided into several disciplines," the only word that makes sense is "thus" – "nevertheless" and "instead" imply that this statement is the opposite of what was shown in the paragraph. The author does not suggest that the study of language is straightforward.

32 *Organization.* A. The preceding paragraph discusses the study of the structure of the language, which is simply different from the study of language uses. The author had previously, in the third paragraph, explained that the field of linguistics is thought of in two parts that deal with different aspects of language. The two parts are different; it would be incorrect to use the words "similar" or "despite" or "in spite of" to signal the difference, as they imply more of a contrast.

33 *Parenthetical Expressions & Nonrestrictive Clauses.* B. By using a comma after "factors" followed by "which," the meaning of the sentence changes to suggest that it is the study of psycholinguistics itself that develops linguistic skills. This is contrary to the meaning of the sentence, which is to define psycholinguistics and explain what the field does. The best way to accomplish this is to omit the comma, and use "that" or "which." However, the tense of "allow" matters, as "the psychological and neurological factors" are treated as singular, requiring the use of the word "allow" instead of "allows."

34 *Parenthetical Expressions & Nonrestrictive Clauses.* D. Using a phrase that begins with "which" or "that" between two commas is a sign of a nonrestrictive clause – something that can be removed from the sentence without changing the basic meaning. In this case, both choices that suggest creating a nonrestrictive clause are incorrect, as taking the clause out would fundamentally alter the meaning of the sentence. Choice B results in a grammatically incorrect sentence, splitting "which" into a parenthetical expression.

35 *Noun Agreement.* B. The sentence provides examples of different administrators, as it speaks generally about all administrators, not a specific administrator of a specific government. Thus, the number of the noun "administrator" must match the number of the nouns in "the prime...statesmen." To do so, the noun must be plural, but not possessive, as in choices C and D.

36 *Support.* A. The paragraph is concerned with explaining how a government by rule of law differs from a government based on different forms of power. The author attempts to clarify the differences with examples and by how each type of government works. The sentence would improve the clarification of these differences. The sentence does not support the notion that power is greater than rule of law, and does in fact flow logically from the preceding sentence.

37 *Focus.* A. The primary focus of the paragraph is to describe how many ancient societies were organized and to provide an example of such organizations. The paragraph does not concern itself with describing the histories of these societies. Therefore, the sentence is irrelevant, blurs the focus of the paragraph, and should be deleted.

38 *Concision.* B. The author implies that societies governed by legal principles are more stable, unlike some regimes that can do whatever it is they want. The author writes "changing whims and feelings" is embodied by "caprices," which means "sudden and unaccountable changes in mood or behavior." This is a more succinct way to convey this point than the other answer choices.

39 *Complete Sentences.* D. As written "often...law" is a fragment, with no subject. Though choice B corrects this, it ignores the rest of the sentence which becomes illogical in the absence of further edits. A semicolon effectively serves the same purpose of a period in this case, and strands "often...law" as a fragmented dependent clause.

40 *Precision.* C. The author intends to show how Aristotle supported the rule of law, stating that he was "fervent" and an "advocate." The author does not provide evidence supporting the notion that Aristotle was flexible on the matter, as "pliant" would suggest; nor does the author suggest that Aristotle was "authoritarian" in his support. Rather, the author suggests that Aristotle was a strong supporter of the rule of law, as "outspoken" suggests.

41 *Noun Agreement.* A. The core of the sentence is "rumblings surfaced" ("similar" modifies "rumblings," and "in ancient China" modifies "surfaced"). However, "rumblings" is also modified by the rest of the sentence "form of...thought." In this sentence, "rumblings" is treated like a unit of measurement to express the quantity and proliferation of the rule of law (how much "rule of law" is in the ancient world? "Rumblings of it.") As such, the noun (though plural) is treated as singular for purposes of matching noun number. It is unnecessary to use "forms" or "schools." "Thought," in any case, should not be pluralized, since it is part of "school of thought."

42 *Style/Tone.* D. The author previously likens the rule of law to a seed being sown, but also indicates that for a long time there were no practicing societies that followed the rule of law (paragraph four). In paragraph five, the author

describes how the Magna Carta served as a framework allowing powerful countries to develop around the rule of law. The comparison to a seed helps to draw the two paragraphs together and is a metaphor for the role the Magna Carta played in terms of developing the rule of law; a seed takes root after it is sown/planted.

43 *Within-Sentence Punctuation.* D. No commas are necessary in this sentence. All of the information in the sentence is important to the meaning of the sentence (i.e. a contrast between what was once obscure becoming prevalent) so there is no parenthetical expression here (so no bookended commas are needed). The sentence does not have any subordinated clauses that would require the use of a single comma.

44 *Logical Sequence.* B. The paragraph serves to introduce the notion of the rule of law and belongs at the beginning of the passage. Paragraphs 2 and 4 discuss historical societies and their forms of government; paragraph 3 interrupts this flow, as it would if placed before paragraph 5. Paragraph 1 expands upon the idea of the rule of law, and would benefit if there was an introduction before it – in this case, paragraph 3.

Math – No Calculator

1 Solving Linear Equations & Inequalities: D. $w = \frac{5}{4} \div \frac{2}{3} = \frac{5}{4} \times \frac{3}{2} = \frac{15}{8}$.

2 Relationships Between Algebraic & Graphical Representations of Functions: A. This parabola intersects the x-axis at –2 and 4. So the x-coordinate of the vertex of the parabola is halfway between these points: $\frac{-2+4}{2} = 1$. So this is the value of c. To find the y coordinate of the vertex, substitute 1 for x in the equation: $y = a(1+2)(1-4) = -9a$.

3 Linear Equations, Inequalities, & Systems in Word Problems: B. Substituting $\frac{5}{9}(F-32)$ for C in the equation yields $K = \frac{5}{9}(F-32)+273$.

4 Linear Equations, Inequalities, & Systems in Word Problems: B. If x is the number of years since 2000, then 12 is the amount the population increases each year.

5 Linear Equations, Inequalities, & Systems in Word Problems: B. Substituting the point into the equation yields $-2 = 5(-1) + b$. This simplifies to $b = 3$.

6 Exponential & Radicals: A. One way to solve is to plug $x^2 - 3$ into each answer choice and see which results in $\sqrt{x^2-6}$. Choice A works because then $f(g(x)) = \sqrt{(x^2-3)-3} = \sqrt{x^2-6}$. Choice D is a commonly chosen incorrect answer that results from mistakenly plugging $x^2 - 3$ in for x in the expression $\sqrt{x^2-6}$.

7 Working with Polynomials: C. Distribute the negative sign to the second set of parentheses: $6x^2 + 2x - 3 - 10x^2 + 2x - 5$. Combine like terms: $6x^2 - 10x^2 = -4x^2$, for $2x + 2x = 4x$, and $-3 - 5 = -8$.

8 Quadratic Functions & Equations: A. Factoring the equation gives $(5m + 2)(5m - 2) = 0$. The solutions are $\frac{2}{5}$ and $-\frac{2}{5}$ which have a sum of 0.

9 Exponential & Radicals: A. This shows the initial population multiplied by the rate of growth plus 1 to the power of the number of years. This will show an increase of 6% in the population each year.

10 Working with Polynomial Factors in Expressions & Equations: A. Remembering the Polynomial Remainder Theorem (see page 413), we know that for any divisor $(x - a)$, when you plug a into the polynomial dividend, the result will be the remainder. Here, since we are plugging in –2, the divisor must be $(x + 2)$ and the remainder must be 6. This matches choice A.

11 More Word Problems: D. We want to solve the equation $R = \frac{T}{T+L}$ for the variable T. First, get rid of the fraction by multiplying both sides by $T + L$. The equation becomes $RT + RL = T$. Now we can subtract RT to get both T terms on the same side. The equation becomes $RL = T - RT$, which factors as $RL = T(1 - R)$. Finally, we can divide both sides by $1 - R$ to get $T = \frac{RL}{R-1}$.

12 Absolute Value: D. Absolute value can never be negative, so the value of the function could never be less than –6, –4, or 0, leaving only choice D. You could also try plugging in each answer choice. For choice A, $f(-6) = |3(-6) - 15| = |-18 - 15| = 33$, so in this case, $f(a) > a$. For choice B, $f(-4) = |-12 - 15| = 27$, so $f(a) > a$. For choice C, $f(0) = 15$, so $f(a) > a$. But for choice D, $f(6) = |18 - 15| = 3$, so $f(a) < a$.

13 Function Notation: B. Replace the variable x in the $f(x)$ equation with the function $g(x)$. We get $f(g(x)) = (3x)^2 + 1 = 9x^2 + 1$.

14 Working with Polynomials: A. Cross-multiply the original equation and we get $7a + 7b = 9a - 9b$. Gathering like terms, we get $16b = 2a$. Dividing both sides by 2 and by b, we get $\frac{a}{b} = \frac{16}{2} = 8$.

15 Imaginary & Complex Numbers: B. To rewrite in $a + bi$ form, we must rationalize the denominator. Multiply the numerator and denominator by the complex conjugate of the denominator: $\frac{(-16+4i)}{(5+3i)} \cdot \frac{(5-3i)}{(5-3i)} = \frac{-80+48i+20i-12i^2}{25-9i^2} = \frac{-80+68i-12(-1)}{25-9(-1)} = \frac{-68+68i}{34} = -2+2i$. Therefore, $a = -2$.

16 Solving Systems of Equations: 5/18 or 0.28: The two equations must be equivalent. Multiplying the first equation by 6 yields $6ax + 6by = 72$. Since both equations now equal 72, equating the coefficients gives $6a = 2$ and $6b = 5$. Therefore, $a = \frac{1}{3}$ and $b = \frac{5}{6}$. $ab = \frac{1}{3} \times \frac{5}{6} = \frac{5}{18}$.

17 Linear Equations, Inequalities, & Systems in Word Problems: 18. To maximize the amount of paint Steve can buy, we must buy the smallest amount of lumber possible, which is 1 bundle. If he buys 1 bundle of lumber for \$100, he has \$900 left for paint, so $50x = 900$, where x is equal to the number of cans of paint Steve can buy if cans of paint are \$50 each. Solving for x results in $x = 18$.

18 Working with Polynomials: 4. Factoring the equation by grouping, we get $x^2(x - 4) + 9(x - 4) = 0$ and then $(x^2 + 9)(x - 4) = 0$. Setting $x^2 + 9 = 0$ results in an imaginary value of x; the only real solution is $x = 4$.

19 Trigonometry: (3/5 or 0.6) It is worth remembering the identities $\sin(x°) = \cos(90° - x°)$ and $\cos(x°) = \sin(90° - x°)$. Therefore, the answer to this problem is the same as the given value.

20 Geometry: 57. In a circle, the degree measure of an arc is equal to the measure of the central angle that intercepts the arc. If central angle BOC is 114°, then arc BC must be 114° as well. The degree measure of an inscribed angle is half the measure of the arc it intercepts. Therefore, if arc BC is 114°, then angle BAC must be 57°.

Math – Calculator

1 Solving Linear Equations & Inequalities: B. We could try to solve for a variable, but there are two, and we don't need to anyway. Take note of the relationship between $2 + 3mx$ and $6 + 9mx$. $2 + 3mx$ is $6 + 9mx$ divided by 3. If we divide the whole inequality by 3 (which is also the greatest common factor of all the terms in the inequality), we get
$4 < 2 + 3mx$, which can be flipped to say $2 + 3mx > 4$.

2 Graphs of Functions: C. We can choose either of the given coordinate points and plug the xy-coordinates into the function. If we choose the first point, substituting -2 for x and 0 for y yields:
$0 = (-2)^3 + 2(-2)^2 + k(-2) + 2$. This gives us $0 = -8 + 8 - 2k + 2$, which simplifies to $0 = -2k + 2$, and finally $k = 1$.

3 Geometry: B. As given on the reference sheet at the beginning of each math section, the volume of each conical cup is $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)^2(6) \approx 56.55$ cubic centimeters. 1000 cubic centimeters divided by 56.55 cubic centimeters ≈ 17.68 cups. Since only full cups count, the answer is 17 cups.

4 Graphs of Functions: D. Any quadratic function q can be written in the form $q(x) = a(x - h)^2 + k$, where a, h, and k are constants and (h,k) is the vertex of the parabola when q is graphed in the coordinate plane. The vertex is at $(2,-1)$, which gives the value of h and k. This can be checked: $(x-2)^2 - 1 = x^2 - 4x + 3$.

5 Trigonometry & Radians: A. The angles of a triangle add up to 180 degrees or π radians. Therefore, $\frac{\pi}{4} + \frac{\pi}{3} + C = \pi$. Simplifying this, we get $\frac{7\pi}{12} + C = \pi$ so $C = \frac{5\pi}{12}$. Remembering that a full circle (360°) is equal to 2π radians, we can also remember that π radians is equal to 180°, in which case we can figure out that one fourth of that is 45° and one third is 60°.

6 Linear Equations, Inequalities, & Systems in Word Problems: C. If m is the number of comic books Mike owns, then Dylan owns $m + 14$ and their total is represented as $m+m+14=128$. Solving for m yields 57, so Dylan owns 71 comic books.

7 Ratios & Proportions: B. Let x be the number of expected red peas in the 210,000 sample. Set up the proportion $\frac{12}{700}=\frac{x}{210{,}000}$. Cross multiplying produces $700x=(12)(210{,}000)$. Solving for x results in $x=3{,}600$.

8 Systems of Equations: A. Replace x in the second equation with the expression that is equal to x in the first equation. The new equation is $y=(2y+4)^2+\frac{1}{2}(2y+4)-2$ which simplifies to $y=(4y^2+16y+16)+(y+2)-2$. After canceling terms, we get $0=4y^2+16y+16$. This is equivalent to $4(y+2)(y+2)=0$, so we get $y=-2$.

9 Linear Equations, Inequalities, & Systems in Word Problems: B. If $F = kx$, then plugging in values for force and distance gives us
$2{,}450 = k(1.5)$. Solving for x results in $x = 1{,}633.3$.

10 Ratios & Proportions: B. The question asks us to find the percentage change between two numbers. We can use the formula $\frac{new-old}{old}$ to determine this. The old price is \$30. To find the new price, there are two calculations to perform. A 20% decrease can be determined by multiplying 0.8 by the price; in this case, $30 \times 0.8 = 24$. From 24, the price was increased 30%, which can be determined by multiplying 1.3 by the price; in this case, $24 \times 1.3 = 31.2$. Substituting into the percentage change formula: $\frac{31.2-30}{30}=0.04$ or 4%.

11 Linear Equations, Inequalities, & Systems in Word Problems: A. There are 60 minutes in an hour, so 0.15×60 is equal to the hourly rate for the Wi-Fi. Multiplying by h to determine the total cost for a certain number of hours will yield a value for c that is equal to $0.15(60h)$.

12 Graphs of Functions: A. A parallel line would have the same slope. The above equation intercepts the y-axis at (5,0) and there's a point on this graph at (1,2). One way to find the slope is to use substitution in $y = mx + b$, where m is equal to the slope and b is equal to the y-intercept. So, $2 = m(1) + 5$ and $m = -3$. So the parallel equation would have the same slope, which rules out choice C and D. Choice A rewritten in terms of y is equal to $y = -3x + 4$ and choice B is equal to $y = 3x + 4$, so only choice A is parallel.

13 Ratios & Proportions: C. 2 hours is equal to 120 minutes. After 120 minutes, 28,800 cubic meters have been drained (found by multiplying 120 by 240), leaving 391,200 cubic meters left.

14 Graphs of Functions: B. Since both models are linear equations, the year they had the same average number of students per classroom can be determined by solving for x when $0.45x+22.5=0.55x+22.1$, which simplifies to $x = 4$. So the year both schools had the same average number of students per classroom was 2004.

15 Solving Linear Equations & Inequalities: C. If we translate the first sentence into an algebraic equation, we get
$7 + 5x = -13$, so $x = -4$. But that's not the answer, because the questions asks us to find $25 + 3x$, which is $25 + 3(-4) = 13$.

16 More Word Problems: C. Since t is measured in days, and the problem refers to one week, we should plug in $t = 7$ days. We get $S = B(0.95)^7 = 0.6983B \approx 0.70B$. This means that 70% of the bananas are fresh enough to sell, so 30% are no longer fresh enough to sell.

17 More Word Problems: B. Last year the value of S was $B(0.95)^d$. This year it is $B(0.95)^{2d}$, which is equivalent to $B(0.95)^{d+d}$, which is also $B(0.95)^d \times B(0.95)^d$. Therefore, the value from last year has been multiplied by 0.95^d.

18 Equations of Circles: A. Since we see that all the answer choices look almost the same except for the signs, we know this will be a process-of-elimination problem. Since the center of the circle is (7,1), we know the equation must start with $(x-7)^2+(y-1)^2$. If we distribute both sets of parentheses, we get $x^2-14x+49+y^2-2y+1$. Answer choice A is the only one that has the terms $-14x$ and $-2y$. We can eliminate all other answer choices and not even worry about the radius.

19 Function Notation: B. Observe that $f(x)=14x-7=7(2x-1)$ so $f(x)$ is divisible by $2x-1$. Therefore, choice A is wrong. Similarly, $g(x)=10x-5=5(2x-1)$ so $g(x)$ is divisible by $2x-1$. Since both $f(x)$ and $g(x)$ are divisible by $2x-1$, it follows that their sum and product (choices D and C) must be divisible by $2x-1$. The only remaining choice is B. This is not divisible by $2x-1$ because $f(g(x))=14(10x-5)-7=70(2x-1)-7$. Thus, $f(g(x))$ has a remainder of -7 when divided by $2x-1$.

20 Additional Data Analysis & Statistics: D. According to the table there were 82 + 33 students who passed the exam. Of those, 33 did not attend the review session. Therefore, the probability that the person chosen did *not* attend the review session is 33 out of 115.

21 Additional Data Analysis & Statistics: B. There are a total of 400 data points provided, so the median will be the average of the 200th and 201st data points. When the data points are sorted in order, there will be one hundred forty 0's, and ninety 1's. Therefore, the 200th and 201st values are 1, so the median is 1.

22 Additional Data Analysis & Statistics: B. When survey participants are selected at random from a larger population, the sample statistics calculated from the survey can be generalized to the larger population. Since 25 out of 200 students (or 1/8) surveyed at each school have 4 pets, one can estimate that this ratio holds for all students. So, to estimate the number of students with 4 pets at Pembroke school: $\frac{1}{8}\times 1{,}800=225$ and Elkwood School: $\frac{1}{8}\times 2{,}000=250$. Since $250-225=25$, the researcher can estimate that there are 25 more students at Elkwood School with 4 pets than at Pembroke school.

23 Working with Polynomials: D. $(2x+5)^3=(2x+5)^2(2x+5)=(4x^2+20x+25)(2x+5)$. This equals $8x^3+20x^2+40x^2+100x+50x+125=8x^3+60x^2+150x+125$.

24 Additional Data Analysis & Statistics: D. It should be clear that the numbers are decreasing, so we can eliminate choices A and C. The problem says that subscribership is decreasing by 25% each year. Whenever growth or decay is by a percentage, that is an exponential change, not a linear one, since a linear change occurs when the data changes by a fixed number at each interval. We can also see in the chart that the decrease in subscribership is by a smaller amount every year, not a consistent amount, so the decrease, or decay, in subscribership is exponential, not linear.

25 Linear Equations, Inequalities, & Systems in Word Problems: C. To solve, find the number of weeks after delivery when $g=q$ by solving for x, so $60+0.3x=62.25+0.15x$. Simplifying yields $0.15x=2.25$, so $x=15$. The price of granite 15 weeks after delivery is $60+0.3(15)=64.5$, so the price is \$64.50/sq. ft.

26 Interpreting Graphs & Tables: B. The substance decays by a constant amount, so that eliminates choices C and D. In the table, the weight decreases by 1.15 ounces every 5 days, so the weight will decrease 0.23 ounces each day. Therefore, the weight of the sample can be shown by a function of t as the original weight minus 0.23 multiplied by the number of days, t.

27 Linear & Exponential Growth: C. Answer A is incorrect because it results in a constant amount of annual pay. Answer B is incorrect because it results in a fixed amount of increased annual pay. Answer D is incorrect because the number of employees increases by a fixed amount each year. Answer C is correct because the amount of increase in pay depends on the amount of increase from the previous day, so it is compounding.

28 Interpreting Graphs & Tables: D. Even though the recorded temperatures are different, the data points for both thermometers go up and down by roughly the same amount.

29 Interpreting Graphs & Tables: C. One way to solve is to use the equation for the line of best: Since x is equal to the customer rating, when $x=4.5$, $y=75-10(4.5)=30$. We can also find 4.5 on the horizontal axis of the graph, follow it up to the line of best fit, then over to the vertical axis. 30 is the closest approximation.

30 Interpreting Graphs & Tables: C. Count how many data points are above the line of best fit. Choice A provides the number of data points on the line, while choice B provides the amount below and choice D the total number of data points.

31 Ratios & Proportions: 160. If $y = 128$ and $x = 16$, then we just plug those values into the equation to find k: $128 = k(16)$, which results in $k = 8$. Now that we know what the value of k is, we want to find the value of y when x is increased by 25% (which means it is increased from 16 to 20). $y = 8(20) = 160$.

32 Linear Equations, Inequalities, & Systems in Word Problems: 47. Set up an inequality to solve: $240x \leq 12{,}000 \leq 260x$, where x is equal to the number of pages. The least number of pages would require the most words per page, so $12{,}000 \leq 260x$, giving us $46.15 \leq x$. Since whole pages would be necessary to complete the document, 47 is the smallest number of pages needed.

33 More Word Problems: 9/16. The taller person's body mass index is $\frac{m}{h^2}$. The shorter person's body mass index is $\frac{m}{\left(\frac{3}{4}h\right)^2}$. Therefore, the ratio is $\frac{m}{h^2} \div \frac{m}{\left(\frac{3}{4}h\right)^2} = \frac{m}{h^2} \cdot \frac{\left(\frac{3}{4}h\right)^2}{m} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$.

34 Additional Data Analysis & Statistics: 4. If $\frac{3}{4}$ of the distance Sadie ran on Monday was in the park, and she ran 3 miles in the park Tuesday, then she ran a total of 4 miles on Monday. $(3 \div \frac{3}{4} = 4)$. If she ran 10 miles over the course of both days, then she ran 6 miles Tuesday. If 2 of those miles were on the street, then 4 were in the park.

35 Geometry: 9.9. This problem is describing a right triangle. It has given the lengths of two sides and is asking you to find the third side. Using the reference information at the beginning of each math section, we can use the formula $a^2 + b^2 = c^2$, where c is the hypotenuse, to find the missing side: $a^2 + 13^2 = 29^2$. Solving for a results in approximately 25.923. The problem is asking how many MORE miles the detour is, so we add the two legs of his detour and subtract the sum from his normal commute: $29 - (13 + 25.9) = 9.9$.

36 Ratios &Proportions: 4.5. Write the given rate as a ratio, $\frac{90 \text{ yards}}{1 \text{ min}}$, and multiply by identity fractions in order to cancel out unwanted units and replace with the units that the question is asking for: $\frac{90 \text{ yards}}{1 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{3 \text{ ft}}{1 \text{ yard}} = \frac{270 \text{ ft}}{60 \text{ sec}}$. This simplifies to $4.5 \frac{\text{ft}}{\text{sec}}$.

37 Linear & Exponential Growth: 7,639. University A grew by 3% per year for 35 years, so its 2015 student body is $10{,}000(1.03)^{35} \approx 28{,}138.6 \approx 28{,}139$. University B grew by 300 students per year for 35 years, so their 2015 student body is $10{,}000 + 300(35) = 20{,}500$. The difference is $28{,}139 - 20{,}500 = 7{,}639$.

38 Exponents & Radicals: 1296. If $y = 6$, then $36 = \sqrt{x}$. Squaring both sides of the equation yields $1{,}296 = x$.

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Lexington, KY
20 June 2016